Slopes, tangents and turning points

Leaving Certificate · Ordinary Level

Understand the idea

How to start

A derivative is a rule for the slope. Put an x-value into it to find the slope at that point.

1. Find the derivative before substituting any x-value.
2. For a slope, substitute x. For a stationary point, set the derivative equal to zero.
3. Use the original function for y and check maximum or minimum separately.

Key terms & definitions

Stationary point
A point where the first derivative is zero.

Tangent
A line having the same slope as the curve at the point of contact.

Key ideas

• A stationary point has $f'(x) = 0$. Solve this equation for x, then use the original function to find y.

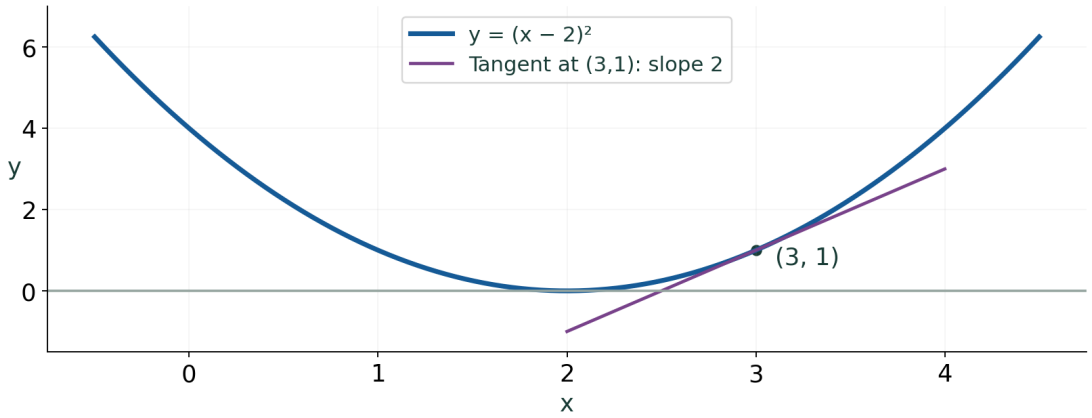
• At a stationary point, $f''(x) > 0$ gives a minimum and $f''(x) < 0$ gives a maximum. A zero second derivative needs another check.

• A tangent through (x_1, y_1) with slope m has equation $y - y_1 = m(x - x_1)$.

• A positive first derivative means the function is increasing locally; a negative one means decreasing.

• In a context, use the allowed domain and compare endpoints where necessary.

A derivative gives the tangent slope



For $y = (x - 2)^2$ the derivative is $2x - 4$. At $x = 3$ the slope is 2, matching the drawn tangent $y = 2x - 5$.

Quick reminder

• A stationary point has $f'(x) = 0$. Solve this equation for x, then use the original function to find y.

• At a stationary point, $f''(x) > 0$ gives a minimum and $f''(x) < 0$ gives a maximum. A zero second derivative needs another check.

Worked examples

Example 01 · Start here: what a slope value means

The distance of a trolley is modelled by $s(t) = t^2$, in metres, with t in seconds. Find its instantaneous speed at $t = 3$.

Why this method?

The derivative of distance with respect to time is speed in this increasing-distance model. Find the derivative first, then put in the time.

$s'(t) = 2t$	**Step 1** Differentiate t squared.
$s'(3) = 2(3) = 6$	**Step 2** Only now replace t with 3.
Speed $= 6$ m/s	**Step 3** State what the number means and include units.

Example 02 · Slope at one point

Find the slope of $y = x^2$ at $x = 3$.

Why this method?

The derivative gives slope; the original function gives the height of a point.

$\frac{dy}{dx} = 2x$	**Step 1** Differentiate the function first.
$m = 2(3) = 6$	**Step 2** Put $x = 3$ into the derivative. The slope is 6.

Example 03 · Find a minimum

Find the minimum point of $f(x) = x^2 - 4x + 7$.

Why this method?

The derivative gives slope; the original function gives the height of a point.

$f'(x) = 2x - 4$	**Step 1** Differentiate.
$2x - 4 = 0$	**Step 2** The slope at the minimum is zero.
$x = 2$	**Step 3** Add 4 and divide by 2.
$f(2) = 4 - 8 + 7 = 3$	**Step 4** Use the original function to find the height.
$f''(x) = 2 > 0$	**Step 5** The positive second derivative confirms a minimum at $(2, 3)$.

Example 04 · Equation of a tangent

Find the tangent to $y = x^2$ at $(2, 4)$.

Why this method?

The derivative gives slope; the original function gives the height of a point.

$m = 2(2) = 4$	**Step 1** The derivative $2x$ gives slope 4.

$y - 4 = 4(x - 2)$	**Step 2** Use the given point and slope.
$y = 4x - 4$	**Step 3** Expand and add 4 to both sides.

Example 05 · Find a maximum step by step

Find the maximum point of $f(x) = -x^2 + 6x + 1$.

Why this method?

At a turning point the tangent is horizontal, so its slope is zero. Solve $f'(x) = 0$, then find the height using the original function.

$f'(x) = -2x + 6$	**Step 1** Differentiate each term.
$-2x + 6 = 0$	**Step 2** Set the first derivative equal to zero.
$-2x = -6$	**Step 3** Subtract 6 from both sides.
$x = 3$	**Step 4** Divide both sides by -2.
$f(3) = -(3)^2 + 6(3) + 1 = -9 + 18 + 1 = 10$	**Step 5** Use the original function to find the y-coordinate.
$f''(x) = -2 < 0$	**Step 6** The negative second derivative confirms a maximum.
$\text{Maximum point} = (3, 10)$	**Step 7** A point needs both coordinates.

Example 06 · Show a stationary point is a minimum

Find and classify the stationary point of $f(x) = x^2 - 6x + 5$.

$f'(x) = 2x - 6$	**Step 1** Differentiate to get the slope.
$2x - 6 = 0 \quad \Rightarrow \quad x = 3$	**Step 2** A stationary point has zero slope.

$f''(3) = 2 > 0$	**Step 3** The curve bends upwards.
$f(3) = -4$	**Final step** The minimum point is $(3, -4)$.

Common mistake

Using the derivative to find a point's y-coordinate. For $f(x) = x^2 + 1$ at $x = 2$, the slope is $f'(2) = 4$, but the height is $f(2) = 5$. Use the original function for y.

Exam technique

State both coordinates when asked for a turning point.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Find the slope of $x^2 + 2x$ at $x = 1$.

Question 2

Find the minimum point of $x^2 - 6x + 10$.

Question 3

Find the slope of $y = x^2 + 1$ at $x = 2$.

Watch an explanation

Corbettmaths — Stationary points: maximum and minimum

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

4

Question 2

$(3, 1)$

Question 3

$\frac{dy}{dx} = 2x$, so the slope is 4.

Differentiate first, then substitute 2.

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