Changing the subject of a formula

Leaving Certificate · Ordinary Level

Understand the idea

How to start

The subject is the letter on its own. Making t the subject means finishing with $t = \ldots$.

1. Circle the letter you need to isolate.
2. Undo operations in reverse order, doing the same to both sides.
3. Keep a whole sum in brackets or above the same fraction bar when dividing it.

Key terms & definitions

Subject
The variable written alone on one side of a formula.

Rearrange
Write an equivalent equation with a different structure.

Restriction
A condition needed for a division or other operation to be valid.

Key ideas

• The subject is the variable isolated on one side of a formula.

• Use inverse operations on both sides. Keep letters as quantities rather than trying to guess values.

• Clear denominators by multiplying every term. Use brackets when dividing a whole sum.

• If the required variable occurs more than once, collect those terms and factor it out.

• Check restrictions before dividing by an expression that could be zero.

Quick reminder

• Identify the variable that must be the subject.

• Undo addition / subtraction, then multiplication / division as needed.

• Keep a whole numerator together with a fraction bar or brackets.

Worked examples

Example 01 · Start here: undo one multiplication

Make w the subject of $A = lw$, where $l \neq 0$.

Why this method?

The letter w is multiplied by l. Divide both sides by l to leave w on its own.

$\frac{A}{l} = \frac{lw}{l}$	**Step 1** Divide the whole left side and the whole right side by l.
$w = \frac{A}{l}$	**Step 2** Write w on the left. We have not put numbers into the formula.

Example 02 · Start here: undo two operations

Make t the subject of $v = u + at$. Assume $a \neq 0$.

Why this method?

Work backwards through the operations around t: undo adding u, then undo multiplying by a.

$v - u = at$	**Step 1** Our goal is t on its own. Subtract u from both sides to remove the added term.
$\frac{v-u}{a} = t$	**Step 2** The remaining t is multiplied by a. Divide the whole left-hand side and the right-hand side by a.
$t = \frac{v-u}{a}$	**Step 3** Write the subject on the left if preferred. The numerator must stay together.

Example 03 · Keep the whole difference above the bar

Make x the subject of $y = 3x + 4$.

Why this method?

The last operation on x is adding 4, so undo that before dividing by 3.

$y - 4 = 3x$	**Step 1** Subtract 4 from both sides.
$\frac{y-4}{3} = x$	**Step 2** Divide the whole difference $y - 4$ by 3.
$x = \frac{y-4}{3}$	**Step 3** Keep both terms in the numerator. The 3 divides all of $y - 4$.

Example 04 · A formula with a square

Make r the subject of $A = \pi r^2$, where r is a non-negative radius.

Why this method?

First undo multiplication by π. Then undo the square by taking the positive square root because a radius cannot be negative.

$\frac{A}{\pi} = r^2$	**Step 1** Divide both sides by π.
$r = \sqrt{\frac{A}{\pi}}$	**Step 2** The square root applies to the whole fraction. Assume A is non-negative.

Example 05 · Make a denominator the subject

Make t the subject of $v = \frac{d}{t}$, with positive quantities.

$vt = d$	**Step 1** Multiply both sides by t.
$t = \frac{d}{v}$	**Final step** Divide both sides by the nonzero speed v.

Common mistake

Dividing only one part of a sum when changing the subject. From $y = 3x + 4$, subtract 4 first, then divide the whole difference: $x = \frac{y-4}{3}$, not $y - \frac{4}{3}$.

Exam technique

Check the rearranged formula using a simple numerical example.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

The formula is $y = a + bx$. Find x when $y = 31$, $a = 7$ and $b = 4$.

Question 2

The formula is $A = \frac{h(a+b)}{2}$. Find h when $A = 30$, $a = 4$ and $b = 8$.

Question 3

Make h the subject of $V = Ah$, with $A \neq 0$.

Question 4

Make x the subject of $y = 2x - 5$.

Before you move on

Try these without looking at a worked example. Revisit any part that is still difficult.

- Identify the letter that must be on its own.
- Undo operations in reverse order.
- Keep a whole sum or difference above the fraction bar.

Watch an explanation

Cognito — Rearranging Formulas (Subject Appears Once)

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

6

Question 2

5

Question 3

$h = \frac{V}{A}$.

Divide both sides by the area A.

Question 4

$x = \frac{y + 5}{2}$.

Add 5 to both sides, then divide the entire sum by 2.

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