Equations of circles

Leaving Certificate · Ordinary Level

Understand the idea

How to start

A circle consists of points at a fixed distance from its centre. That fixed distance is the radius.

1. Compare the equation with the standard circle form.
2. Read the centre carefully; a bracket $(x - a)$ uses centre coordinate a.
3. Take the positive square root of the right-hand number to find the radius.

Key terms & definitions

Centre
The fixed point in the middle of the circle.

Radius
Distance from the centre to the circle.

Key ideas

• Centre (h, k) and radius r give $(x - h)^2 + (y - k)^2 = r^2$.

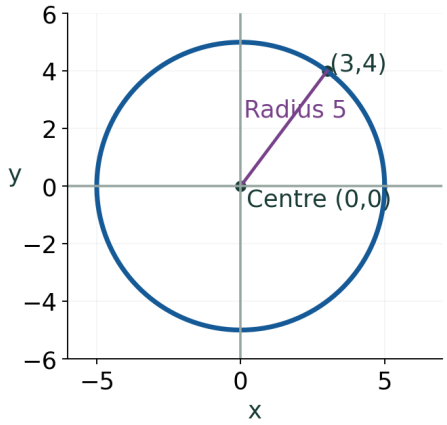
• The signs inside the brackets are opposite to the centre coordinates.

• A circle centred at the origin has equation $x^2 + y^2 = r^2$.

• To test a point, substitute both coordinates. It lies on the circle only if the equation is true.

• To find line–circle intersections, substitute the line relationship into the circle and solve.

A circle: every point is one radius from the centre



The circle $x^2 + y^2 = 25$ has centre $(0, 0)$ and radius 5. The point $(3, 4)$ lies on it because $3^2 + 4^2 = 25$.

Quick reminder

• Centre (h, k) and radius r give $(x - h)^2 + (y - k)^2 = r^2$.

• The signs inside the brackets are opposite to the centre coordinates.

Worked examples

Example 01 · Start here: read a radius from the equation

Find the centre and radius of $x^2 + y^2 = 36$.

Why this method?

Compare with $x^2 + y^2 = r^2$. The number on the right is the radius squared, not the radius.

$r^2 = 36$	**Step 1** The centre is $(0, 0)$ because neither x nor y has a shift inside a bracket.
$r = \sqrt{36} = 6$	**Step 2** Take the positive square root.
Centre $= (0, 0)$, $\quad r = 6$	**Step 3** Give both pieces of information.

Example 02 · Circle at the origin

Write the equation of a circle with centre $(0, 0)$ and radius 5.

Why this method?

Compare the equation with the standard circle form before calculating.

$x^2 + y^2 = 5^2$	**Step 1** The centre is the origin, so no shifts are needed.
$x^2 + y^2 = 25$	**Step 2** Square the radius.

Example 03 · Read centre and radius

Find the centre and radius of $(x - 2)^2 + (y + 3)^2 = 16$.

Why this method?

Compare the equation with the standard circle form before calculating.

$h = 2, \quad k = -3$	**Step 1** Compare with $x - h$ and $y - k$. A plus 3 means k is -3.
$r = \sqrt{16} = 4$	**Step 2** The right side is radius squared, so take a square root.

Example 04 · A line meets a circle

Find where $y = 3$ meets $x^2 + y^2 = 25$.

Why this method?

Compare the equation with the standard circle form before calculating.

$x^2 + 3^2 = 25$	**Step 1** Substitute $y = 3$ into the circle.
$x^2 = 16$	**Step 2** Subtract 9 from both sides.
$x = 4 \quad \text{or} \quad x = -4$	**Step 3** Both signs square to 16.

$(4, 3), \quad (-4, 3)$	**Step 4** Use $y = 3$ for both points.

Example 05 · Find the points where a line crosses a circle

Find the intersections of $x = 3$ with $x^2 + y^2 = 25$.

$3^2 + y^2 = 25$	**Step 1** Put the stated x value into the circle equation.
$y^2 = 16$	**Step 2** Subtract 9.
$(3, 4)$ and $(3, -4)$	**Final step** Both signs work; write both coordinate pairs.

Common mistake

Reading the number on the right as the radius. In $x^2 + y^2 = 25$, 25 is r^2, so the radius is $\sqrt{25} = 5$, not 25.

Exam technique

Check each intersection point in both original equations.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Write the circle with centre $(1, 2)$, radius 3.

Question 2

Find the centre and radius of $(x - 2)^2 + (y + 1)^2 = 9$.

Question 3

Does $(0, 5)$ lie on $x^2 + y^2 = 25$?

Self-check answers

Question 1

$(x-1)^2+(y-2)^2=9$

Question 2

Centre $(2,\ -1)$ and radius 3.

The signs inside the brackets are opposite to the centre coordinates.

Question 3

Yes: $0^2+5^2=25$.

Substitute both coordinates into the equation.

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