Circle angles and cyclic quadrilaterals

Leaving Certificate · Ordinary Level

Understand the idea

How to start

A radius joins the centre to the circle. A diameter passes through the centre and has length two radii.

1. Mark the centre, diameter or chord named in the question.
2. Check which arc or chord the angles stand on before choosing a fact.
3. Write the angle relationship and explain why it applies to these angles.

Key terms & definitions

Chord
A segment joining two points on a circle.

Arc
A part of the circumference.

Cyclic quadrilateral
A quadrilateral with all four vertices on one circle.

Key ideas

• The angle at the centre is twice the angle at the circumference standing on the same arc.

• Angles standing on the same chord and in the same segment are equal. The side-of-chord condition matters.

• An angle in a semicircle is a right angle. Conversely, a chord subtending a right angle at the circumference is a diameter.

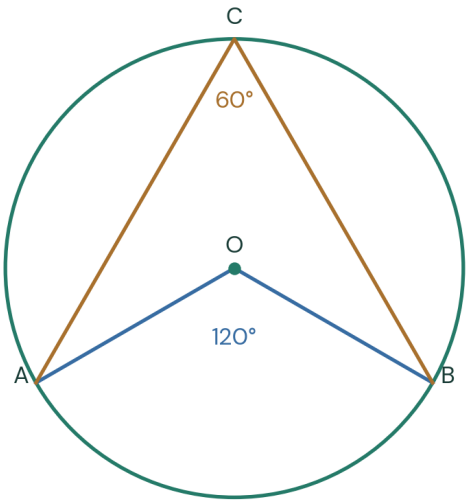
• A cyclic quadrilateral has all four vertices on a circle. Its opposite angles total 180°.

• Explain a circle angle by naming the relevant arc or diameter; do not assume a line through a sketch is a diameter unless given.

• A tangent is perpendicular to the radius at the point of contact.

• The angle between a tangent and a chord equals the angle in the alternate segment.

• The perpendicular from the centre to a chord bisects the chord.



On the same lower arc AB, angle AOB at the centre is 120° and angle ACB at the circumference is 60°.

Quick reminder

• Use the same arc when comparing centre and circumference angles.

• An angle in a semicircle is 90°.

• Opposite angles in a cyclic quadrilateral total 180°.

Worked examples

Example 01 · Centre and circumference

The angle at the centre standing on an arc is 124°. Find the angle at the circumference on the same arc.

Why this method?

Identify the arc, chord or diameter named in the question. Use the circle result whose conditions match that information.

$2\theta = 124^\circ$	**Step 1** Let θ be the angle at the circumference. The angle at the centre standing on the same arc is twice θ.
$\theta = \frac{124^\circ}{2}$	**Step 2** Divide both sides by 2 because we want one θ.

$\theta = 62^\circ$	**Step 3** The circumference angle is half the given centre angle.
$2(62^\circ) = 124^\circ$	**Step 4** Check the angle relationship.

Example 02 · Cyclic quadrilateral

ABCD is cyclic. Angle A is 108° and angle B is 76°. Find C and D.

Why this method?

Identify the arc, chord or diameter named in the question. Use the circle result whose conditions match that information.

$C = 180^\circ - 108^\circ = 72^\circ$	**Step 1** Opposite angles are supplementary
$D = 180^\circ - 76^\circ = 104^\circ$	**Step 2** Use the other opposite pair
$108 + 76 + 72 + 104 = 360$	**Step 3** Add the four interior angles. A quadrilateral should have a total of 360°.

Example 03 · An angle in a semicircle

AB is a diameter and C is on the circle. Angle CAB is 34°. Find angle ABC.

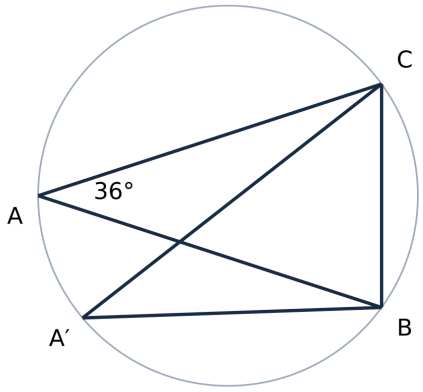
Why this method?

Identify the arc, chord or diameter named in the question. Use the circle result whose conditions match that information.

$\angle ACB = 90^\circ.$	**Step 1** The stated diameter makes the angle at the circumference a right angle. We can then use the triangle angle sum.
$\angle ABC = 180^\circ - 90^\circ - 34^\circ = 56^\circ$	**Step 2** Use the triangle angle sum

Example 04 · Corollaries 1, 2 and 5 — another example

A, A□, B and C lie on a circle. A and A□ are on the same side of line BC. Angle BAC $= 36^\circ$. Find angle BA□C.



A and A′ are on the same side of BC.

Angles standing on the same arc. Diagram not to scale.

Why this method?

Identify the arc, chord or diameter named in the question. Use the circle result whose conditions match that information.

A and A□ are on the same side of BC; all four points are distinct.	**Step 1** Check the same-arc condition
angle BAC = angle BA□C.	**Step 2** Apply corollary 2
angle BA□$C = 36^\circ$.	**Step 3** Substitute

Example 05 · Find arc length and sector area

$A72^\circ$ sector has radius 10 cm. Find its arc length and area.

$\frac{72}{360} = \frac{1}{5}$	**Step 1** The sector is one fifth of the full circle.
$L = \frac{1}{5}(2\pi \cdot 10) = 4\pi$ cm	**Step 2** Take one fifth of the circumference.

$A = \frac{1}{5}(\pi \cdot 10^2) = 20\pi \text{ cm}^2$	**Final step** Take one fifth of the area.

Common mistake

Adding adjacent angles of a cyclic quadrilateral to get 180°. It is the opposite angles that add to 180°: if one angle is 110°, its opposite angle is 70°.

Exam technique

Draw the radii to see which central angle stands on the same arc.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

The angle at the centre standing on an arc is 136°. Find the angle at the circumference on the same arc.

Question 2

ABCD is cyclic. Angle A is 112° and angle B is 82°. Find C and D.

Self-check answers

Question 1

$136^\circ \div 2 = 68^\circ$.

The centre angle is twice the circumference angle on the same arc.

Question 2

$C = 68^\circ$ and $D = 98^\circ$.

Subtract each given angle from 180°. Pair opposite angles: A with C, and B with D.

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