Indices and scientific notation

Leaving Certificate · Ordinary Level

Understand the idea

How to start

A power counts repeated multiplication: $x^3 = x \times x \times x$.

1. Identify the base: the number or expression being multiplied repeatedly.
2. Choose the rule for the operation shown. Adding powers and multiplying powers are different.
3. Keep brackets around a negative base, because they change what the power applies to.

Key terms & definitions

Base
The number or expression being raised to a power.

Index
The exponent indicating a power.

Scientific notation
A number written as a coefficient between 1 and 10 multiplied by an integer power of 10.

Key ideas

• For the same non-zero base, $a^m a^n = a^{m+n}$ and $\frac{a^m}{a^n} = a^{m-n}$. Also $(a^m)^n = a^{mn}$.

• A zero power is 1 for a non-zero base. A negative index means reciprocal: $a^{-n} = \frac{1}{a^n}$.

• For positive real bases, $a^{p/q} = \sqrt[q]{a^p}$. Check the real domain before evaluating roots of negative numbers.

• An even root of a negative number is not real. An odd root can be negative. Zero cannot have a negative power.

• Scientific notation is $a \times 10^n$ with $1 \le a < 10$ for positive values and integer n. Small positive numbers use negative powers.

Quick reminder

- $a^m a^n = a^{m+n}$: multiply same bases → add indices.
- $\frac{a^m}{a^n} = a^{m-n}$: divide same non-zero bases → subtract.
- $(a^m)^n = a^{mn}$: power of a power → multiply indices.

Worked examples

Example 01 · Start here: multiply powers of the same base

Simplify $x^3 \times x^2$.

Why this method?

The bases are both x and the operation is multiplication. Count the factors to see why their indices are added.

$(x \times x \times x)(x \times x)$	**Step 1** Write out what each power means. There are three x factors and then two more.
x^5	**Step 2** Count the five factors. Multiplication keeps them all in the product.
$x^{3+2} = x^5$	**Step 3** This is why we add the indices for multiplication with the same base.

Example 02 · Scientific notation calculation

Calculate $(3 \times 10^{-4})(2 \times 10^7)$ in scientific notation.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

$3 \times 2 = 6$	**Step 1** Multiply coefficients
$10^{-4} \times 10^7 = 10^3$	**Step 2** Add indices of the common base

6×10^3	**Step 3** Put the coefficient and the power of ten together, then check that the coefficient is between 1 and 10.

Example 03 · Powers with negative and zero integer bases

Evaluate $(-6)^9$. The minus sign is inside the brackets.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

9 factors, each equal to -6.	**Step 1** Read the brackets before multiplying
Result : -10077696.	**Step 2** An odd number of negative factors gives a negative product. Calculate 6 to the ninth power, then apply that sign.

Think about it: How does a bracket change whether a minus sign is raised to the power?

Example 04 · Powers with negative and zero integer bases — another example

Evaluate -7^8. The exponent is applied before the leading minus sign.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

First evaluate 7^8, then negate it.	**Step 1** Read the brackets before multiplying
Result : -5764801.	**Step 2** The power applies only to 7. Calculate 7 to the eighth power, then apply the leading negative sign.

Example 05 · Negative indices

Evaluate 3^{-3}. Give your answer as a fraction.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

$3^{-3} = 1 \div 3^3$	**Step 1** A negative index means the reciprocal of the positive power
$3^3 = 27$, so the answer is $\frac{1}{27}$	**Step 2** Work out the power

Think about it: Before moving on: why does a negative index give a fraction, and not a negative number?

Example 06 · Negative indices — another example

Evaluate 5^{-3}. Give your answer as a fraction.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

$5^{-3} = 1 \div 5^3$	**Step 1** A negative index means the reciprocal of the positive power
$5^3 = 125$, so the answer is $\frac{1}{125}$	**Step 2** Work out the power

Example 07 · Combine a fractional index and a negative index

Calculate $27^{-\frac{2}{3}}$.

$27^{\frac{1}{3}} = 3$	**Step 1** The denominator 3 means cube root.
$27^{\frac{2}{3}} = 3^2 = 9$	**Step 2** The numerator 2 means square.
$27^{-\frac{2}{3}} = \frac{1}{9}$	**Final step** The minus sign in the index means reciprocal.

Example 08 · Multiply in scientific notation

Calculate $(3 \times 10^5)(4 \times 10^{-2})$ in standard form.

12×10^3	**Step 1** Multiply coefficients and add indices.
1.2×10^4	**Final step** Move the decimal point to make the first factor lie from 1 up to, but not including 10.

Common mistake

Adding powers when adding terms. $x^2 + x^3$ is not x^5: these are unlike terms. For multiplication with the same base, $x^2 \times x^3 = x^5$.

Exam technique

Use brackets to distinguish $(-3)^2$ from -3^2.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Evaluate -7^3. The exponent is applied before the leading minus sign.

A. -343

B. 343

C. -4

D. -21

Question 2

Evaluate 2^{-5}. Give your answer as a fraction.

A. $\frac{1}{32}$

B. 32

C. -32

D. $\frac{1}{10}$

Watch an explanation

Cognito — Multiplying & Dividing Powers

Cognito — Negative Powers

Cognito — Fractional Powers

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

-343

Correct.

Question 2

$\frac{1}{32}$

Correct : $2^{-5} = 1 \div 2^{5} = \frac{1}{32}$.

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