Complex numbers and Argand diagrams

Leaving Certificate · Ordinary Level

Understand the idea

How to start

A complex number has a real part and an imaginary part. In $3 + 2i$, these are 3 and 2.

1. Identify the real part and the coefficient of i.
2. Combine matching parts; when multiplying, replace i squared with -1.
3. For division, multiply top and bottom by the denominator's conjugate.

Key terms & definitions

Real part
The number without i.

Conjugate
The same real part with the opposite imaginary part.

Modulus
Distance from the origin on an Argand diagram.

Key ideas

• $i^2 = -1$. This is the rule used when multiplying imaginary parts.

• For $z = a + bi$, the real part is a and the imaginary part is b. For example, $\text{Im}(5 - 3i) = -3$.

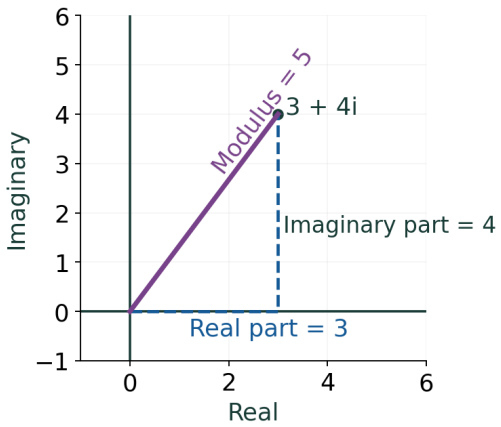
• Add real parts to real parts and imaginary parts to imaginary parts.

• On an Argand diagram, real is horizontal and imaginary is vertical.

• The conjugate of $a + bi$ is $a - bi$. Only the imaginary sign changes.

• The modulus is distance from the origin: $|a + bi| = \sqrt{a^2 + b^2}$.

• For division, multiply the numerator and denominator by the conjugate of the denominator.



Plot $3+4i$ at $(3,4)$ on real and imaginary axes. Its distance from the origin is the modulus: square root of $3^2+4^2=5$.

Quick reminder

- $i^2=-1$. This is the rule used when multiplying imaginary parts.
- Add real parts to real parts and imaginary parts to imaginary parts.

Worked examples

Example 01 · Start here: subtract matching parts

Calculate $(5+4i)-(2+i)$.

Why this method?

Remove the second bracket carefully, then subtract real parts and imaginary parts separately.

$5+4i-2-i$	**Step 1** The minus before the second bracket applies to both terms.
$(5-2)+(4-1)i$	**Step 2** Keep the real and imaginary parts in separate groups.
$3+3i$	**Step 3** State the real part first.

Example 02 · Add two complex numbers

Calculate $(2 + 3i) + (4 + i)$.

Why this method?

Work with real and imaginary parts separately. Use i squared $= -1$ whenever it appears.

$2 + 4 + 3i + i$	**Step 1** Put real terms together and imaginary terms together.
$6 + 4i$	**Step 2** Three i plus one i is four i.

Example 03 · Multiply and use i squared

Calculate $(1 + 2i)(3 + i)$.

Why this method?

Work with real and imaginary parts separately. Use i squared $= -1$ whenever it appears.

$3 + i + 6i + 2i^2$	**Step 1** Multiply every term in the first bracket by every term in the second.
$3 + 7i + 2(-1)$	**Step 2** Replace i^2 by -1.
$1 + 7i$	**Step 3** Collect the real numbers.

Example 04 · Conjugate and modulus

Find the conjugate and modulus of $3 + 4i$.

Why this method?

Work with real and imaginary parts separately. Use i squared $= -1$ whenever it appears.

$\overline{z} = 3 - 4i$	**Step 1** Change the sign of the imaginary part.
$\lvert z\rvert = \sqrt{3^2 + 4^2}$	**Step 2** Use distance from the origin.
$\lvert z\rvert = \sqrt{9 + 16} = 5$	**Step 3** Square, add, then take the positive root.

Example 05 · Divide using a conjugate

Calculate $\frac{3+i}{1+i}$.

Why this method?

Work with real and imaginary parts separately. Use i squared $=-1$ whenever it appears.

$\frac{3+i}{1+i} \times \frac{1-i}{1-i}$	**Step 1** Multiply top and bottom by $1-i$. This whole multiplier equals 1.
$\frac{3-3i+i-i^2}{1-i^2}$	**Step 2** Multiply both brackets on top and on the bottom.
$\frac{4-2i}{2}=2-i$	**Step 3** Use $i^2=-1$, then divide both terms by 2.

Example 06 · Divide in rectangular form

Write $\frac{4+i}{1-2i}$ in the form $a+bi$.

$\frac{(4+i)(1+2i)}{(1-2i)(1+2i)}$	**Step 1** Multiply numerator and denominator by the denominator's conjugate.
$\frac{2+9i}{5}$	**Step 2** Expand, then replace i^2 by -1.
$\frac{2}{5}+\frac{9}{5}i$	**Final step** Divide both parts by 5.

Common mistake

Including i when stating the imaginary part. For $z=3+2i$, the imaginary part is 2, not $2i$. The coefficient of i is the imaginary part.

Exam technique

Write the real part first in a final rectangular-form answer.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Find the conjugate of $2 - 5i$.

Question 2

Find $|6 + 8i|$.

Question 3

Calculate $(4 + 3i) - (1 + i)$.

Question 4 · Build confidence

For $z = 5 - 3i$, state the real part and imaginary part.

Question 5 · Apply or explain

Calculate $(2 + i)(3 - i)$.

Before you move on

Try these without looking at a worked example. Revisit any part that is still difficult.

- Identify the real part and the coefficient of i.
- Use i squared $= -1$ when multiplying.
- Find the conjugate and use it in division.

Self-check answers

Question 1

$2 + 5i$

Question 2

10

Question 3

$3 + 2i$.

Subtract each corresponding part.

Question 4

$\mathrm{Re}(z) = 5$ and $\mathrm{Im}(z) = -3$.

The imaginary part is the real coefficient of i, not $-3i$.

Question 5

$6 - 2i + 3i - i^2 = 7 + i$.

Replace i squared by -1, so $-i$ squared contributes $+1$.

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