Differentiation: the first steps

Leaving Certificate · Ordinary Level

Understand the idea

How to start

Differentiation finds how quickly a quantity changes. On a graph it gives the slope at a point.

1. Identify the power on each x term.
2. Multiply by that power, then reduce the power by 1.
3. A constant becomes zero. Combine the resulting terms.

Key terms & definitions

Derivative
A rate of change; the slope of a tangent on a graph.

Second derivative
The derivative of the first derivative.

Key ideas

• For a power, multiply by the old power and reduce the power by 1 : $\frac{d}{dx}(x^n) = nx^{n-1}$.

• Differentiate each term separately. Keep its coefficient.

• A constant differentiates to zero because it does not change.

• $f'(x)$ and $\frac{dy}{dx}$ are ways of writing the derivative.

• Differentiate again to find the second derivative.

• For this Ordinary Level course, practise linear, quadratic and cubic polynomial functions by rule.

Quick reminder

• For a power, multiply by the old power and reduce the power by 1 : $\frac{d}{dx}(x^n) = nx^{n-1}$.

• Differentiate each term separately. Keep its coefficient.

Worked examples

Example 01 · Start here: a straight line and a constant

Differentiate $y = 4x + 7$.

Why this method?

A straight line has a constant slope. Each increase of 1 in x increases $4x + 7$ by 4; adding 7 does not change the slope.

$\frac{d}{dx}(4x) = 4$	**Step 1** The derivative of x is 1, so $4x$ differentiates to 4.
$\frac{d}{dx}(7) = 0$	**Step 2** The constant 7 does not change with x.
$\frac{dy}{dx} = 4 + 0 = 4$	**Step 3** Combine the two derivatives.

Example 02 · Differentiate x squared

Find the derivative of $y = x^2$.

Why this method?

Differentiate one term at a time using its power; a constant contributes zero.

$\frac{dy}{dx} = 2x^{2-1}$	**Step 1** Bring the power 2 in front. Reduce the power by 1.
$\frac{dy}{dx} = 2x$	**Step 2** A power of 1 is normally left unwritten.

Example 03 · Keep the coefficient

Differentiate $y = 3x^2 + 4x - 5$.

Why this method?

Differentiate one term at a time using its power; a constant contributes zero.

$\frac{dy}{dx} = 3(2)x + 4 - 0$	**Step 1** The derivative of x^2 is $2x$. The derivative of $4x$ is 4. The constant disappears.

$\frac{dy}{dx} = 6x + 4$	**Step 2** Multiply 3 by 2.

Example 04 · **A cubic and a second derivative**

Find both derivatives of $f(x) = 2x^3 - 3x^2 + 5$.

Why this method?

Differentiate one term at a time using its power; a constant contributes zero.

$f'(x) = 2(3)x^2 - 3(2)x$	**Step 1** Differentiate each non-constant term.
$f'(x) = 6x^2 - 6x$	**Step 2** Simplify the coefficients.
$f''(x) = 12x - 6$	**Step 3** Differentiate the first derivative once more.

Example 05 · **First and second derivatives**

Find both derivatives of $f(x) = 2x^3 - 3x^2 + 4$.

$f'(x) = 6x^2 - 6x$	**Step 1** Multiply by each power and reduce the power by one. The constant disappears.
$f''(x) = 12x - 6$	**Final step** Differentiate the first derivative again.

Common mistake

Reducing the power without multiplying by the original power. The derivative of x^3 is $3x^2$, not x^2.

Exam technique

Write the derivative before putting in a particular x-value.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Differentiate $y = 4x^2 + 3x + 7$.

Question 2

Find the second derivative of x^3.

Question 3

Differentiate $y = 5x + 2$.

Watch an explanation

Corbettmaths — Differentiation: using the power rule

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

$8x + 3$

Question 2

$6x$

Question 3

$\frac{dy}{dx} = 5.$

The constant 2 differentiates to zero.

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