Combined events, trees and tables

Leaving Certificate · Ordinary Level

Understand the idea

How to start

For two events, keep track of what happens first and what happens second.

1. List the complete paths that give the event you want.
2. For independent events, multiply the probabilities along each path.
3. Add different successful paths when they cannot happen together.

Key terms & definitions

Independent events
Knowing one outcome does not change the probability of the other.

Mutually exclusive
Events that cannot happen together.

Tree diagram
A branching display of successive outcomes and their probabilities.

Key ideas

• The counting principle multiplies the number of choices at successive stages when each stage has the stated choices.

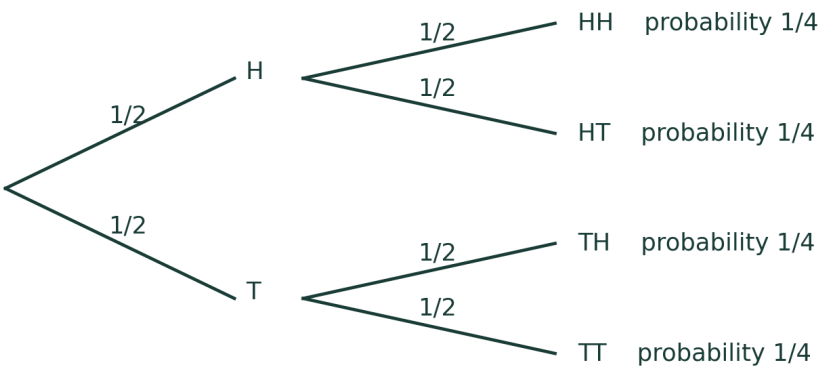
• For independent events, multiply probabilities along a tree branch.

• For mutually exclusive complete paths to the required event, add the path probabilities.

• A two-way table shows overlaps. Use the correct total for the group named in the question.

• Ordinary Level focuses on independent events, up to three Bernoulli trials and the first success on a specified trial. The dependent-event examples are optional extension.

Two fair coin tosses: four equally likely paths



Two independent fair coin tosses give HH, HT, TH and TT. Each complete path has probability one half times one half, or one quarter.

Quick reminder

- Multiply along an independent sequence of events.
- Add mutually exclusive successful paths.
- Use the total of the group actually selected.

Worked examples

Example 01 · Start here: count choices in stages

A café offers 3 starters and 4 main courses. How many different starter-and-main meals can be chosen?

Why this method?

Each starter can be paired with any of the 4 mains. Count 4 meals for each of the 3 starters.

$4 + 4 + 4 = 12$	**Step 1** Three groups, each containing four possible meals.
$3 \times 4 = 12$	**Step 2** Multiplication counts the same choices more quickly. Order the choices consistently: starter, then main.

Example 02 · Two independent spins

A spinner has 10 equal-sized sections, with 3 red sections. The probability of red is 0.3. It is spun twice independently. Find the probability of exactly one red.

Blue follows red then not red. Purple follows not red then red. Add the two complete paths for exactly one red.

Why this method?

List the possible paths or groups first. Decide when we are multiplying along a path and when we are adding different successful paths.

$P(\text{notred}) = 1 - 0.3 = 0.7$	**Step 1** Red and not red cover all possible outcomes of one spin.
Successful paths: red then not red, or not red then red.	**Step 2** Exactly one red means one red spin and one non-red spin. Both orders count.
$P(\text{redthennotred}) = 0.3 \times 0.7 = 0.21$	**Step 3** The spins are independent, so multiply along this path.
$P(\text{notredthenred}) = 0.7 \times 0.3 = 0.21$	**Step 4** Calculate the other successful order.
$P(\text{exactlyonered}) = 0.21 + 0.21 = 0.42$	**Step 5** Add these paths because the same pair of spins cannot follow both orders.

Example 03 · Two dice

Two fair six-sided dice are rolled. Find the probability of a total of 8.

Why this method?

List the possible paths or groups first. Decide when we are multiplying along a path and when we are adding different successful paths.

$6 \times 6 = 36$	**Step 1** Count equally likely ordered outcomes
$(2,6), (3,5), (4,4), (5,3), (6,2).$	**Step 2** List favourable pairs
$P(8) = \frac{5}{36}$	**Step 3** Form the probability

Example 04 · A two-way table

Of 24 students , 14 cycle. Of those 14, 6 wear glasses. If a cyclist is selected, what is the probability they wear glasses?

Why this method?

List the possible paths or groups first. Decide when we are multiplying along a path and when we are adding different successful paths.

Choose from the 14 cyclists.	**Step 1** Only cyclists can be selected here, so use 14 as the total, not 24.
$P(\text{glassesamongcyclists}) = \frac{6}{14}$	**Step 2** Six of the fourteen cyclists wear glasses.
$\frac{6}{14} = \frac{3}{7}$	**Step 3** Divide numerator and denominator by 2. The denominator still represents the cyclist group.

Example 05 · First success on trial 3

A trial succeeds with probability 0.4, independently each time. Find the probability of first success on trial 3.

Why this method?

Identify whether each stage changes the available outcomes before multiplying probabilities.

$P(\text{failure}) = 1 - 0.4 = 0.6$	**Step 1** The first two trials must fail.
$P = 0.6(0.6)(0.4)$	**Step 2** The required path is failure, failure, success.
$P = 0.144$	**Step 3** Multiply along this independent path.

Example 06 · Exactly two successes in three trials

Each trial succeeds independently with probability 0.4. Find the chance of exactly two successes.

SSF, SFS, FSS	**Step 1** List the three different orders.
$0.4^2(0.6) = 0.096$	**Step 2** Multiply along one order.

$3(0.096) = 0.288$	**Final step** Add the three equally probable orders.

Example 07 · Arrange, rather than choose

How many ways can 8 runners fill gold, silver and bronze?

$8 \times 7 \times 6 = 336$	**Final step** Order matters, and a runner cannot occupy a second place.

Optional extension · not required for Ordinary Level

Example 08 · Without replacement: the second probability changes

A bag contains 3 red counters and 2 blue counters. Two counters are taken without replacement. Find the probability that both are red.

Why this method?

Follow the red-then-red path. The first red counter is kept out, so the second draw has fewer counters and fewer reds.

$P(\text{first red}) = \frac{3}{5}$	**Step 1** At the start, 3 of the 5 counters are red.
$P(\text{second red} \mid \text{first red}) = \frac{2}{4}$	**Step 2** After taking one red, 2 red counters remain among 4 counters.
$P(\text{both red}) = \frac{3}{5} \times \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$	**Step 3** Multiply along this complete path. Do not reuse $\frac{3}{5}$ for the second draw.

Common mistake

Confusing independent with mutually exclusive. Two coin tosses are independent. Getting heads and tails on the same single toss is mutually exclusive: both cannot happen on that toss.

Exam technique

Check that the probabilities leaving each tree point add to one.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

A spinner has probability 0.2 of red. It is spun twice independently. Find the probability of exactly one red.

Question 2

Two fair six-sided dice are rolled. Find the probability of a total of 9.

Question 3 · Build confidence

Choose one of 2 soups and one of 5 sandwiches. How many meals?

Question 4 · Optional extension

A bag contains 4 red and 2 blue counters. Two are taken without replacement. Find the probability of two reds.

Before you move on

Try these without looking at a worked example. Revisit any part that is still difficult.

- Count successive choices systematically.
- Change the second probability when an item is not replaced.
- Multiply probabilities along one complete path.

Watch an explanation

Cognito — AND / OR Rules

Cognito — Tree Diagrams

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

$0.2(0.8) + 0.8(0.2) = 0.32$.

There are two disjoint paths: red then not red, or not red then red. Multiply along each path and add the results.

Question 2

$\frac{4}{36} = \frac{1}{9}$.

The ordered pairs are $(3, 6), (4, 5), (5, 4)$ and $(6, 3)$. Count outcomes out of 36, not out of the possible totals.

Question 3

$2 \times 5 = 10$.

Each of the two soups can be paired with all five sandwiches.

Question 4

$\frac{4}{6} \times \frac{3}{5} = \frac{2}{5}$.

After the first red, there are 3 reds among 5 remaining counters.

Open the full chapter online