Functions and graphs

Leaving Certificate · Ordinary Level

Understand the idea

How to start

$f(3)$ means the output of the rule f when the input is 3. It does not mean f multiplied by 3.

1. Decide whether you are finding an input, an output or a point on a graph.
2. For an output, replace x by the input and calculate.
3. On a graph, read the horizontal coordinate first, then the vertical coordinate.

Key terms & definitions

Domain
The set of allowed inputs.

Range
The set of outputs actually produced.

Intercept
A point at which a graph meets an axis.

Turning point
A point at which a curve changes from increasing to decreasing, or vice versa.

Key ideas

• A function assigns exactly one output to each input in its domain. $f(x)$ means the output for input x.

• The domain contains allowed inputs; the codomain is the stated set of possible outputs, and the range contains outputs actually reached.

• Make a table of values, label axes and choose consistent scales before plotting.

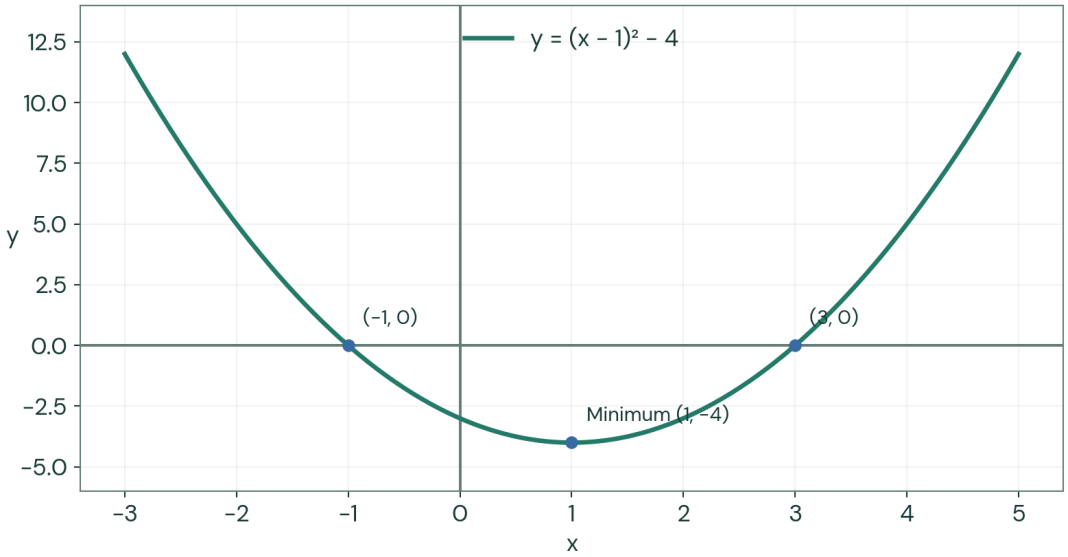
• A root is an x-value where the graph crosses or touches the x-axis. The y-intercept occurs when x is zero.

• An intersection solves $f(x) = g(x)$. A maximum or minimum describes an extreme output within the given domain.

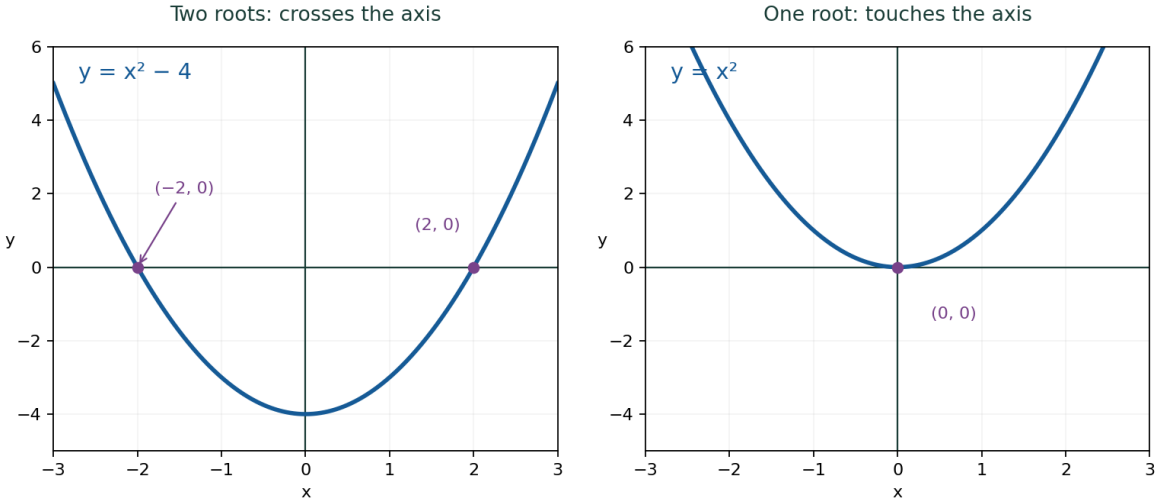
• For contextual graphs, interpret coordinates with units. A model may only make sense for non-negative or whole-number inputs.

• A composite function uses one output as the next input. $(f \circ g)(x) = f(g(x))$: do g first.

• For an informal limit, investigate the outputs as x approaches a value from both sides. The approached value need not equal the value of the function at that point.



The quadratic has roots -1 and 3 and a minimum at $(1, -4)$.



A root is an x-value for which y is zero. Crossing is not required: a graph can touch the x-axis at a root. The two panels use the same scales.

Quick reminder

• Input x first, output y second.

• Roots: where $y = 0$. Y-intercept: where $x = 0$.

• An intersection has the same coordinates on both graphs.

Worked examples

Example 01 · Evaluating a function

The function f is given by $f(x) = 3x + 4$. What is $f(3)$?

Purple follows the input 3 into the rule, then the multiplication result 9. The coefficient 3 stays in place.

Why this method?

Decide whether the input or the output is known. For graphs, label the axes and read the relevant coordinate in the right order.

$f(x) = 3x + 4$	**Step 1** The rule says multiply the input by 3, then add 4.
$f\ (3)\ = 3\ (3)\ + 4$	**Step 2** Replace x by the input 3. The 3 outside the brackets is the coefficient.
$f(3) = 9 + 4$	**Step 3** Multiply before adding.
$f(3) = 13$	**Step 4** Thirteen is the output when the input is 3.

Think about it: Before moving on: what does $f(3)$ mean, and how is it different from $3 \times f$?

Example 02 · Finding the input for a given output

The function f is given by $f(x) = 8x + 7$. For what value of x is $f(x) = 87$?

Why this method?

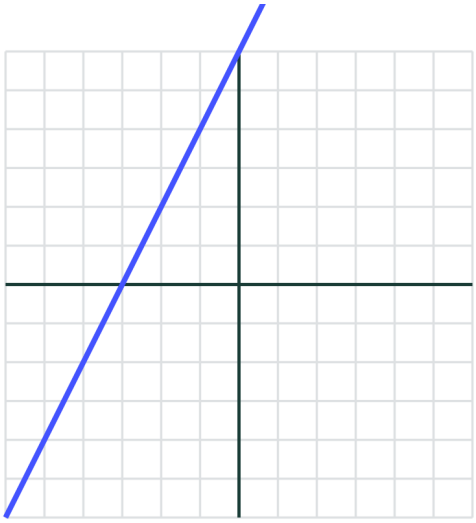
Decide whether the input or the output is known. For graphs, label the axes and read the relevant coordinate in the right order.

$8x + 7 = 87$	**Step 1** Write the equation
$8x = 87 - 7 = 80$	**Step 2** Take the constant away from both sides
$x = 80 \div 8 = 10$	**Step 3** Divide every term on both sides by the coefficient so that the coefficient of the variable becomes 1.

Think about it: Before moving on: how can you check your answer for x by evaluating the function?

Example 03 · Reading the y-intercept from a graph

Graph 1 : The straight line in the graph crosses the y-axis. What is the y-value where it crosses?



Straight line rising from left to right on axes marked from -6 to 6 on each side. The line is drawn across the whole grid.

Why this method?

Decide whether the input or the output is known. For graphs, label the axes and read the relevant coordinate in the right order.

The line crosses the y-axis at $y = 6$

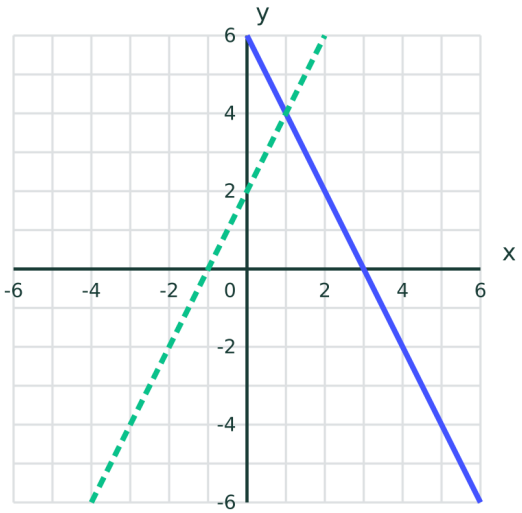
Step 1

The y-intercept is where the line crosses the y-axis, at $x = 0$

Think about it: Before moving on: which point on the graph gives the y-intercept, and why is it the only one you need?

Example 04 · Reading where two graphs cross

Graph 1 : The two straight lines in the graph cross at one point. What is the x-value of the point where they cross?



f: solid blue g: dashed green

Two straight lines drawn on axes marked from -6 to 6 on each side. The lines cross at one point.

Why this method?

Decide whether the input or the output is known. For graphs, label the axes and read the relevant coordinate in the right order.

The lines cross at $x = 1$	**Step 1** The lines cross where their y-values are equal. Read the x-value of that point from the horizontal axis

Think about it: Before moving on: why is the x-value where the lines cross the solution, and not the y-value?

Example 05 · Two functions in succession

$g(x) = x + 1$ and $f(x) = 2x$. Find $f(g(3))$.

Why this method?

Substitute carefully into the stated rule and keep brackets around a substituted expression.

$g(3) = 3 + 1 = 4$	**Step 1** Do the inside function first.
$f(4) = 2(4) = 8$	**Step 2** Put its output 4 into f.

Example 06 · A simple exponential table

Find values of $f(x) = 2^x$ at $x = 0, 1, 2$.

Why this method?

Substitute carefully into the stated rule and keep brackets around a substituted expression.

$f(0) = 2^0 = 1$	**Step 1** A non-zero number to the power zero is 1.
$f(1) = 2$	**Step 2** A power of 1 leaves the base unchanged.
$f(2) = 2(2) = 4$	**Step 3** Multiply two factors of 2. Plot $(0, 1), (1, 2), (2, 4)$.

Example 07 · A limit from nearby inputs

Investigate $f(x) = x + 1$ as x approaches 2.

Why this method?

Substitute carefully into the stated rule and keep brackets around a substituted expression.

$f(1.9) = 2.9, \quad f(1.99) = 2.99$	**Step 1** Approach 2 using slightly smaller inputs.
$f(2.1) = 3.1, \quad f(2.01) = 3.01$	**Step 2** Approach from larger inputs.
$\lim_{x \to 2}(x+1)=3$	**Step 3** The outputs approach 3 from both sides.

Example 08 · Build a table for a quadratic graph

Make a table for $y = x^2 - 2$ using $x = -2, -1, 0, 1, 2$.

x	-2	-1	0	1	2
y	2	-1	-2	-1	2

Why this method?

Choose an input, replace x, square it and then subtract 2. Repeat the same procedure for every input.

$x = -2: \quad y = (-2)^2 - 2 = 4 - 2 = 2$	**Step 1** Keep the negative input in brackets before squaring.
$x = -1: \quad y = (-1)^2 - 2 = 1 - 2 = -1$	**Step 2** The square of negative 1 is positive 1.
$x = 0: \quad y = 0^2 - 2 = -2$	**Step 3** This gives the point on the y-axis.
$x = 1: \quad y = 1^2 - 2 = -1$	**Step 4** The inputs 1 and -1 give the same output.
$x = 2: \quad y = 2^2 - 2 = 2$	**Step 5** The inputs 2 and -2 also give the same output.
$(-2, 2), (-1, -1), (0, -2), (1, -1), (2, 2)$	**Step 6** Plot x first and y second. Draw a smooth curve through the points, rather than straight segments.

Example 09 · A graph can touch the axis at a root

Compare the roots of $y = x^2 - 4$ and $y = x^2$.

Why this method?

A root is an input where the output is zero. The graph may cross the x-axis or just touch it.

$x^2 - 4 = 0 \quad \Rightarrow \quad x^2 = 4$	**Step 1** For the first graph, set y equal to zero.
$x = -2 \quad \text{or} \quad x = 2$	**Step 2** Both positive 2 and negative 2 square to 4. This graph crosses the axis twice.

$x^2 = 0 \quad \Rightarrow \quad x = 0$	**Step 3** For the second graph, zero is the only root. This graph touches the axis at $(0, 0)$.

Example 10 · Read a cost graph in context

A taxi fare is modelled by $C = 5 + 2d$, where C is in euro and d is in kilometres. Interpret the intercept and slope.

Why this method?

Read what each axis measures. The intercept gives the cost at distance zero; the slope gives the increase in cost for each kilometre.

$d = 0: \quad C = 5 + 2(0) = 5$	**Step 1** The vertical intercept is $(0, 5)$. The fixed charge is $€5$.
$\frac{\text{change in cost}}{\text{change in distance}} = \frac{2}{1} = 2$	**Step 2** Each extra kilometre adds $€2$. The slope is $€2$ per km.
$d = 3: \quad C = 5 + 2(3) = 11$	**Step 3** At 3 km the model predicts a fare of $€11$.

Example 11 · Compose in the right order

If $f(x) = 2x + 1$ and $g(x) = x^2$, find $g(f(3))$.

$f(3) = 2(3) + 1 = 7$	**Step 1** Work from the inside out.
$g(7) = 7^2 = 49$	**Final step** The first output is the next input.

Example 12 · Read an approaching value

For $f(x) = \frac{x^2 - 4}{x - 2}$, what does f(x) approach as x approaches 2?

$f(x) = x + 2, \quad x \neq 2$	**Step 1** Factor the numerator and cancel only away from 2.
The values approach 4.	**Final step** The limit is about nearby values; $f(2)$ itself is undefined.

Example 13 · Plot a cubic using a table

Sketch the graph of $f(x) = x^3 - 4x$ near its roots.

$f(-3) = -15,\ f(-1) = 3,\ f(1) = -3,\ f(3) = 15$	**Step 1** Substitute selected negative and positive inputs.
$f(x) = x(x-2)(x+2)$	**Step 2** Factor to locate the crossings exactly.
Plot the values and join with a smooth curve.	**Final step** The curve crosses the axis at $-2, 0$ and 2. Use closer inputs if a turning point is hard to see.

Common mistake

Swapping input and output coordinates. The point $(2, 5)$ means input 2 gives output 5. It does not mean that input 5 gives output 2.

Exam technique

Read the scale before estimating a crossing and label a graphical estimate as approximate.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

The function f is given by $f(x) = 6x + 11$. What is $f(7)$?

Question 2

The function f is given by $f(x) = 5x + 3$. For what value of x is $f(x) = 38$?

Question 3 · Build confidence

For $y = x^2 - 2$, find y when $x = -3$.

Question 4 · Apply or explain

Find the root of $y = (x-2)^2$. Does the graph cross or touch the x-axis?

Question 5 · Apply or explain

The cost model is $C = 8 + 3d$, in euro, for distance d in kilometres. Interpret both numbers.

Before you move on

Try these without looking at a worked example. Revisit any part that is still difficult.

- Make a table by substituting each input.
- Plot coordinates in the order (x,y).
- Find roots where the graph crosses or touches the axis.
- Interpret slope and intercept using the axis units.

Watch an explanation

Corbettmaths — Function Machines

Cognito — What are Quadratic Graphs

Tulla Maths — Calculator: make a table of values

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

53

Question 2

7

Question 3

$y = (-3)^2 - 2 = 7.$

Square the negative input in brackets.

Question 4

The root is $x = 2$. The graph touches the axis at $(2, 0)$.

A square is never negative, so the graph stays on or above the axis.

Question 5

€8 fixed charge and €3 for each kilometre.

The fixed charge remains when d is zero; 3 is the cost increase per kilometre.

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