Inequalities and solution sets

Leaving Certificate · Ordinary Level

Understand the idea

How to start

The sign $<$ means less than. The sign $\leq$ means less than or equal to.

1. Read whether the endpoint is included.
2. Use the same operations on both sides. Reverse the sign if you multiply or divide by a negative number.
3. Check whether the answer needs all real numbers or only whole numbers.

Key terms & definitions

Strict inequality
A comparison using $<$ or $>$, excluding equality.

Interval
A continuous set of numbers between boundaries.

Boundary
An endpoint at which the truth of an inequality can change.

Key ideas

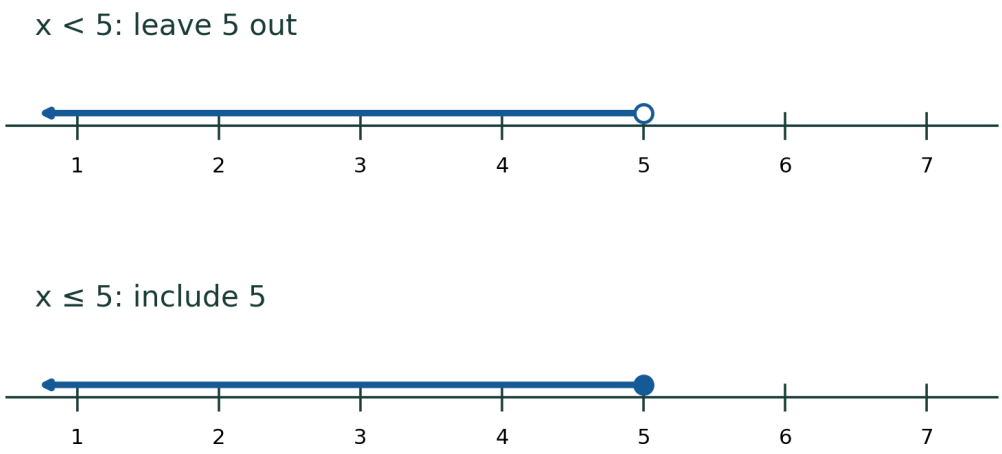
• Solve an inequality much like an equation, but reverse its sign when multiplying or dividing by a negative number.

• Use an open dot for a strict endpoint and a filled dot when the endpoint is included.

• State the domain. Integer solutions and real solutions are different sets.

• On a graph, $f(x) > g(x)$ where the graph of f is above the graph of g.

• For a strict inequality, crossings and touching points are excluded. For an inclusive inequality, include points where the graphs agree.



The arrow continues towards smaller numbers. An open endpoint excludes 5; a filled endpoint includes it. These sketches show part of the real number line.

Quick reminder

• Reverse the inequality when multiplying or dividing by a negative.

• Open endpoint: equality excluded. Filled endpoint: equality included.

• Check whether the domain is integers or real numbers.

Worked examples

Example 01 · Start here: one change to both sides

Solve $x + 3 < 8$.

Why this method?

Subtracting 3 undoes the addition. Subtract the same amount from both sides; subtraction does not reverse the inequality.

$x + 3 - 3 < 8 - 3$	**Step 1** Subtract 3 from both sides.
$x < 5$	**Step 2** Every real number below 5 works. The endpoint 5 is excluded.
$4 + 3 = 7 < 8$	**Step 3** Check an allowed value. At $x = 5$, the left side is 8, so the strict inequality would fail.

Example 02 · Linear inequalities: the largest whole solution

What is the largest whole number n that satisfies $7n + 1 < 85$?

Purple marks the same subtraction on both sides. Subtracting 1 leaves the inequality sign unchanged.

Why this method?

Isolate the variable or compare the heights of the graphs. Keep track of whether equality is included at the boundary.

$7n + 1 < 85$	**Step 1** We want n on its own. The sign means strictly less than.
$7n + 1 \color{#7c3aed}{-1} < 85 \color{#7c3aed}{-1}$	**Step 2** Subtract 1 from both sides.
$7n < 84$	**Step 3** Simplify the subtraction.
$n < 12$	**Step 4** Divide both sides by positive 7. The inequality sign stays the same.
$n = 11$	**Step 5** The question asks for the largest whole number below 12. Twelve itself is excluded.
$7(11) + 1 = 78 < 85$	**Step 6** Check 11. The next whole number gives $7(12) + 1 = 85$, which is not less than 85.

Think about it: Before moving on: why is the bound itself not a solution when the inequality is strict?

Example 03 · Divide by a positive number

Solve $2x + 1 \leq 9$.

Why this method?

Undo adding 1 first, then undo multiplying by 2. Dividing by positive 2 keeps the sign pointing the same way.

$2x \leq 8$	**Step 1** Subtract 1 from both sides.
$\frac{2x}{2} \leq \frac{8}{2}$	**Step 2** Divide both sides by positive 2.

$x \leq 4$	**Step 3** Include 4 because the original sign allows equality.

Example 04 · Why a negative divisor reverses the sign

Solve $-2x < 6$.

Why this method?

Division by a negative reverses the order of numbers. For example, $2 < 3$ becomes $-2 > -3$ after multiplication by -1.

$\frac{-2x}{-2} > \frac{6}{-2}$	**Step 1** Divide both sides by -2 and reverse $<$ to $>$.
$x > -3$	**Step 2** Every number greater than -3 works.
$-2(0) = 0 < 6$	**Step 3** Check $x = 0$. The boundary $x = -3$ gives $6 < 6$, which is false.

Example 05 · Reverse the inequality when dividing by a negative

Solve $7 - 3x \geq 1$.

$-3x \geq -6$	**Step 1** Subtract 7 from both sides.
$x \leq 2$	**Step 2** Divide by -3 and reverse the inequality.
Filled dot at 2; shade left.	**Final step** 2 is included, and all smaller real numbers work.

Common mistake

Forgetting to reverse the sign when multiplying or dividing both sides by a negative number. For example, $-2x < 6$ gives $x > -3$, not $x < -3$. Adding or subtracting does not reverse the sign.

Exam technique

Test one point in each region when reading a graphical inequality.

Try it yourself

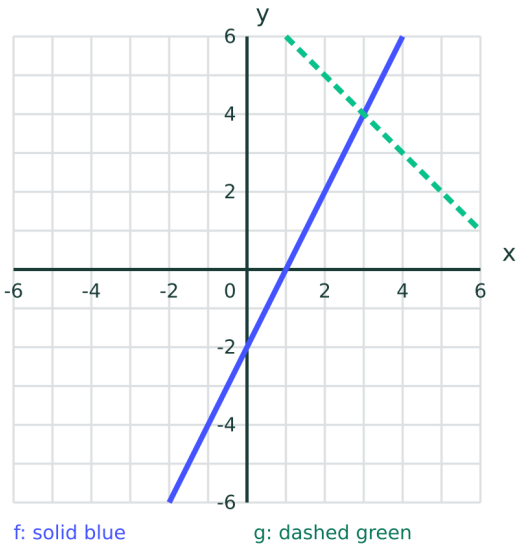
Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

What is the largest whole number n that satisfies $4n + 3 < 35$?

Question 2

The blue line is f(x) and the green line is g(x). Using the graph, find the solution set of $f(x) < g(x)$.



f: solid blue g: dashed green

Two straight lines drawn on axes marked from -6 to 6 on each side. The lines cross at one point.

A. $x < 3$

B. $y < 4$

C. $x \leq 3$

D. $x > 3$

Question 3

Solve $3x < 12$.

Question 4

Solve $-x \leq 5$.

Watch an explanation

Cognito — Algebra with Inequalities 1

Cognito — Introduction to Inequalities

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

7

Question 2

$x < 3$

Correct: the lines cross at $x = 3$, and the blue line is below the green line for $x < 3$.

Question 3

$x < 4$.

Divide both sides by positive 3; do not reverse the sign.

Question 4

$x \geq -5$.

Divide by -1 and reverse the inequality.

Open the full chapter online