# Averages, spread and grouped data

Leaving Certificate · Ordinary Level

## Understand the idea

### How to start

Mean is a shared-out total. Median is the middle of an ordered list. Mode is the most common value.

1. Read which measure the question asks for.
2. For the mean, add the values and divide by the number of values.
3. For the median, order the list first. Range is largest minus smallest.

### Key terms & definitions

**Mean**
The total shared equally across all observations.

**Median**
The middle of the ordered data.

**Mode**
The most frequent value.

**Range**
The largest value minus the smallest value.

### Key ideas

• The mean is $\frac{\text{sumofvalues}}{\text{numberofvalues}}$. For frequencies, use $\frac{\sum fx}{\sum f}$.

• Find the median after ordering the data. With an even number of values, average the two middle values.

• The mode is the most frequent value; data can have more than one mode or none. The range is maximum minus minimum.

• An outlier can change the mean strongly. The median is often more useful for very skewed data.

• For grouped data, use class midpoints to estimate the mean. It is an estimate because the individual values are not known.

• Quartiles split ordered data into four parts. The interquartile range is the upper quartile minus the lower quartile. State your convention if asked to calculate quartiles.

# Quick reminder

• Mean: add and divide by the number of values.

• Median: order first, then find the middle.

• Grouped mean uses midpoints and is an estimate.

# Worked examples

### Example 01 · Mean, median, mode and range

The data set is $5, 18, 6, 8, 18$. Find the mean.

#### Why this method?

For a mean, add and count. Ordering is needed for the median, but it is not needed to calculate this mean.

| | |
|---|---|
| $5 + 18 + 6 + 8 + 18 = 55$ | **Step 1**<br>Add every value, including the repeated $18$. |
| Number of values $= 5$. | **Step 2**<br>Count observations, including repeats. There are five values, not four different values. |
| $\text{mean} = \frac{55}{5}$ | **Step 3**<br>The mean shares the total equally among the five observations. |
| $\text{mean} = 11$ | **Step 4**<br>Divide $55$ by $5$. The mean lies between the smallest and largest values. |

Think about it: Before moving on: which of these four measures is not an average, and why?

### Example 02 · Reading a stem-and-leaf plot

Using the stem-and-leaf plot, what is the range of the data?

| Stem | Leaves |
|---|---|
| 1 | 117 |
| 2 | 146 |
| 3 | 33578 |

### Why this method?

Identify the measure requested. Order values for a median; add and count for a mean; use the extremes for a range.

| | |
|---|---|
| $11$ and $38$ | **Step 1** Find the smallest and largest values |
| $38 - 11 = 27$ | **Step 2** Subtract the smallest from the largest |

Think about it: Before moving on: which value is the smallest, and how do you read it from the plot?

### Example 03 · Mean from grouped data

Intervals $0 - <10, 10 - <20$ and $20 - <30$ have frequencies $2, 5$ and $3$. Estimate the mean.

### Why this method?

Identify the measure requested. Order values for a median; add and count for a mean; use the extremes for a range.

| | |
|---|---|
| $5, 15$ and $25$. | **Step 1** The individual values are unknown. Use the middle of each class as a representative value, so the final mean will be an estimate. |
| $\sum fx = 2(5) + 5(15) + 3(25) = 160$ | **Step 2** Multiply each representative value by its frequency because it stands for that many observations. |
| $\bar{x} \approx \frac{160}{10} = 16$ | **Step 3** Divide by the number of observations, not the number of classes, to share the total equally. |

| | |
|---|---|
| This is an estimate because each class is represented by its midpoint. | **Step 4**<br>State why this is an estimate: we used a midpoint to represent all the unknown individual values in each class. |

### Example 04 · Quartiles and interquartile range

Find the quartiles and IQR of $1, 2, 3, 4, 5, 6, 7, 8$.

#### Why this method?

Read the requested quantity, choose the matching rule and follow the reason beside each line.

| | |
|---|---|
| $Q_1 = \frac{2+3}{2} = 2.5$ | **Step 1**<br>The lower half is $1, 2, 3, 4$. Its median is halfway between $2$ and $3$. |
| $Q_2 = \frac{4+5}{2} = 4.5$ | **Step 2**<br>The overall median is halfway between the two middle values. |
| $Q_3 = \frac{6+7}{2} = 6.5$ | **Step 3**<br>The upper half is $5, 6, 7, 8$. Find its median. |
| $\text{IQR} = 6.5 - 2.5 = 4$ | **Step 4**<br>Subtract the lower quartile from the upper quartile. |

### Example 05 · Compare spread using the IQR

Find the interquartile range of $2, 3, 5, 7, 8, 11, 13, 19$.

| | |
|---|---|
| $Q_1 = \frac{3+5}{2} = 4$ | **Step 1**<br>Find the median of the lower four values. |
| $Q_3 = \frac{11+13}{2} = 12$ | **Step 2**<br>Find the median of the upper four values. |
| $\text{IQR} = 12 - 4 = 8$ | **Final step**<br>The IQR describes the width of the middle half. |

## Common mistake

Finding the middle value before sorting the data. In $8, 2, 5$, the median is $5$ after ordering as $2, 5, 8$; it is not the middle entry $2$ in the original list.

# Exam technique

State whether a calculated mean is exact or estimated from grouped data.

# Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

**Question 1**

The data set is $2, 5, 5, 9, 14$. Find the median.

**Question 2**

Using the same stem-and-leaf plot, how many values are $25$ or more?

| Stem | Leaves |
|---|---|
| 1 | 117 |
| 2 | 146 |
| 3 | 33578 |

# Watch an explanation

Cognito — Averages - Mean, Median, Mode & Range

Cognito — Grouped Frequency Tables

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

# Self-check answers

### Question 1

5

### Question 2

6

Open the full chapter online