Coordinate geometry

Leaving Certificate · Ordinary Level

Understand the idea

How to start

In $(3, 2)$, move 3 units horizontally first and 2 units vertically second.

1. Write the coordinates of both points clearly.
2. For midpoint, average matching coordinates. For slope or distance, find the changes between the points.
3. Keep the same point order in both subtractions.

Key terms & definitions

Coordinate
A number locating a point relative to axes.

Midpoint
The point halfway between two endpoints.

Slope
Vertical change divided by horizontal change.

Key ideas

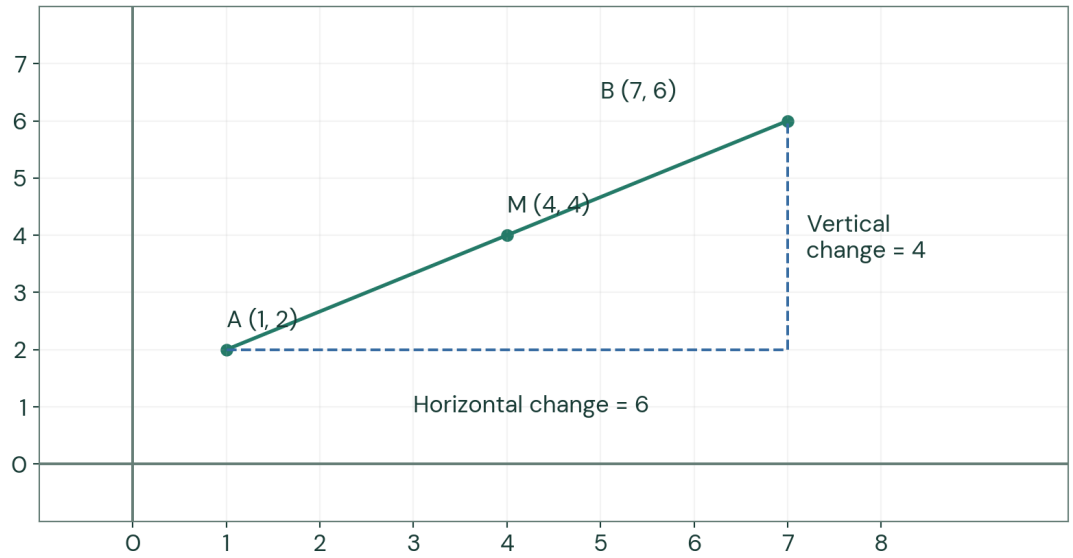
• An ordered pair (x, y) gives horizontal movement first and vertical movement second.

• The midpoint is $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$.

• Distance is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, from Pythagoras.

• Slope is $m = \frac{y_2 - y_1}{x_2 - x_1}$ when the denominator is non-zero.

• A horizontal line has slope zero. A vertical line has undefined slope and equation $x = k$.



From A to B: slope is $\frac{4}{6} = \frac{2}{3}$; the midpoint averages the coordinates to give $(4, 4)$.

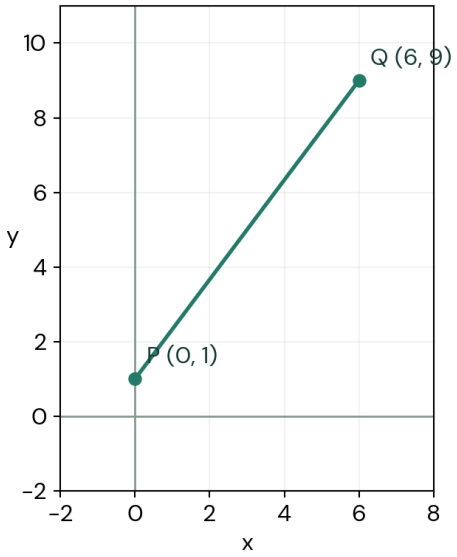
Quick reminder

- Keep the same point order in each subtraction.
- Midpoint: average the x-values and the y-values.
- Slope: vertical change ÷ horizontal change.

Worked examples

Example 01 · Distance between two points

$P(0, 1)$ and $Q(6, 9)$ are two points. Find the distance between them.



Coordinates matching this question : $P(0, 1), Q(6, 9)$.

Blue follows the horizontal change. Purple follows the vertical change. They become the two shorter sides used in Pythagoras.

Why this method?

Write the coordinates in the same point order. Decide whether we need a distance, a midpoint or a slope before substituting.

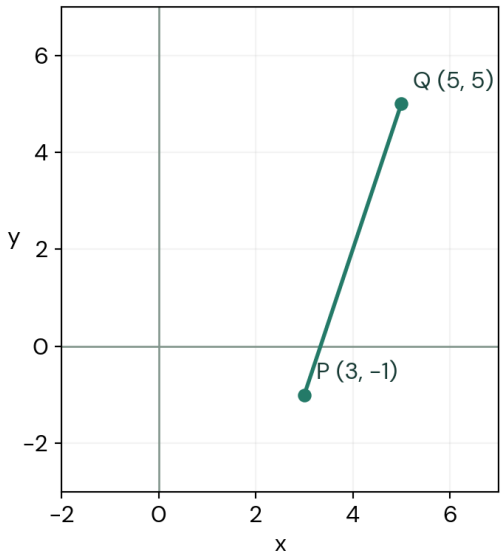
$6 - 0 = 6$	**Step 1** Find the horizontal change from P to Q. The x-coordinate changes from 0 to 6.
$9 - 1 = 8$	**Step 2** Find the vertical change in the same point order. The y-coordinate changes from 1 to 9.
$d^2 = 6^2 + 8^2$	**Step 3** These changes are the perpendicular sides of a right triangle. The straight-line distance is its hypotenuse.
$d^2 = 36 + 64 = 100$	**Step 4** Square each change and add.

$d = \sqrt{100} = 10$	**Step 5** Take the positive square root because a distance cannot be negative.

Think about it: Before moving on: why do the horizontal and vertical distances not simply add up to the distance?

Example 02 · Midpoint of a line segment

$P(3, -1)$ and $Q(5, 5)$ are two points. What is the x-coordinate of the midpoint of PQ?



Coordinates matching this question : $P(3, -1), Q(5, 5)$.

Why this method?

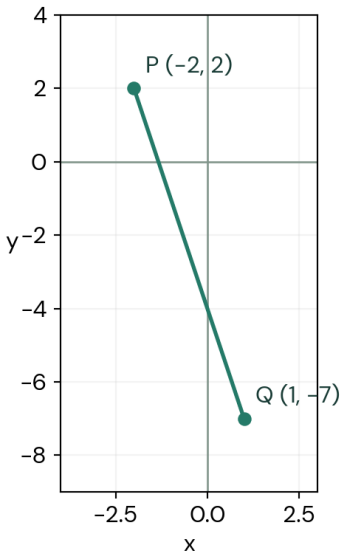
Write the coordinates in the same point order. Decide whether we need a distance, a midpoint or a slope before substituting.

$x_M = \frac{x_1 + x_2}{2}$	**Step 1** The midpoint x-coordinate is halfway between the two x-coordinates.
$x_M = \frac{3 + 5}{2}$	**Step 2** Use 3 and 5, the x-coordinates of P and Q. The whole sum stays above the fraction bar.
$x_M = \frac{8}{2} = 4$	**Step 3** Add first, then divide by 2.
The x-coordinate is 4.	**Step 4** It lies halfway between 3 and 5. The question does not ask for the y-coordinate.

Think about it: Before moving on: why must the sum of the coordinates be halved?

Example 03 · Slope of a line through two points

$P(-2, 2)$ and $Q(1, -7)$ lie on a straight line. What is its slope?



Coordinates matching this question : $P(-2, 2), Q(1, -7)$.

Why this method?

Write the coordinates in the same point order. Decide whether we need a distance, a midpoint or a slope before substituting.

$(x_1, y_1) = (-2, 2)$ and $(x_2, y_2) = (1, -7)$.	**Step 1** Label the first point and second point. Keep the same order for both subtractions.
$m = \frac{-7-2}{1-(-2)}$	**Step 2** Slope is vertical change divided by horizontal change. Put the whole differences above and below the fraction bar.
$m = \frac{-9}{3}$	**Step 3** Subtracting -2 is adding 2, so the horizontal change is $+3$.
$m = -3$	**Step 4** Divide. A negative slope means the line falls as x increases.

Think about it: Before moving on: why does a falling line have a negative slope?

Example 04 · Area from three coordinate points

Find the area of the triangle with vertices $(1, 2), (7, 2)$ and $(4, 6)$.

Base $= 7 - 1 = 6$	**Step 1** The first two points have the same y-coordinate.
Height $= 6 - 2 = 4$	**Step 2** Use perpendicular distance above the horizontal base.
$A = \frac{1}{2}(6)(4) = 12$ square units	**Final step** Use half base times height.

Common mistake

Reversing the point order in only half of the slope formula. For $(1, 2)$ and $(3, 6)$, use $\frac{6-2}{3-1} = 2$, not $\frac{6-2}{1-3} = -2$. Keep the same point order in both differences.

Exam technique

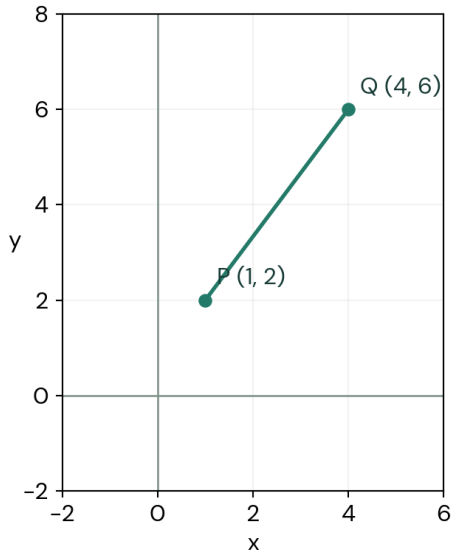
Sketch the two points to check the signs and approximate distance.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

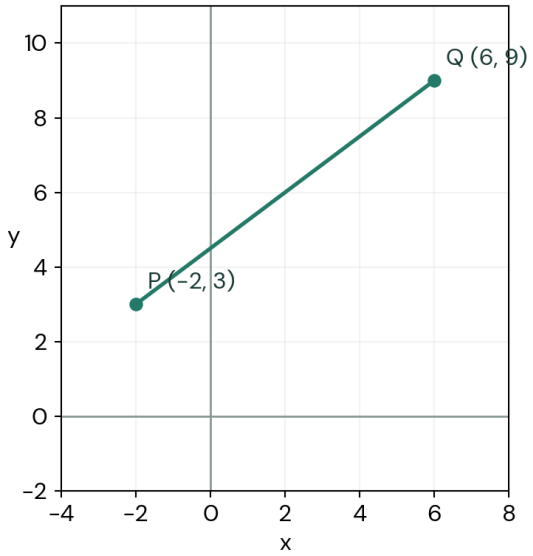
$P(1, 2)$ and $Q(4, 6)$ are two points. Find the distance between them.



Coordinates matching this question : $P(1, 2), Q(4, 6)$.

Question 2

$P(-2, 3)$ and $Q(6, 9)$ are two points. What is the x-coordinate of the midpoint of PQ?



Coordinates matching this question : $P(-2, 3), Q(6, 9)$.

Watch an explanation

Corbettmaths — Linear graphs: length of a line

Cognito — Finding the Gradient Using Coordinates

Cognito — Finding the Midpoint of a Line

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

5

Question 2

2

Open the full chapter online