Number sets and signed numbers

Leaving Certificate · Ordinary Level

Understand the idea

How to start

A negative number is below zero. On a number line, numbers get larger as you move right.

1. Read the operation sign separately from the sign of each number.
2. For addition, move right for a positive number and left for a negative number.
3. For multiplication or division, decide the sign first, then calculate the size.

Key terms & definitions

Integer
A whole number, positive, negative or zero.

Reciprocal
The number that multiplies a non-zero number to give 1.

Absolute value
Distance from zero; it is never negative.

Key ideas

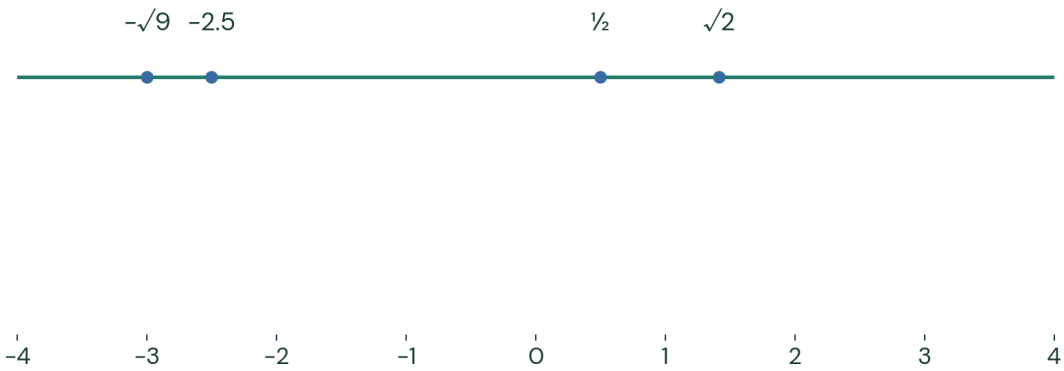
• Natural numbers count whole objects; integers also include negative whole numbers and zero. State whether your convention includes zero in $\mathbb{N}$.

• Rational numbers can be written as $\frac{a}{b}$ with integers a, b and $b \neq 0$. Irrational numbers, such as $\sqrt{2}$ and π, cannot.

• Real numbers include rational and irrational numbers. Place them on a number line to compare their size.

• For mixed operations, do brackets first, then powers, then multiplication and division from left to right, then addition and subtraction from left to right.

• Subtracting a negative is adding its opposite. For multiplication and division, equal signs give a positive result and different signs give a negative result.



Numbers increase from left to right. Negative numbers are to the left of zero.

Quick reminder

• Check the sign before calculating.

• Use brackets around negative numbers in powers.

• Example: $(-3)^2 = 9$, but $-3^2 = -9$.

Worked examples

Example 01 · Adding and subtracting signed numbers

Work out $(-6) + 11$.

Why this method?

Use the number line and the sign of each number to decide what the calculation means before calculating.

$-6 + 6 = 0$	**Step 1** Start at -6. The first 6 steps to the right bring you to zero.
$11 - 6 = 5$	**Step 2** Of the 11 steps, 5 are still left.
$-6 + 11 = 5$	**Step 3** Move the remaining 5 steps right from zero. The answer is positive.

Think about it: Before moving on: which way does the number move when you add a negative number?

Example 02 · Multiplying and dividing signed numbers

Work out $4 \times (-7)$.

Why this method?

Use the number line and the sign of each number to decide what the calculation means before calculating.

$4 \times (-7) = -(4 \times 7)$	**Step 1** A positive times a negative gives a negative result.
$4 \times 7 = 28$	**Step 2** Calculate the size without the negative sign.
$4 \times (-7) = -28$	**Step 3** Put back the negative sign.

Think about it: Before moving on: what sign does a negative times a negative give?

Example 03 · Order and number sets

Order $-\sqrt{9}$, -2.5, $\frac{1}{2}$ and $\sqrt{2}$ from smallest to largest.

Why this method?

Use the number line and the sign of each number to decide what the calculation means before calculating.

$-\sqrt{9} = -3$ and $\sqrt{2} \approx 1.414$.	**Step 1** Simplify or estimate for comparison
$-3 < -2.5 < \frac{1}{2} < \sqrt{2}$.	**Step 2** Place them in order
-3 is an integer; -2.5 and $\frac{1}{2}$ are rational; $\sqrt{2}$ is irrational. All are real.	**Step 3** Use the definitions to distinguish integer, rational and irrational values. All of them belong to the real numbers.

Example 04 · Order signed numbers

Place $-3, 0$ and -1 in increasing order.

-3 lies left of -1.	**Step 1** Farther left means smaller.

$-3 < -1 < 0$	**Final step** Zero is larger than either negative number.

Common mistake

Using the multiplication sign rule when adding: “a negative and a positive make a negative.” Addition depends on their values. For example, $-2 + 7 = 5$, while $-7 + 2 = -5$.

Exam technique

Use brackets when substituting a negative number into a power.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Start at -4 on the number line. Add 7. Where do you finish?



Number line from -6 to 6 with -4 marked.

Question 2

Work out $(-3) \times (-5)$.

Watch an explanation

Corbettmaths — Negatives: addition/subtractions

Cognito — How to use BODMAS

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

3

Question 2

15

Open the full chapter online

Decimals, rounding and accuracy

Leaving Certificate · Ordinary Level

Understand the idea

How to start

In 3.47, the 4 means four tenths and the 7 means seven hundredths.

1. Read whether the question asks for decimal places or significant figures.
2. Mark the last digit you will keep, then look at the digit immediately after it.
3. Round only at the end of a calculation.

Key terms & definitions

Estimate
An approximate value used to judge whether an answer is sensible.

Significant figures
Digits starting at the first non-zero digit that show the stated precision.

Recurring decimal
A decimal with a repeating digit or block.

Key ideas

• To convert a fraction to a decimal, divide the numerator by the denominator.

• A terminating decimal ends. A recurring decimal repeats a digit or block; both are rational.

• Decimal places count positions after the decimal point. Significant figures begin at the first non-zero digit.

• For rounding, look at the next digit : 5 or more rounds up; 4 or less leaves the retained digit unchanged.

• Keep extra digits during a calculation. Round the final answer to the accuracy requested.

Quick reminder

• Decimal places start after the decimal point.

• Significant figures start at the first non-zero digit.

• Keep extra digits and round the final result.

Worked examples

Example 01 · Fractions and decimals

Write $\frac{3}{4}$ as a decimal.

Why this method?

Identify the place value or conversion being asked for before moving or rounding any digits.

$\frac{3}{4} = 3 \div 4$	**Step 1** The fraction bar means divide
$3.00 \div 4 = 0.75$	**Step 2** Think of 3 as 3.00 and divide by 4
$\frac{3}{4} = 0.75$	**Step 3** So the two forms name the same number

Think about it: Why is $0.75a$ sensible answer for $\frac{3}{4}$?

Example 02 · Decimal and percentage

Write $\frac{7}{20}$ as a decimal and percentage.

Why this method?

Identify the place value or conversion being asked for before moving or rounding any digits.

$7 \div 20 = 0.35$	**Step 1** The fraction bar means division. Divide the numerator by the denominator to find the decimal value.
$0.35 \times 100\% = 35\%$	**Step 2** Convert to hundredths

Example 03 · Rounding to a stated accuracy

Round 9.995 to 2 decimal places. For halfway values, round away from zero; show the stated precision.

Why this method?

Identify the place value or conversion being asked for before moving or rounding any digits.

Rounding unit : 0.01.	**Step 1** Identify the rounding unit
10.00	**Step 2** Round the magnitude, then restore the sign and show the precision

Think about it: Why does 0.00450 have three significant figures?

Example 04 · Percentage error

A length of 80 cm is measured as 78 cm. Find the percentage error.

$\lvert 78 - 80 \rvert = 2$ cm	**Step 1** Find the size of the error.
$\frac{2}{80} \times 100 = 2.5\%$	**Final step** Divide by the actual value, not the measured value.

Example 05 · Add measurement tolerances

Two lengths are stated as 12.0 ± 0.1 cm and 7.0 ± 0.2 cm. Bound their difference.

$11.9 - 7.2 = 4.7$ cm	**Step 1** Smallest difference: smallest first value minus largest second value.
$12.1 - 6.8 = 5.3$ cm	**Final step** Largest difference: largest first value minus smallest second value.

Common mistake

Counting the zeros before the first non-zero digit as significant figures. In 0.00470, the significant digits are 4, 7 and the final 0 : three significant figures. That final zero records precision.

Exam technique

Write the requested accuracy beside your answer.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Round 0.62721 to 3 significant figures. For halfway values, round away from zero; show the stated precision.

A. 0.630

B. 0.627

C. 0.6272

D. 0.628

Question 2

Write $\frac{7}{10}$ as a decimal.

Watch an explanation

Cognito — Converting Fractions to Decimals

Cognito — Rounding Decimals

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

0.627

Correct.

Question 2

0.7

Compare your method with the worked examples.

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Fractions: all four operations

Leaving Certificate · Ordinary Level

Understand the idea

How to start

The numerator counts parts. The denominator tells you how many equal parts make one whole.

1. For adding or subtracting, make the denominators match first.
2. For multiplying, multiply the tops and multiply the bottoms.
3. For dividing, turn the second fraction upside down, then multiply.

Key terms & definitions

Numerator
The top number: the number of equal parts.

Denominator
The bottom number: the number of equal parts in a whole; it cannot be zero.

Improper fraction
A fraction whose numerator is at least as large as its positive denominator.

Key ideas

• Equivalent fractions name the same number. Multiply or divide numerator and denominator by the same non-zero number.

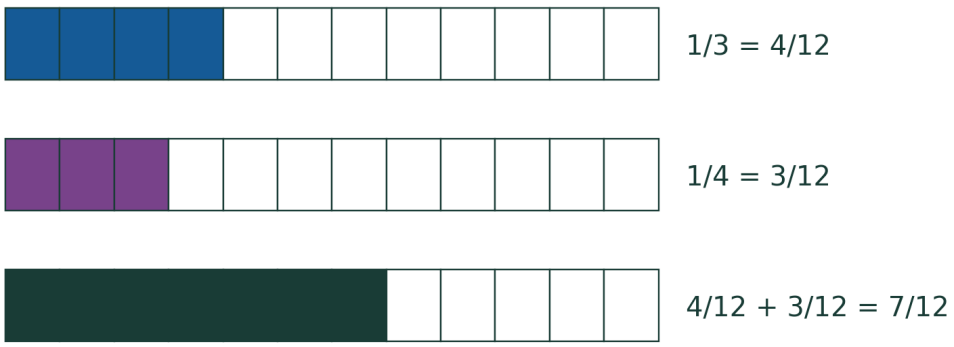
• Simplify using a common factor. For addition or subtraction, use a common denominator first.

• To multiply, multiply numerators and multiply denominators. Cancel common factors before multiplying if helpful.

• To divide by a non-zero fraction, multiply by its reciprocal. Convert mixed numbers to improper fractions first.

• A fraction bar groups its whole numerator and denominator, like brackets.

Add fractions: count the same-sized parts



Each bar is the same whole, divided into twelve equal parts. Four twelfths plus three twelfths gives seven twelfths.

Quick reminder

• Add / subtract: use the same denominator first.

• Multiply: top × top, bottom × bottom.

• Divide: multiply by the reciprocal.

Worked examples

Example 01 · Fractions as numbers

Which is greater : $\frac{3}{5}$ or $\frac{3}{8}$?

Why this method?

Identify the operation first. For addition, use equal-sized parts; for multiplication or division, change mixed numbers to improper fractions before calculating.

3 pieces each	**Step 1** Both fractions have 3 pieces
$\frac{1}{5} > \frac{1}{8}$	**Step 2** Compare the size of one piece: a fifth is bigger than an eighth
$\frac{3}{5} > \frac{3}{8}$	**Step 3** So three bigger pieces are more than three smaller pieces

Think about it: Before moving on: why does a bigger bottom number make smaller pieces?

Example 02 · Equivalent fractions

$\frac{2}{3} = \frac{?}{12}$

Why this method?

Identify the operation first. For addition, use equal-sized parts; for multiplication or division, change mixed numbers to improper fractions before calculating.

$3 \times 4 = 12$	**Step 1** What times 3 gives 12?
$2 \times 4 = 8$	**Step 2** Do the same to the top
$\frac{2}{3} = \frac{8}{12}$	**Step 3** So the missing top number is 8

Think about it: Before moving on: why is multiplying top and bottom by 4 allowed, but adding 4 to each is not?

Example 03 · Adding and subtracting fractions

$\frac{1}{3} + \frac{1}{4}$

Blue follows the first fraction as it changes into twelfths. Purple follows the second. The denominator stays 12 when they are added.

Why this method?

Identify the operation first. For addition, use equal-sized parts; for multiplication or division, change mixed numbers to improper fractions before calculating.

common denominator $= 12$	**Step 1** The pieces must be the same size before we can add them. We need a number that both 3 and 4 divide into. The smallest one is 12, so use twelfths.
$\frac{1}{3} = \frac{1 \times 4}{3 \times 4} = \frac{4}{12}$	**Step 2** Change $\frac{1}{3}$ into twelfths. The bottom 3 is multiplied by 4, so the top must also be multiplied by 4.
$\frac{1}{4} = \frac{1 \times 3}{4 \times 3} = \frac{3}{12}$	**Step 3** Change $\frac{1}{4}$ into twelfths. The bottom 4 is multiplied by 3, so the top must also be multiplied by 3.

$\frac{4}{12} + \frac{3}{12} = \frac{7}{12}$	**Step 4** Now both fractions are made of twelfths, so add the top numbers and keep the denominator 12.
$\frac{7}{12} < 1$	**Step 5** Check the size of the answer

Think about it: Why must you multiply the numerator by the same number whenever you multiply the denominator?

Example 04 · Multiply mixed numbers

Calculate $1\frac{1}{2} \times 2\frac{2}{3}$.

Why this method?

Identify the operation first. For addition, use equal-sized parts; for multiplication or division, change mixed numbers to improper fractions before calculating.

$\frac{3}{2} \times \frac{8}{3}$	**Step 1** Convert each mixed number to an improper fraction first. Then multiply the numerators and multiply the denominators.
$\frac{1 \times 8}{2 \times 1}$	**Step 2** Divide a numerator and a denominator by the same common factor. This keeps the value unchanged and makes the multiplication easier.
$\frac{8}{2} = 4$	**Step 3** Multiply and simplify

Example 05 · Divide fractions

Calculate $\frac{5}{6} \div \frac{10}{9}$.

Why this method?

Identify the operation first. For addition, use equal-sized parts; for multiplication or division, change mixed numbers to improper fractions before calculating.

$\frac{5}{6} \div \frac{10}{9} = \frac{5}{6} \times \frac{9}{10}$	**Step 1** Dividing by a fraction is multiplying by its reciprocal. Only the second fraction is turned over.

$\frac{5 \times 9}{6 \times 10} = \frac{45}{60}$	**Step 2** Multiply the numerators and multiply the denominators.
$\frac{45 \div 15}{60 \div 15} = \frac{3}{4}$	**Step 3** Divide top and bottom by the same common factor , 15.

Example 06 · Subtract a mixed number

Calculate $3\frac{1}{4} - 1\frac{5}{6}$.

Why this method?

Identify the operation first. For addition, use equal-sized parts; for multiplication or division, change mixed numbers to improper fractions before calculating.

$\frac{13}{4} - \frac{11}{6}$	**Step 1** Write each mixed number as one improper fraction so the subtraction rule can be applied.
$\frac{39}{12} - \frac{22}{12} = \frac{17}{12}$	**Step 2** Use denominator 12
$1\frac{5}{12}$	**Step 3** Write as a mixed number

Example 07 · Subtract using a common denominator

Calculate $\frac{5}{6} - \frac{1}{4}$.

$\frac{10}{12} - \frac{3}{12}$	**Step 1** Use twelfths for both fractions.
$\frac{7}{12}$	**Final step** Subtract numerators; keep the denominator.

Common mistake

Adding the denominators as well as the numerators. $\frac{1}{2} + \frac{1}{3}$ is not $\frac{2}{5}$. Use a common denominator: $\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$.

Exam technique

Estimate first: multiplying a positive number by a positive proper fraction makes it smaller.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Which fraction is greatest : $\frac{1}{2}, \frac{3}{4}$ or $\frac{5}{8}$? Give your answer as a fraction.



Number line from 0 to 1 split into eighths, with one half marked.

Question 2

$\frac{?}{15} = \frac{4}{5}$. Find the missing top number.

Watch an explanation

Cognito — Adding & Subtracting Fractions

Cognito — Multiplying & Dividing Fractions

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

$\frac{3}{4}$

Compare your method with the worked examples.

Question 2

12

Compare your method with the worked examples.

Open the full chapter online

Ratio, proportion and rates

Leaving Certificate · Ordinary Level

Understand the idea

How to start

The order matters: red : blue compares red counters first and blue counters second.

1. Put quantities in the same units before comparing them.
2. To share a total, add the ratio parts and find the value of one part.
3. To find a cost or rate, find one unit first when possible.

Key terms & definitions

Ratio
A comparison of quantities by division.

Unit rate
The amount for one unit, such as euro per kilogram.

Constant of proportionality
The fixed multiplier in a directly proportional relationship.

Key ideas

• A ratio compares quantities in the same units. Simplify it by dividing every part by a common factor.

• To share an amount in a ratio, add the ratio parts, find one part, then scale each share.

• Direct proportion means one quantity is a constant multiple of another: $y = kx$. Its graph passes through the origin.

• A rate compares quantities with different units. Average speed is $\frac{\text{totaldistance}}{\text{totaltime}}$.

• Use the stated exchange rate in the correct direction. A fixed starting charge means a relationship is not direct proportion.



Sharing €84 in the ratio 2 : 5 means seven equal parts of €12.

Quick reminder

• Keep the quantities in the stated order.

• Sharing: add the parts, find one part, then scale.

• Rates: put all measurements into compatible units.

Worked examples

Example 01 · What a ratio means

There are 2 red and 5 blue counters. Write red : blue.

Why this method?

Keep the quantities in the stated order. Find the value of one equal part or one unit, then scale to the amount requested.

red : blue	**Step 1** Read the order: red first, then blue
2 : 5	**Step 2** Put in the two numbers for those colours
2 : 5 is not 2 : 7	**Step 3** Do not add the counters together, because that compares red with all counters

Think about it: Before moving on: why is 5 : 2 not the same ratio as 2 : 5?

Example 02 · Equivalent ratios

$2:3 = ? :12$

Why this method?

Keep the quantities in the stated order. Find the value of one equal part or one unit, then scale to the amount requested.

$\times 4$	**Step 1** Find the scale factor : $3 \times 4 = 12$
$2 \times 4 = 8$	**Step 2** Do the same to the other part : $2 \times 4 = 8$
$8:12$	**Step 3** So $2:3 = 8:12$

Think about it: Before moving on: what would have happened if you added 4 to both parts instead?

Example 03 · Find one, then scale

6 tickets cost €24. How much do 9 tickets cost?

Why this method?

Keep the quantities in the stated order. Find the value of one equal part or one unit, then scale to the amount requested.

$24 \div 6 = 4$	**Step 1** Divide the total price by 6 to find the cost of one ticket.
One ticket costs €4.	**Step 2** This is the unit price. We assume each ticket has the same price.
$9 \times 4 = 36$	**Step 3** Nine tickets cost nine times the unit price.
Nine tickets cost €36.	**Step 4** State what the calculated number means.

Think about it: Before moving on: why is the price of one ticket found first?

Example 04 · Scale a ratio

Share €630 in the ratio $2:5$.

$2 + 5 = 7$ parts	**Step 1** Find the total number of parts.

€630 ÷ 7 = €90	**Step 2** Find one part.
€180 and €450	**Final step** Multiply by 2 and 5; check that they total €630.

Common mistake

Treating the first number in a ratio of $2:3$ as two-thirds of the total. There are $2+3=5$ parts, so the first share is $\frac{2}{5}$ of the total.

Exam technique

Convert minutes to hours before calculating speed in kilometres per hour.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Pears : apples $= 9:3$. How many pears for each apple?

Question 2

$2:9=6:?$. Find the missing number.

Watch an explanation

Cognito — Ratios - Introduction & how to Simplify

Cognito — Proportional Division

Corbettmaths — Speed, distance and time

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

3

Compare your method with the worked examples.

Question 2

27

Compare your method with the worked examples.

Open the full chapter online

Surds and exact answers

Leaving Certificate · Ordinary Level

Understand the idea

How to start

A square root asks which non-negative number squares to give the number inside. For example, $\sqrt{9} = 3$.

1. To simplify a root, look for a square factor such as $4, 9, 16$ or 25.
2. Take the square root of that factor, leaving the other factor inside.
3. Add surds only when their root parts match. Keep exact roots until a decimal is requested.

Key terms & definitions

Exact answer
An answer without rounding, such as $3\sqrt{2}$.

Like surds
Terms with the same simplified root part.

Conjugates
A pair such as $a + \sqrt{b}$ and $a - \sqrt{b}$.

Key ideas

• A surd is an irrational root written exactly. Keep it exact unless a decimal approximation is requested.

• Find a square factor to simplify: $\sqrt{72} = \sqrt{36}\sqrt{2} = 6\sqrt{2}$.

• Collect like surds after simplifying. Different surds cannot usually be added into one term.

• For non-negative real a, b, $\sqrt{a}\sqrt{b} = \sqrt{ab}$. Expand brackets carefully when multiplying surd expressions.

• The positive square root symbol means the non-negative root. Solving $x^2 = 9$ gives $x = \pm 3$.

Quick reminder

• Find a square factor to simplify a root.

• Collect only terms with the same simplified root part.

• Keep exact surds until a decimal answer is requested.

Worked examples

Example 01 · Simplifying a surd

Simplify $\sqrt{8}$.

Why this method?

Keep the answer exact. Look for square factors and matching root parts before combining any terms.

$8 = 4 \times 2$	**Step 1** Look for a square factor. Four is a square because $2 \times 2 = 4$.
$\sqrt{8} = \sqrt{4} \times \sqrt{2}$	**Step 2** The square root of a product of non-negative numbers can be split into the product of the roots.
$\sqrt{8} = 2\sqrt{2}$	**Step 3** The root of 4 is 2. Two has no square factor greater than 1, so its root stays inside.

Think about it: Why can 2 come out of the square root, while the other factor stays inside?

Example 02 · Simplifying a surd — another example

Simplify $\sqrt{12}$.

Why this method?

Keep the answer exact. Look for square factors and matching root parts before combining any terms.

$12 = 2^2 \times 3$	**Step 1** Find the largest square factor of 12

$\sqrt{12} = \sqrt{2^2} \times \sqrt{3} = 2\sqrt{3}$	**Step 2** Take the square root of the square factor out

Example 03 · Multiplying surds

Simplify $\sqrt{2} \times \sqrt{3}$.

Why this method?

Keep the answer exact. Look for square factors and matching root parts before combining any terms.

$\sqrt{2} \times \sqrt{3} = \sqrt{6}$	**Step 1** Multiply the numbers under one root
$\sqrt{6}$	**Step 2** There is no square factor greater than 1 to take outside, so this is already simplified.

Think about it: Before moving on: why can the two numbers be multiplied under one root, but not added?

Example 04 · Give an exact trigonometric length

A right triangle has hypotenuse 8 cm and an angle of 45°. Find the opposite side exactly.

$x = 8\sin 45^\circ$	**Step 1** Use sine: opposite over hypotenuse.
$x = 8\left(\frac{\sqrt{2}}{2}\right) = 4\sqrt{2}$ cm	**Final step** Use the exact value rather than a rounded decimal.

Common mistake

Splitting a square root across addition. $\sqrt{9+16} = \sqrt{25} = 5$, but $\sqrt{9} + \sqrt{16} = 3 + 4 = 7$. These are different calculations.

Exam technique

Simplify each root before deciding whether terms can be collected.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Simplify $\sqrt{98}$.

A. $\sqrt{14}$

B. $7\sqrt{14}$

C. $49\sqrt{2}$

D. $7\sqrt{2}$

Question 2

Simplify $\sqrt{3} \times \sqrt{12}$.

A. $\sqrt{15}$

B. 36

C. $3\sqrt{12}$

D. 6

Watch an explanation

Cognito — Surds - Introduction & how to Simplify

Cognito — Surds - Rationalising the Denominator

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

$7\sqrt{2}$

Correct : $98 = 7^2 \times 2$, so $\sqrt{98} = 7\sqrt{2}$.

Question 2

6

Correct : $\sqrt{3} \times \sqrt{12} = \sqrt{36} = 6$.

Open the full chapter online

Percentages and percentage change

Leaving Certificate · Ordinary Level

Understand the idea

How to start

Percent means out of 100. So 20% is 20 out of 100, or 0.20.

1. Underline the amount the percentage is based on.
2. Find the percentage amount, then add or subtract it if the question asks for a change.
3. If you know the final price and need the original price, work backwards by dividing by the multiplier.

Key terms & definitions

Percentage
An amount expressed as a number of hundredths.

Multiplier
The factor used to calculate a percentage increase or decrease.

Original amount
The starting value before a change.

Key ideas

• Percent means out of 100. Divide a percentage by 100 to make a decimal multiplier.

• Find a percentage of an amount by multiplying. Increase by 15% using 1.15; decrease by 15% using 0.85.

• Percentage change uses the original amount as the denominator: $\frac{\text{change}}{\text{original}} \times 100\%$.

• For a reverse percentage, divide the final amount by the multiplier. Do not subtract the percentage from the final amount.

• Distinguish an absolute change from a relative change: the same increase can be a very different percentage of different starting values.

Quick reminder

• Decide what represents 100%.

• Forward change: multiply by the percentage multiplier.

• Reverse change: divide the final amount by the multiplier.

Worked examples

Example 01 · Fractions, decimals and percentages

Write 0.4 as a percentage.

Why this method?

Decide which amount represents 100%. This tells us what to multiply or divide by.

$0.4 = 0.40 = \frac{40}{100}$	**Step 1** Write four tenths as forty hundredths.
$\frac{40}{100} = 40\%$	**Step 2** Percent means out of 100, so the numerator gives the percentage.

Think about it: Why is 0.4 equal to 40%, not 4%?

Example 02 · Find a percentage of an amount

Find 20% of $€60$.

Why this method?

Decide which amount represents 100%. This tells us what to multiply or divide by.

$20\% = \frac{20}{100} = 0.2$	**Step 1** Change the percentage into a multiplier.
$0.2 \times 60 = 12$	**Step 2** Multiply the whole amount, $€60$, by 0.2.
20% of $€60$ is $€12$.	**Step 3** Include the euro unit because we found an amount of money.

Think about it: Why does dividing by 5 work for 20%?

Example 03 · Percentage increase and decrease

A €50 item increases by 20%. Find the new price.

Why this method?

Decide which amount represents 100%. This tells us what to multiply or divide by.

20% of $50 = 10$	**Step 1** Find 20% of the original €50
increase $= €10$	**Step 2** This €10 is the increase, not the final price
$€50 + €10 = €60$	**Step 3** Add the increase to the original price

Think about it: Why is €10 not the final answer?

Example 04 · Absolute and relative comparison

A score rises from 70 to 80. Compare the change.

Why this method?

Decide which amount represents 100%. This tells us what to multiply or divide by.

$80 - 70 = 10$ points	**Step 1** Find the absolute increase
$\frac{10}{70} \approx 0.143$	**Step 2** For the relative increase, compare the change with the starting value
relative increase $\approx 14.3\%$	**Step 3** Write as a percentage

Think about it: Why is 70, not 80, the reference value for the increase?

Example 05 · Reverse a discount

A jacket costs €68 after $a15\%$ discount. Find its original price.

Why this method?

Decide which amount represents 100%. This tells us what to multiply or divide by.

The final price is 85% of the original.	**Step 1** Identify the remaining percentage

$0.85P = 68$ so $P = \frac{68}{0.85} = 80$.	**Step 2** Write and solve
15% of €80 is €12; €80 minus €12 is €68. Original price: €80.	**Step 3** Apply the stated percentage change to the original amount and check that it gives the stated final price.

Example 06 · Reverse a percentage change

After a 15% increase a price is €138. Find the original price.

$1.15 \times \text{original} = 138$	**Step 1** The new price is 115% of the original.
€138 ÷ 1.15 = €120	**Final step** Reverse the multiplier by dividing.

Common mistake

Assuming equal percentage increases and decreases cancel. Starting at €100, a 20% decrease gives €80; a 20% increase on €80 gives €96. The second percentage uses a different starting amount.

Exam technique

Label which amount represents 100% before beginning.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Write 0.7 as a percentage.

A. 700%

B. 0.7%

C. 7%

D. 70%

Question 2

Find 25% of 80.

Watch an explanation

Corbettmaths — Percentages: of an amount (non-calc)

Cognito — Percentage Increases & Decreases (Multiplier Method)

Cognito — Reverse Percentage - Finding Price Before a % Discount

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

70%

Compare your method with the worked examples.

Question 2

20

Compare your method with the worked examples.

Open the full chapter online

Money, bills, interest and tax

Leaving Certificate · Ordinary Level

Understand the idea

How to start

Profit is money gained: selling price minus cost price. A percentage needs a clearly named starting amount.

1. Write down the amount, rate and time given in the question.
2. Choose the starting amount for the percentage: cost price, selling price or original balance.
3. Calculate each charge or deduction separately before finding the final total.

Key terms & definitions

Principal
The original sum invested or borrowed.

Gross pay
Pay before deductions.

Net pay
Pay after deductions.

Tax credit
An amount subtracted from calculated tax, subject to the rules stated.

Key ideas

• Profit is selling price minus cost price. Loss is cost price minus selling price.

• Mark-up uses cost price as its base; margin uses selling price. State which base you use.

• For compound growth, multiply by the same growth factor each period: $A = P(1 + r)^n$, where r is a decimal rate.

• For depreciation, use $A = P(1 - r)^n$. Simple interest is $I = Prt$ when the rate and time units match.

• A bill may have a standing charge, usage charges, discounts and tax. Follow the order stated.

• Net pay is gross pay minus deductions. Tax bands apply only to the part of pay within each band. All rates here are fictional teaching examples.

Quick reminder

• Mark-up uses cost price; margin uses selling price.

• Compound interest applies the multiplier once per period.

• Use only the fictional tax rules and bands given in the question.

Worked examples

Example 01 · Profit, loss and percentage profit on cost price

A shop buys an item for €20 and sells it for €25. What is the profit?

Cost price	€20
Selling price	€25
Profit	€5

Why this method?

Identify the starting amount and the base for each percentage. Apply each charge or change in the order stated.

Selling price $= €25$. Cost price $= €20$.	**Step 1** Identify the two prices before subtracting.
$\text{profit} = 25 - 20 = 5$	**Step 2** Profit is the selling price minus the original cost.
The profit is €5.	**Step 3** The positive result shows the shop gained money.

Think about it: Before moving on: which price is the percentage profit worked out from, and why?

Example 02 · VAT and net pay

A price before VAT is €40. VAT at 10% is added. What is the price including VAT?

Price before VAT	€40
VAT at 10%	€4
Price including VAT	€44

Why this method?

Identify the starting amount and the base for each percentage. Apply each charge or change in the order stated.

$€40 \times 10\% = €4$	**Step 1** VAT = price before VAT × rate
$€40 + €4 = €44$	**Step 2** Price including VAT = price before VAT + VAT

Think about it: Before moving on: why is the price before VAT not found by taking 20% off the price including VAT?

Example 03 · Bills with a standing charge and a charge per unit

An electricity bill has a standing charge of €20. Each unit costs 25 cent, and 100 units are used. What is the total bill?

Standing charge	€20
100 units at 25 cent	€25
Total bill	€45

Why this method?

Identify the starting amount and the base for each percentage. Apply each charge or change in the order stated.

$100 \times 25 \div 100 = €25$	**Step 1** Charge for the units = number of units × cost per unit (in cent) ÷ 100
$€20 + €25 = €45$	**Step 2** Total bill = standing charge + charge for the units

Think about it: Before moving on: why must the cost per unit be multiplied by the number of units before anything else is added?

Example 04 · Splitting a bill equally

A bill of €120 is shared equally between 3 people. How much does each person pay?

Why this method?

Identify the starting amount and the base for each percentage. Apply each charge or change in the order stated.

$€120 \div 3 = €40$	**Step 1** Each person pays an equal share

Think about it: Before moving on: how can you check that each share is right?

Example 05 · Compound interest

€100 is invested at 20% compound interest per year. What is the amount after 2 years?

Start	End of year 1	End of year 2
€100	€120	€144

Why this method?

Identify the starting amount and the base for each percentage. Apply each charge or change in the order stated.

$20\% = 0.20$	**Step 1** Change the annual percentage to a decimal.
$1 + 0.20 = 1.20$	**Step 2** Keep the whole balance, 100%, and add 20%. This gives the growth multiplier.
$100 \times 1.20 = 120$	**Step 3** At the end of year 1, the balance is €120.
$120 \times 1.20 = 144$	**Step 4** For year 2, apply the interest to the new balance, not just the original €100.
The amount after 2 years is €144.	**Step 5** Compound interest includes interest on earlier interest.

Think about it: Before moving on: why is the amount after two years not the same as the start amount plus two years of simple interest?

Example 06 · Cost of one item (unit price)

Pack A costs €300 and contains 6 items. What is the cost of one item?

Why this method?

Identify the starting amount and the base for each percentage. Apply each charge or change in the order stated.

$€300 \div 6 = €50$	**Step 1** Cost per item = pack cost ÷ number of items

Think about it: Before moving on: why can the cheaper pack not always be the better value?

Example 07 · Mark-up on cost price

An item costs €200 to buy. The shop marks it up by 10% on the cost price. What is the selling price?

Why this method?

Identify the starting amount and the base for each percentage. Apply each charge or change in the order stated.

10% of $€200 = €20$	**Step 1** Mark-up is a percentage of the cost price
$€200 + €20 = €220$	**Step 2** Add the mark-up to the cost price

Think about it: Before moving on: why is the mark-up a percentage of the cost price, and why is it added rather than taken off?

Example 08 · Margin on selling price

An item sells for €200. The margin is 10% of the selling price. What is the cost price?

Why this method?

Identify the starting amount and the base for each percentage. Apply each charge or change in the order stated.

10% of $€200 = €20$	**Step 1** Margin is a percentage of the selling price, so the profit is
$€200 - €20 = €180$	**Step 2** Take the profit off the selling price

Think about it: Before moving on: why is margin measured against the selling price, and how does that differ from mark-up?

Example 09 · Compound interest over more than 3 years

€300 is invested at 2% compound interest per year. What is the amount after 6 years? Give your answer to the nearest cent.

Why this method?

Identify the starting amount and the base for each percentage. Apply each charge or change in the order stated.

$€300 \times (1.02)^6$	**Step 1** Multiply the amount by (1) rate) once for each year
$= €337.85$	**Step 2** Round to the nearest cent

Think about it: Before moving on: why is the power the number of years, and not the number of years minus one?

Example 10 · Net pay with two tax rates and other deductions

In this fictional tax system, gross weekly pay is €800. Income tax is 20% on the first €700 and 40% on the rest. A further deduction of 2% of gross pay is taken. What is the net weekly pay?

Why this method?

Identify the starting amount and the base for each percentage. Apply each charge or change in the order stated.

$0.20 \times 700 = 140$	**Step 1** Tax the first €700 at 20%.
$800 - 700 = 100$	**Step 2** Find the part of gross pay above the first band.
$0.40 \times 100 = 40$	**Step 3** Tax only that remaining part at 40%.
$140 + 40 = 180$	**Step 4** Add the two income-tax amounts.
$0.02 \times 800 = 16$	**Step 5** The further deduction is 2% of gross pay.
$800 - 180 - 16 = 604$	**Step 6** Subtract income tax and the other deduction from gross pay. Net weekly pay is €604.

Think about it: Before moving on: why is the 40% rate applied only to the part of the pay above the band?

Example 11 · Reducing-balance depreciation

A van worth €18,000 loses 12% in value each year. Estimate its value after three years.

$1 - 0.12 = 0.88$	**Step 1** Each year 88% of the previous value remains.
$18000(0.88)^3 = 12266.496$	**Step 2** Apply the multiplier three times.
€12,266.50	**Final step** Round to cents only at the end.

Example 12 · Cost materials, wastage and labour

A job uses $24m^2$ at €15 per m^2, with 10% extra material and €160 labour. Find the cost.

$24 \times 1.10 = 26.4m^2$	**Step 1** Add wastage to the area before pricing.
$26.4 \times €15 + €160 = €556$	**Final step** Price the materials and add labour.

Common mistake

Applying the higher tax rate to the whole income. When a question gives two tax bands, calculate tax on each band separately. Only the amount above the stated threshold uses the higher rate.

Exam technique

Show money to two decimal places at the end; retain calculator precision in intermediate steps.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

A shop buys an item for €30 and sells it for €42. What is the profit?

Cost price	€30
Selling price	€42
Profit	€12

Question 2

A price before VAT is €70. For this question, use a VAT rate of 20%. What is the price including VAT?

Price before VAT	€70
VAT at 20%	€14
Price including VAT	€84

Watch an explanation

Tulla Maths — Income tax and USC: the calculation method

Corbettmaths — Percentages: compound interest

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

12

Question 2

84

Open the full chapter online

Indices and scientific notation

Leaving Certificate · Ordinary Level

Understand the idea

How to start

A power counts repeated multiplication: $x^3 = x \times x \times x$.

1. Identify the base: the number or expression being multiplied repeatedly.
2. Choose the rule for the operation shown. Adding powers and multiplying powers are different.
3. Keep brackets around a negative base, because they change what the power applies to.

Key terms & definitions

Base
The number or expression being raised to a power.

Index
The exponent indicating a power.

Scientific notation
A number written as a coefficient between 1 and 10 multiplied by an integer power of 10.

Key ideas

• For the same non-zero base, $a^m a^n = a^{m+n}$ and $\frac{a^m}{a^n} = a^{m-n}$. Also $(a^m)^n = a^{mn}$.

• A zero power is 1 for a non-zero base. A negative index means reciprocal: $a^{-n} = \frac{1}{a^n}$.

• For positive real bases, $a^{p/q} = \sqrt[q]{a^p}$. Check the real domain before evaluating roots of negative numbers.

• An even root of a negative number is not real. An odd root can be negative. Zero cannot have a negative power.

• Scientific notation is $a \times 10^n$ with $1 \le a < 10$ for positive values and integer n. Small positive numbers use negative powers.

Quick reminder

- $a^m a^n = a^{m+n}$: multiply same bases → add indices.
- $\frac{a^m}{a^n} = a^{m-n}$: divide same non-zero bases → subtract.
- $(a^m)^n = a^{mn}$: power of a power → multiply indices.

Worked examples

Example 01 · Start here: multiply powers of the same base

Simplify $x^3 \times x^2$.

Why this method?

The bases are both x and the operation is multiplication. Count the factors to see why their indices are added.

$(x \times x \times x)(x \times x)$	**Step 1** Write out what each power means. There are three x factors and then two more.
x^5	**Step 2** Count the five factors. Multiplication keeps them all in the product.
$x^{3+2} = x^5$	**Step 3** This is why we add the indices for multiplication with the same base.

Example 02 · Scientific notation calculation

Calculate $(3 \times 10^{-4})(2 \times 10^7)$ in scientific notation.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

$3 \times 2 = 6$	**Step 1** Multiply coefficients
$10^{-4} \times 10^7 = 10^3$	**Step 2** Add indices of the common base

6×10^3	**Step 3** Put the coefficient and the power of ten together, then check that the coefficient is between 1 and 10.

Example 03 · Powers with negative and zero integer bases

Evaluate $(-6)^9$. The minus sign is inside the brackets.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

9 factors, each equal to -6.	**Step 1** Read the brackets before multiplying
Result : -10077696.	**Step 2** An odd number of negative factors gives a negative product. Calculate 6 to the ninth power, then apply that sign.

Think about it: How does a bracket change whether a minus sign is raised to the power?

Example 04 · Powers with negative and zero integer bases — another example

Evaluate -7^8. The exponent is applied before the leading minus sign.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

First evaluate 7^8, then negate it.	**Step 1** Read the brackets before multiplying
Result : -5764801.	**Step 2** The power applies only to 7. Calculate 7 to the eighth power, then apply the leading negative sign.

Example 05 · Negative indices

Evaluate 3^{-3}. Give your answer as a fraction.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

$3^{-3} = 1 \div 3^3$	**Step 1** A negative index means the reciprocal of the positive power
$3^3 = 27$, so the answer is $\frac{1}{27}$	**Step 2** Work out the power

Think about it: Before moving on: why does a negative index give a fraction, and not a negative number?

Example 06 · Negative indices — another example

Evaluate 5^{-3}. Give your answer as a fraction.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

$5^{-3} = 1 \div 5^3$	**Step 1** A negative index means the reciprocal of the positive power
$5^3 = 125$, so the answer is $\frac{1}{125}$	**Step 2** Work out the power

Example 07 · Combine a fractional index and a negative index

Calculate $27^{-\frac{2}{3}}$.

$27^{\frac{1}{3}} = 3$	**Step 1** The denominator 3 means cube root.
$27^{\frac{2}{3}} = 3^2 = 9$	**Step 2** The numerator 2 means square.
$27^{-\frac{2}{3}} = \frac{1}{9}$	**Final step** The minus sign in the index means reciprocal.

Example 08 · Multiply in scientific notation

Calculate $(3 \times 10^5)(4 \times 10^{-2})$ in standard form.

12×10^3	**Step 1** Multiply coefficients and add indices.
1.2×10^4	**Final step** Move the decimal point to make the first factor lie from 1 up to, but not including 10.

Common mistake

Adding powers when adding terms. $x^2 + x^3$ is not x^5: these are unlike terms. For multiplication with the same base, $x^2 \times x^3 = x^5$.

Exam technique

Use brackets to distinguish $(-3)^2$ from -3^2.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Evaluate -7^3. The exponent is applied before the leading minus sign.

A. -343

B. 343

C. -4

D. -21

Question 2

Evaluate 2^{-5}. Give your answer as a fraction.

A. $\frac{1}{32}$

B. 32

C. -32

D. $\frac{1}{10}$

Watch an explanation

Cognito — Multiplying & Dividing Powers

Cognito — Negative Powers

Cognito — Fractional Powers

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

-343

Correct.

Question 2

$\frac{1}{32}$

Correct : $2^{-5} = 1 \div 2^{5} = \frac{1}{32}$.

Open the full chapter online

Sets and Venn diagrams

Leaving Certificate · Ordinary Level

Understand the idea

How to start

A set is a collection of items. An overlap contains items that belong to both sets.

1. Name the whole group, called the universal set.
2. Put the shared items in the overlap first.
3. Fill the remaining regions and check that every item is counted once.

Key terms & definitions

Universal set
All elements under consideration, usually labelled U.

Complement
Elements in U that are not in the named set.

Cardinality
The number of elements in a finite set.

Empty set
A set with no elements.

Key ideas

• A set is a collection of distinct elements. State the universal set before finding a complement.

• $A \cup B$ means in A or B or both. $A \cap B$ means in both. $A \setminus B$ means in A but not B.

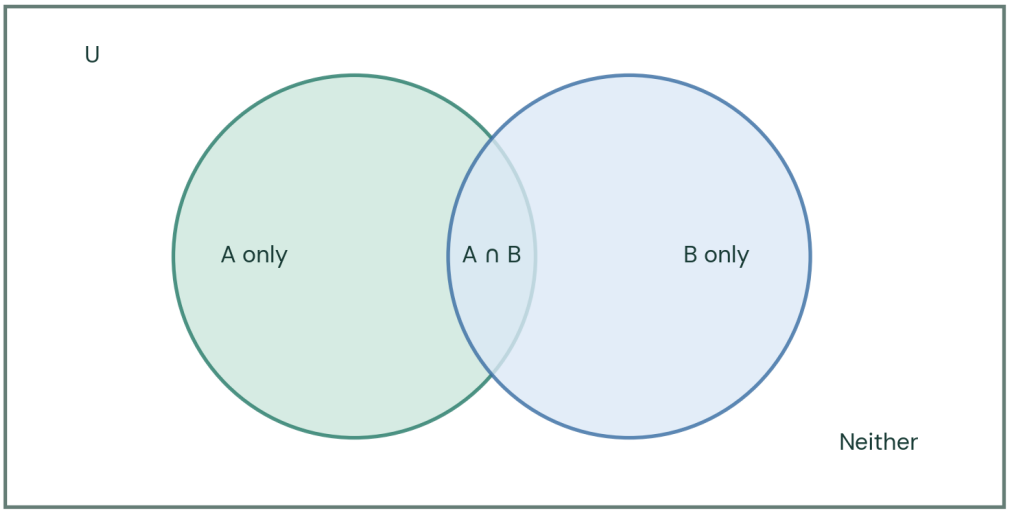
• Fill a Venn diagram from the most specific overlap outwards: the triple intersection first, then pair-only regions, then one-set-only regions.

• For two sets, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. Subtract the union from the universal total to find neither.

• Union and intersection are commutative and associative. These laws regroup sets; they do not change which elements belong.

• A subset contains only elements of another set: $A \subseteq B$. Distinguish an element belonging to a set, $x \in A$, from a set being a subset.

• For three sets,
$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$.



Four separate regions: A only, both A and B, B only, and neither. Together they make the universal set.

Quick reminder

• Union: in either set or both. Intersection: in both.

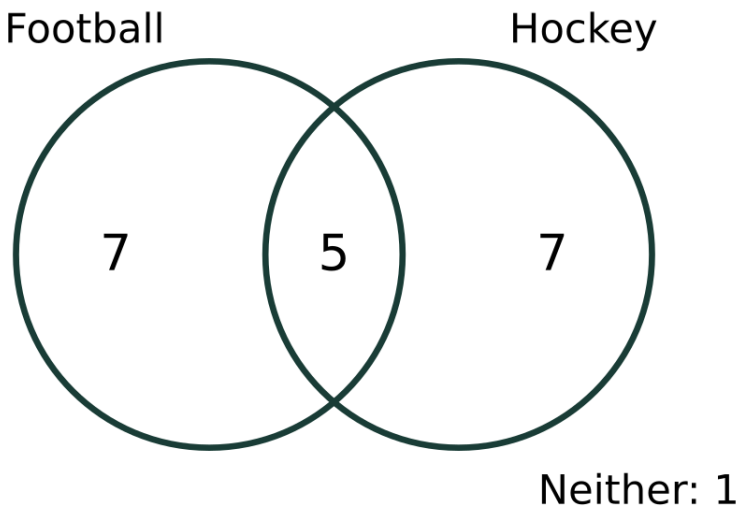
• In Venn diagrams, fill the deepest overlap first.

• Neither = universal total minus the union.

Worked examples

Example 01 · Two-set Venn diagrams

In a group of 20 students, 12 play football, 12 play hockey, and 5 play both. How many play football only?



Two-set Venn diagram for 20 students. Football only : 7. Both : 5. Hockey only : 7. Neither : 1.

Why this method?

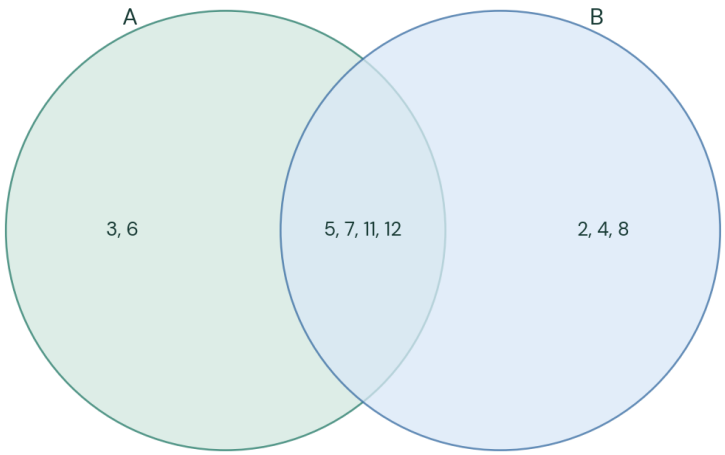
Identify which region or operation the question asks for. Count each element or person only in the regions where they belong.

$\text{Football total} = \text{football only} + \text{both}.$	**Step 1** The 12 football players already include the 5 who play both sports.
$12 = \text{footballonly} + 5$	**Step 2** Put the given numbers into that relationship.
$\text{footballonly} = 12 - 5 = 7$	**Step 3** Subtract the overlap once to leave only the football-only region.

Think about it: Before moving on: why must the overlap be taken away from each circle total?

Example 02 · Set notation: union, intersection and difference

$A = \{3, 5, 6, 7, 11, 12\}$ and $B = \{2, 4, 5, 7, 8, 11, 12\}$. How many elements are in $A \cup B$?



Venn diagram matching the sets in this question. A only : [3, 6]. Both : [5, 7, 11, 12]. B only : [2, 4, 8].

Why this method?

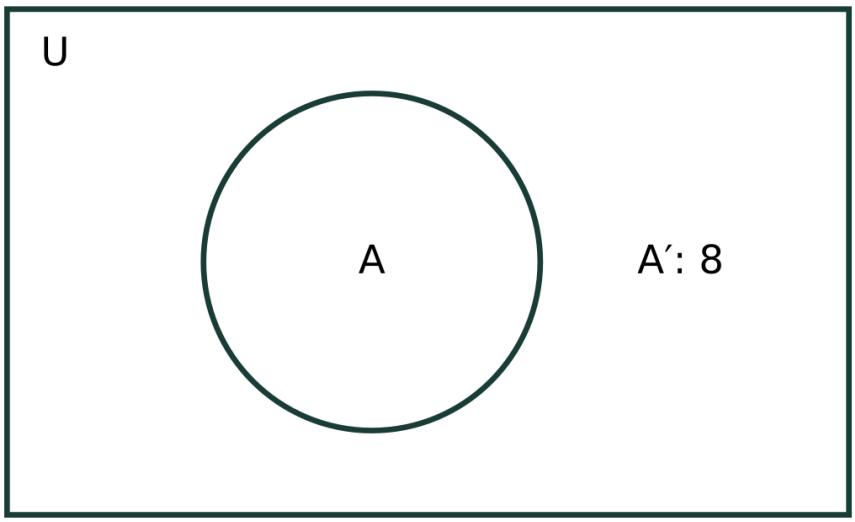
Identify which region or operation the question asks for. Count each element or person only in the regions where they belong.

$A \cup B = \{2, 3, 4, 5, 6, 7, 8, 11, 12\}$	**Step 1** $A \cup B$ contains every element in A or in B, each listed once
9 elements	**Step 2** Count the elements

Think about it: Before moving on: why is an element that is in both sets listed only once in $A \cup B$?

Example 03 · Complement and universal set

The universal set is $U = \{1, 2\} \ldots, 12\}$. $A = \{2, 6, 8, 12\}$. How many elements are in A□, the complement of A?



Universal set $U = \{1\} \ldots, 12\}$ drawn as a rectangle, with $A = \{2, 6, 8, 12\}$ as a circle inside it. The complement A□ is the part of U outside the circle, with 8 elements.

Why this method?

Identify which region or operation the question asks for. Count each element or person only in the regions where they belong.

A□ $= \{1, 3, 4, 5, 7, 9, 10, 11\}$	**Step 1** The complement is the elements of U that are not in A
$12 - 4 = 8$	**Step 2** Count the elements of A□

Think about it: Before moving on: why is A□ found by taking A away from U rather than by adding A to U?

Example 04 · Cardinal number and the null set

Hannah writes the set $A = \{2, 2, 9, 12\}$. What is n(A), the number of elements in A?

Why this method?

Identify which region or operation the question asks for. Count each element or person only in the regions where they belong.

Different elements : $\{2, 9, 12\}$	**Step 1** Each element of a set is counted once, even if it is written more than once

$n(A) = 3$	**Step 2** Count the different elements

Think about it: Before moving on: why is the null set not the same as the set $\{0\}$?

Example 05 · **Count an overlap**

In a group of 32, 18 take French, 15 take Music and 7 take both. How many take neither?

$18 + 15 - 7 = 26$	**Step 1** Subtract the overlap once: it was counted twice.
$32 - 26 = 6$	**Final step** Subtract the union from the total.

Common mistake

Counting the overlap twice when finding how many are in A or B. If $n(A) = 12$, $n(B) = 10$ and $n(A \cap B) = 4$, the total is $12 + 10 - 4 = 18$, not 22.

Exam technique

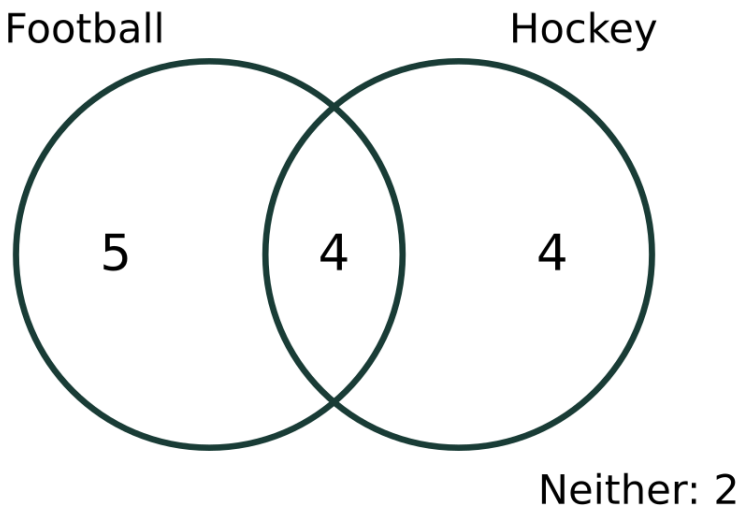
Check that all disjoint regions add to the universal total.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

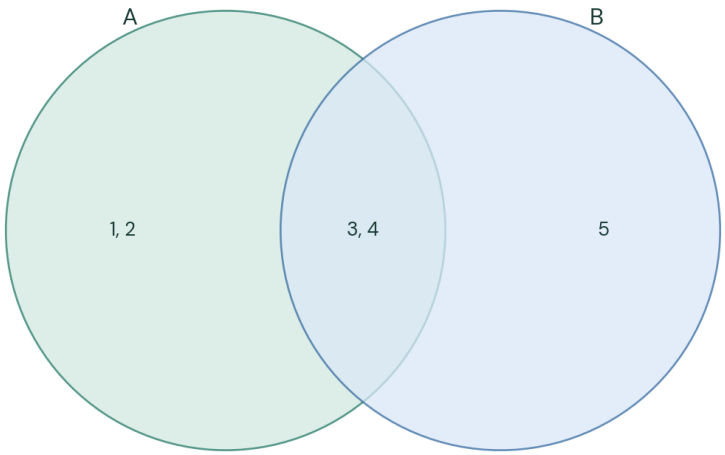
In a group of 15 students, 9 play football, 8 play hockey, and 4 play both. How many play neither sport?



Two-set Venn diagram for 15 students. Football only : 5. Both : 4. Hockey only : 4. Neither : 2.

Question 2

$A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5\}$. How many elements are in $A \cup B$?



Venn diagram matching the sets in this question. A only : [1, 2]. Both : [3, 4]. B only : [5].

Watch an explanation

Cognito — Set Notation & Venn Diagrams

Cognito — Venn Diagrams

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

2

Question 2

5

Open the full chapter online

Complex numbers and Argand diagrams

Leaving Certificate · Ordinary Level

Understand the idea

How to start

A complex number has a real part and an imaginary part. In $3 + 2i$, these are 3 and 2.

1. Identify the real part and the coefficient of i.
2. Combine matching parts; when multiplying, replace i squared with -1.
3. For division, multiply top and bottom by the denominator's conjugate.

Key terms & definitions

Real part
The number without i.

Conjugate
The same real part with the opposite imaginary part.

Modulus
Distance from the origin on an Argand diagram.

Key ideas

• $i^2 = -1$. This is the rule used when multiplying imaginary parts.

• For $z = a + bi$, the real part is a and the imaginary part is b. For example, $\text{Im}(5 - 3i) = -3$.

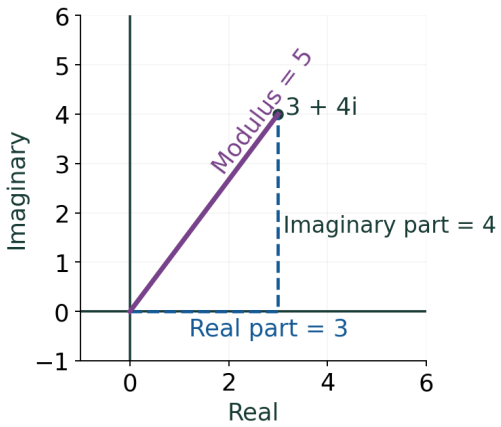
• Add real parts to real parts and imaginary parts to imaginary parts.

• On an Argand diagram, real is horizontal and imaginary is vertical.

• The conjugate of $a + bi$ is $a - bi$. Only the imaginary sign changes.

• The modulus is distance from the origin: $|a + bi| = \sqrt{a^2 + b^2}$.

• For division, multiply the numerator and denominator by the conjugate of the denominator.



Plot $3+4i$ at $(3,4)$ on real and imaginary axes. Its distance from the origin is the modulus: square root of $3^2+4^2=5$.

Quick reminder

- $i^2=-1$. This is the rule used when multiplying imaginary parts.
- Add real parts to real parts and imaginary parts to imaginary parts.

Worked examples

Example 01 · Start here: subtract matching parts

Calculate $(5+4i)-(2+i)$.

Why this method?

Remove the second bracket carefully, then subtract real parts and imaginary parts separately.

$5+4i-2-i$	**Step 1** The minus before the second bracket applies to both terms.
$(5-2)+(4-1)i$	**Step 2** Keep the real and imaginary parts in separate groups.
$3+3i$	**Step 3** State the real part first.

Example 02 · Add two complex numbers

Calculate $(2 + 3i) + (4 + i)$.

Why this method?

Work with real and imaginary parts separately. Use i squared $= -1$ whenever it appears.

$2 + 4 + 3i + i$	**Step 1** Put real terms together and imaginary terms together.
$6 + 4i$	**Step 2** Three i plus one i is four i.

Example 03 · Multiply and use i squared

Calculate $(1 + 2i)(3 + i)$.

Why this method?

Work with real and imaginary parts separately. Use i squared $= -1$ whenever it appears.

$3 + i + 6i + 2i^2$	**Step 1** Multiply every term in the first bracket by every term in the second.
$3 + 7i + 2(-1)$	**Step 2** Replace i^2 by -1.
$1 + 7i$	**Step 3** Collect the real numbers.

Example 04 · Conjugate and modulus

Find the conjugate and modulus of $3 + 4i$.

Why this method?

Work with real and imaginary parts separately. Use i squared $= -1$ whenever it appears.

$\overline{z} = 3 - 4i$	**Step 1** Change the sign of the imaginary part.
$\lvert z\rvert = \sqrt{3^2 + 4^2}$	**Step 2** Use distance from the origin.
$\lvert z\rvert = \sqrt{9 + 16} = 5$	**Step 3** Square, add, then take the positive root.

Example 05 · Divide using a conjugate

Calculate $\frac{3+i}{1+i}$.

Why this method?

Work with real and imaginary parts separately. Use i squared $=-1$ whenever it appears.

$\frac{3+i}{1+i} \times \frac{1-i}{1-i}$	**Step 1** Multiply top and bottom by $1-i$. This whole multiplier equals 1.
$\frac{3-3i+i-i^2}{1-i^2}$	**Step 2** Multiply both brackets on top and on the bottom.
$\frac{4-2i}{2}=2-i$	**Step 3** Use $i^2=-1$, then divide both terms by 2.

Example 06 · Divide in rectangular form

Write $\frac{4+i}{1-2i}$ in the form $a+bi$.

$\frac{(4+i)(1+2i)}{(1-2i)(1+2i)}$	**Step 1** Multiply numerator and denominator by the denominator's conjugate.
$\frac{2+9i}{5}$	**Step 2** Expand, then replace i^2 by -1.
$\frac{2}{5}+\frac{9}{5}i$	**Final step** Divide both parts by 5.

Common mistake

Including i when stating the imaginary part. For $z=3+2i$, the imaginary part is 2, not $2i$. The coefficient of i is the imaginary part.

Exam technique

Write the real part first in a final rectangular-form answer.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Find the conjugate of $2 - 5i$.

Question 2

Find $|6 + 8i|$.

Question 3

Calculate $(4 + 3i) - (1 + i)$.

Question 4 · Build confidence

For $z = 5 - 3i$, state the real part and imaginary part.

Question 5 · Apply or explain

Calculate $(2 + i)(3 - i)$.

Before you move on

Try these without looking at a worked example. Revisit any part that is still difficult.

- Identify the real part and the coefficient of i.
- Use i squared $= -1$ when multiplying.
- Find the conjugate and use it in division.

Self-check answers

Question 1

$2 + 5i$

Question 2

10

Question 3

$3 + 2i$.

Subtract each corresponding part.

Question 4

$\mathrm{Re}(z) = 5$ and $\mathrm{Im}(z) = -3$.

The imaginary part is the real coefficient of i, not $-3i$.

Question 5

$6 - 2i + 3i - i^2 = 7 + i$.

Replace i squared by -1, so $-i$ squared contributes $+1$.

Open the full chapter online

Algebra and substitution

Leaving Certificate · Ordinary Level

Understand the idea

How to start

In $3x - 5$, x is the variable , 3 is its coefficient, and -5 is the constant term.

1. Evaluate means find a numerical value. Substitute means replace a letter by a given number.
2. Replace every occurrence of the letter. Use brackets around a negative replacement.
3. Calculate powers before multiplication, then finish the additions and subtractions.

Key terms & definitions

Term
A part of an expression separated by addition or subtraction.

Coefficient
The numerical factor multiplying a variable.

Variable
A symbol representing a value.

Key ideas

• A variable stands for a number that may change. A coefficient multiplies a variable. A constant has no variable.

• An expression has no equals sign. An equation states that two expressions are equal.

• Substitution means replacing every occurrence of a variable with its value. Put negative values in brackets.

• Follow the order of operations. In $3x^2$, square x before multiplying by 3.

• Use algebra to describe a situation: three euro per item and a fixed fee of two euro gives $C = 3n + 2$.

Quick reminder

• An expression has no equals sign; an equation has one.

• Replace every occurrence of the variable.

• Use brackets for negative substitutions.

Worked examples

Example 01 · Start here: what does $3x$ mean?

Find $3x + 2$ when $x = 4$.

Why this method?

$3x$ means 3 multiplied by x. Replace x by 4, then multiply before adding.

$3x + 2 = 3(4) + 2$	**Step 1** Replace the letter x with 4. The brackets show multiplication.
$= 12 + 2$	**Step 2** Multiply 3 by 4 first.
$= 14$	**Step 3** Now add 2.

Example 02 · Substitute a negative value

Evaluate $2x^2 - 3x + 1$ when $x = -2$.

Purple follows the substituted value and the results calculated from it. Brackets keep each negative value together.

Why this method?

Evaluate means find the number the expression represents when x is -2. Brackets help us keep the negative replacement together.

$2x^2 - 3x + 1$	**Step 1** Start with the expression. We will replace every x by -2.
$2\,(-2)^2 - 3\,(-2) + 1$	**Step 2** Put -2 in brackets each time. The first 2 and the coefficient -3 stay in place.

$2(4) - 3(-2) + 1$	**Step 3** Square the bracket first : $(-2) \times (-2) = 4$.
$8 + 6 + 1$	**Step 4** Multiply : $2 \times 4 = 8$ and $-3 \times (-2) = +6$.
15	**Step 5** Add the three numbers. Evaluating ends with a numerical answer.

Example 03 · Substituting into a linear rule

A taxi charges €14 to start and €7 for each km. What is the cost of a journey of 10 km?

Why this method?

Replace every variable with its given value, using brackets for negative values. Then follow the order of operations.

$7 \times 10 = 70$	**Step 1** Multiply the rate by the number
$70 + 14 = 84$	**Step 2** Add the starting amount

Think about it: Before moving on: why is the starting amount added after the multiplication, not before it?

Example 04 · Substituting into a two-variable rule

A shop sells pens for €6 each and notebooks for €8 each. What do 8 pens and 4 notebooks cost altogether?

Why this method?

Replace every variable with its given value, using brackets for negative values. Then follow the order of operations.

$6 \times 8 = 48$	**Step 1** Cost of the pens
$8 \times 4 = 32$	**Step 2** Cost of the notebooks
$48 + 32 = 80$	**Step 3** Add the two costs

Think about it: Before moving on: why does each price have to be multiplied by the number of its own items?

Example 05 · A negative value that must be squared

Find $2a^2 - 3b$ when $a = -2$ and $b = 5$.

Why this method?

The square applies to a, including its negative sign. Put the replacement value in brackets.

$2(-2)^2 - 3(5)$	**Step 1** Replace each letter. Do not square the coefficient 2.
$= 2(4) - 15$	**Step 2** Negative 2 multiplied by negative 2 is positive 4.
$= 8 - 15$	**Step 3** Multiply before subtracting.
$= -7$	**Step 4** Subtract 15 from 8.

Example 06 · Substitute a negative number

Evaluate $2a^2 - 3b$ for $a = -2$ and $b = 5$.

$2(-2)^2 - 3(5)$	**Step 1** Use brackets around the negative input.
$2(4) - 15 = -7$	**Final step** Square before multiplying and subtracting.

Common mistake

Dropping the brackets when substituting a negative value. If $x = -2$, then $x^2 = (-2)^2 = 4$, not -4. In contrast, $-x^2 = -4$ because the minus is outside the square.

Exam technique

Write the substitution line before calculating.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

A plumber charges €30 call-out plus €45 per hour. What is the charge for 3 hours?

Question 2

Adult tickets cost €6 and child tickets cost €4. How much do 2 adult tickets and 3 child tickets cost?

Question 3 · Build confidence

Find $3x + 2$ when $x = 5$.

Question 4 · Apply or explain

A learner says $2a^2 = 36$ when $a = -3$. Explain the mistake.

Before you move on

Try these without looking at a worked example. Revisit any part that is still difficult.

- Explain what a coefficient means.
- Substitute a negative value using brackets.
- Square the value before multiplying by a coefficient.

Watch an explanation

Corbettmaths — Algebra: substitution

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

165

Question 2

24

Question 3

17.

Multiply 3 by 5, then add 2.

Question 4

$2(-3)^2 = 2(9) = 18$.

Only a is squared; the coefficient 2 is outside the square.

Open the full chapter online

Simplifying, expanding and dividing

Leaving Certificate · Ordinary Level

Understand the idea

How to start

A term includes its sign. In $4x - 3 + 2x$, the terms are $4x$, -3 and $+2x$.

1. Simplify means write an equal expression with fewer unnecessary parts.
2. Expand means multiply out brackets. Every term inside must be multiplied.
3. Keep signs with terms when moving like terms beside one another.

Key terms & definitions

Like terms
Terms with the same variable part and powers.

Expand
Remove brackets by distributing multiplication.

Polynomial
A sum of terms with non-negative whole-number powers of the variables.

Key ideas

• Collect only like terms: their variable parts and powers must match.

• Distribute a factor to every term in a bracket. A minus sign before a bracket changes every sign inside.

• For two brackets, multiply every term in the first by every term in the second. Collect like terms afterwards.

Quick reminder

• Collect only like terms.

• Multiply every term when expanding brackets.

• Check division by multiplying the quotient and divisor.

Worked examples

Example 01 · Start here: collect just two like terms

Simplify $6x + 4x$.

Why this method?

Both terms count the same type of quantity: x. Add their coefficients and keep x.

$6x + 4x = (6 + 4)x$	**Step 1** Six lots of x and four lots of x make ten lots of x.
$= 10x$	**Step 2** The letter part stays x; we have not multiplied x by x.

Example 02 · Collecting like terms

Simplify $9x + 12 + 9x - 10$. What is the constant term?

Blue follows the x terms. Purple follows the constant terms, including their signs.

Why this method?

Identify the terms, brackets or divisor first. Each written line should show how the expression changes without changing its value.

$9x + 12 + 9x - 10$	**Step 1** Start with the whole expression, keeping each sign with its term.
$9x + 9x + 12 - 10$	**Step 2** Put the x terms together and the number terms together.
$18x + 2$	**Step 3** Nine x plus nine x is eighteen x. Twelve minus ten is two.
The constant term is 2.	**Step 4** The question also asks for the term with no x.

Think about it: Before moving on: why can you add the x terms together but not add an x term to a number?

Example 03 · Expanding one bracket

Expand $6(x+9)$. What is the constant term?

Amber follows the outside multiplier. It must multiply both terms inside the bracket.

Why this method?

Identify the terms, brackets or divisor first. Each written line should show how the expression changes without changing its value.

$6\,(x+9)$	**Step 1** Six multiplies everything inside the bracket.
$6\times x+6\times 9$	**Step 2** Multiply each term by 6, including the number 9.
$6x+54$	**Step 3** Write the complete expanded expression.
The constant term is 54.	**Step 4** The number term has no variable.

Think about it: Before moving on: why does the number outside have to multiply the number inside the bracket as well as the x?

Example 04 · Start here: expand two simple brackets

Expand $(x+2)(x+3)$.

Blue follows the products made by x. Purple follows the products made by $+2$. Collect only the like terms.

Why this method?

Every term in one bracket must multiply every term in the other. Write the four products so none is missed.

$x(x+3)=x^2+3x$	**Step 1** The first x multiplies every term in the second bracket.
$2(x+3)=2x+6$	**Step 2** The $+2$ also multiplies every term in the second bracket.
$x^2+3x\ +2x+6$	**Step 3** Put all four products together before collecting terms.

$x^2 + 5x + 6$	**Step 4** Only $3x$ and $2x$ are like terms, so combine them.

Example 05 · Subtract a whole bracket

Simplify $(3x + 5) - (x + 2)$.

Why this method?

Subtract every term of the second bracket. The minus applies to the whole bracket.

$3x + 5 - x - 2$	**Step 1** Change both signs in the second bracket.
$(3x - x) + (5 - 2)$	**Step 2** Group only like terms.
$2x + 3$	**Step 3** Combine the x terms and the numbers separately.

Example 06 · Expand and reduce

Simplify $(x + 4)(x - 3) - 2x$.

$x^2 - 3x + 4x - 12 - 2x$	**Step 1** Multiply each term in the first bracket by both terms in the second.
$x^2 - x - 12$	**Final step** Combine the x terms, keeping the squared term separate.

Common mistake

Combining terms with different letter parts. $3x + 2x^2$ cannot become $5x^3$. Only like terms combine: $3x + 2x = 5x$.

Exam technique

Set out long division in powers of x and include a zero coefficient for any missing power.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Simplify $2x + 9 + 4x - 3$. What is the constant term?

Question 2

Expand $9(x + 6)$. What is the constant term?

Question 3 · **Build confidence**

Simplify $9a - 2a + 5$.

Question 4 · **Apply or explain**

A learner writes $3(x + 4) = 3x + 4$. Correct it and explain.

Before you move on

Try these without looking at a worked example. Revisit any part that is still difficult.

- Collect like terms without changing their letter part.
- Multiply every term when expanding a bracket.
- Explain why x and x squared are not like terms.

Watch an explanation

Cognito — Collecting Like Terms

Cognito — Expanding Double Brackets

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

6

Question 2

54

Question 3

$7a + 5$.

Collect the a terms. The constant remains separate.

Question 4

$3(x + 4) = 3x + 12$.

Multiply both x and 4 by 3.

Open the full chapter online

Factorising expressions and quadratics

Leaving Certificate · Ordinary Level

Understand the idea

How to start

Factorise means write an expression as a multiplication. It reverses expanding brackets.

1. Look for a factor shared by every term before trying another method.
2. For a quadratic, match the actual terms to $ax^2 + bx + c$ and identify the signed numbers.
3. For the guide-number method, the pair must pass both tests: correct product and correct sum.

Key terms & definitions

Factor
An expression multiplied by another to make a product.

Quadratic
An expression whose highest power of its variable is 2.

Common factor
A factor present in every term.

Key ideas

• Factorising writes an expression as a product. Look for a common factor first.

• A difference of squares factors as $a^2 - b^2 = (a - b)(a + b)$.

• For a quadratic $ax^2 + bx + c$, the guide number is $a \times c$. Keep the signs of a, b and c.

• Find two numbers that multiply to the guide number and add to b. Split bx using these two numbers, factor each pair, then take out the shared bracket.

• Factorising is not solving. Only set each factor to zero when the product is equal to zero.

Quick reminder

• Check for a common factor first.

• Guide number: $a \times c$. Pair: product ac, sum b.

• Split the middle term, group, then factor the shared bracket.

Worked examples

Example 01 · Start here: taking out a common factor

Factorise $18 - 6y$.

Blue follows the first term, purple follows the second term, and amber follows the common factor. The words and equations explain every step too.

Why this method?

Find the largest factor shared by both terms. Divide each whole term by it to find what belongs inside the bracket. Expand at the end to check.

$18 \; -6y$	**Step 1** The first term is 18. The second term is $-6y$, including its minus sign.
$\text{HCF} = 6$	**Step 2** The factors of 18 are $1, 2, 3, 6, 9, 18$. The positive factors of 6 are $1, 2, 3, 6$. The largest shared factor is 6.
$\frac{18}{6} = 3$	**Step 3** Divide the first term by 6. This gives the first term inside the bracket.
$\frac{-6y}{6} = -y$	**Step 4** Divide the second whole term by 6. The minus sign stays, giving $-y$.
$6 \; (\; 3 \; -y \;)$	**Step 5** Put the common factor outside the bracket. Keep both results inside it.
$6 \; (\; 3 \; -y \;) = 18 \; -6y$	**Step 6** Expand to check : 6 multiplies every term inside the bracket, giving the original expression.

Example 02 · Guide number : x^2 has coefficient 1

Factorise $x^2 + 5x + 6$.

First identify a, b and c

$x^2 \quad +5x \quad +6$ This question

$ax^2 \quad +bx \quad +c$ Match the form

$a = 1$ — the number multiplying x^2.

$b = 5$ — the number multiplying x, including its sign.

$c = 6$ — the number on its own, including its sign.

x^2 means $1x^2$, so $a = 1$.

Amber marks the identical whole bracket in both groups. That whole bracket is the shared factor.

Guide number : $1 \times (6) = 6$. Target sum : 5.

Chosen pair : $(2) \times (3) = 6;\ 2 + (3) = 5$.

Why this method?

Check for a common factor first. The guide number is $a \times c$. Find two signed numbers that multiply to $a \times c$ and add to b, then split the middle term and group.

$GN = 1 \times 6 = 6$	**Step 1** Multiply the coefficient of x^2 by the constant term.
$2 \times 3 = 6, \quad 2 + 3 = 5$	**Step 2** Choose the factor pair whose sum is the coefficient of x.
$x^2 + 2x + 3x + 6$	**Step 3** Split $5x$ into $2x + 3x$.
$x\,(x + 2) + 3\,(x + 2)$	**Step 4** Factor the first pair and the second pair. The same bracket appears twice.
$(x + 3)(x + 2)$	**Step 5** Take out the shared bracket $(x + 2)$.
$x^2 + 2x + 3x + 6 = x^2 + 5x + 6$	**Step 6** Expand to check.

Example 03 · Guide number: coefficient greater than 1

Factorise $3x^2 + 10x + 8$.

First identify a, b and c

$3x^2$ $+10x$ $+8$ This question

ax^2 $+bx$ $+c$ Match the form

$a = 3$ — the number multiplying x^2.

$b = 10$ — the number multiplying x, including its sign.

$c = 8$ — the number on its own, including its sign.

Amber marks the identical whole bracket in both groups. That whole bracket is the shared factor.

Guide number : $3 \times (8) = 24$**. Target sum** : 10**.**

Chosen pair : $(6) \times (4) = 24$; $6 + (4) = 10$.

Why this method?

Check for a common factor first. The guide number is $a \times c$. Find two signed numbers that multiply to $a \times c$ and add to b, then split the middle term and group.

$GN = 3 \times 8 = 24$	**Step 1** The guide number is 24. The required sum is 10.
$1 \times 24 = 24, \quad 1 + 24 = 25$	**Step 2** Try factor pairs of 24. This pair has the wrong sum.
$2 \times 12 = 24, \quad 2 + 12 = 14$	**Step 3** The product is correct, but the sum is still wrong.
$6 \times 4 = 24, \quad 6 + 4 = 10$	**Step 4** Choose 6 and 4 : both tests now work.
$3x^2 + 6x + 4x + 8$	**Step 5** Split $10x$ into $6x + 4x$.
$3x\,(x + 2) + 4\,(x + 2)$	**Step 6** Factor each pair. Both groups contain $(x + 2)$.
$(3x + 4)(x + 2)$	**Step 7** Factor out the shared bracket.

$3x^2 + 6x + 4x + 8 = 3x^2 + 10x + 8$	**Step 8** Expand to check.

Example 04 · A common factor first

Factorise $6x^2 - 24$.

Why this method?

Look for a common factor first. Then choose a factorisation pattern and check it by expanding.

$6(x^2 - 4)$	**Step 1** Take out the highest common factor
$6(x - 2)(x + 2)$	**Step 2** Use difference of squares
$6(x^2 - 4) = 6x^2 - 24$	**Step 3** Expand the brackets to check that every coefficient agrees with the original expression.

Example 05 · Use the guide number

Factorise $6x^2 + 11x + 3$.

$6 \times 3 = 18; \quad 9 + 2 = 11$	**Step 1** 9 and 2 have product 18 and sum 11.
$6x^2 + 9x + 2x + 3$	**Step 2** Split the middle term.
$3x(2x + 3) + 1(2x + 3)$	**Step 3** Factor each pair.
$(3x + 1)(2x + 3)$	**Final step** Take out the common bracket.

Common mistake

Choosing a pair that has the correct product but the wrong sum. For $x^2 + 5x + 6$, 1 and 6 multiply to 6 but add to 7. Use 2 and 3: $(x + 2)(x + 3)$.

Exam technique

Expand your factors to check all three coefficients.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Factorise $5x^2 + 11x + 2$.

A. $(x + 1)(x + 2)$

B. $(5x + 1)(x + 2)$

C. $(5x - 1)(x - 2)$

D. $(5x + 2)(x + 1)$

Question 2

Factorise fully : $4x^2 - 15x - 4$.

A. $(4x + 1)(x - 4)$

B. $(4x - 1)(x + 4)$

C. $(4x - 1)(x - 4)$

D. $(x + 1)(x - 4)$

Watch an explanation

Whiteboard Math — Why factorising works

Cognito — Factorising Single Brackets

Cognito — Factorising Quadratic Equations (a = 1)

Cognito — Factorising Quadratic Equations (a > 1)

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

$(5x + 1)(x + 2)$

Correct : $5x \times 2 + 1 \times x = 11x$, and $1 \times 2 = 2$.

Question 2

$(4x + 1)(x - 4)$

Correct.

Open the full chapter online

Algebraic fractions

Leaving Certificate · Ordinary Level

Understand the idea

How to start

The letters do not change the fraction rules. The whole numerator must stay above its fraction bar.

1. Check which values would make any original denominator zero.
2. For adding or subtracting, make the denominators match by multiplying top and bottom by the same factor.
3. Combine the numerators. For subtraction, bracket the whole second numerator.

Key terms & definitions

Restriction
A value excluded because it makes an original denominator zero.

Common denominator
A denominator shared by equivalent forms of the fractions.

Rational expression
A ratio of polynomials with a non-zero denominator.

Key ideas

• An algebraic fraction follows the same rules as a numerical fraction, but you must also state excluded values.

• A denominator cannot be zero. Find restrictions from every original denominator before simplifying.

• Cancel common factors of the whole numerator and denominator. Do not cancel terms across addition.

• For addition or subtraction, choose a common denominator and scale each numerator by the missing factor.

• For subtraction, put the whole second numerator in brackets before collecting terms.

Quick reminder

• State excluded values from the original denominators.

• Use equivalent fractions with a common denominator.

• Cancel factors, never separate terms in a sum.

Worked examples

Example 01 · Start here: the same denominator

Simplify $\frac{x}{5} + \frac{2x}{5}$.

Why this method?

Start with equal denominators. Adding x fifths and $2x$ fifths gives $3x$ fifths, just as adding ordinary fractions does.

$\frac{x + 2x}{5}$	**Step 1** Both fractions are already in fifths, so the denominator can stay 5.
$\frac{3x}{5}$	**Step 2** The numerator contains like terms: one x plus two x makes three x.
$\frac{x}{5} + \frac{2x}{5} = \frac{3x}{5}$	**Step 3** We do not add the denominators. They describe the size of the parts we are counting.

Example 02 · Different denominators, one line at a time

Simplify $\frac{x + 3}{5} - \frac{2x - 1}{3}$.

Blue follows the first whole fraction and purple the second as their denominators are made equal. Each numerator stays above its own fraction bar.

Why this method?

The smallest common denominator is 15. Multiply top and bottom of the first fraction by 3 and of the second by 5. Each fraction keeps its value.

$\frac{x + 3}{5} - \frac{2x - 1}{3}$	**Step 1** Start with the original expression.

$\frac{3(x+3)}{15} - \frac{5(2x-1)}{15}$	**Step 2** The factors 3 and 5 multiply the whole numerators and stay above their bars.
$\frac{3x+9}{15} - \frac{10x-5}{15}$	**Step 3** Expand each numerator.
$\frac{3x+9-(10x-5)}{15}$	**Step 4** Combine the numerators. Bracket the whole second numerator.
$\frac{3x+9-10x+5}{15}$	**Step 5** Subtracting -5 gives $+5$.
$\frac{-7x+14}{15}$	**Step 6** Collect the x terms and the number terms.

Example 03 · Adding algebraic fractions with linear denominators

Write as a single fraction : $\frac{4}{(x+4)} + \frac{2}{(x+2)}$.

Blue follows the first whole fraction and purple the second as their denominators are made equal. Each numerator stays above its own fraction bar.

Why this method?

Find values that would make a denominator zero. Then build equivalent fractions with the same denominator before combining them.

$\frac{4}{x+4} + \frac{2}{x+2}$	**Step 1** The denominators differ. Use $(x+4)(x+2)$, keeping $x \neq -4$ and $x \neq -2$.
$\frac{4(x+2)}{(x+4)(x+2)} + \frac{2(x+4)}{(x+4)(x+2)}$	**Step 2** Multiply the whole first fraction by $\frac{(x+2)}{(x+2)}$, and the whole second fraction by $\frac{(x+4)}{(x+4)}$. Keep both multipliers in their numerators.
$\frac{4x+8}{(x+4)(x+2)} + \frac{2x+8}{(x+4)(x+2)}$	**Step 3** Expand each numerator while keeping the common denominator underneath.
$\frac{4x+8+2x+8}{(x+4)(x+2)}$	**Step 4** The denominators now match, so add the numerators.

$\frac{6x+16}{(x+4)(x+2)}$	**Step 5** Collect like terms on top. Keep the original restrictions $x \neq -4, -2$.

Think about it: Before moving on: why can the numerators not simply be added when the denominators are different?

Example 04 · Subtracting algebraic fractions with linear denominators

Write as a single fraction : $\frac{4}{(x+2)} - \frac{2}{(x+4)}$.

Blue follows the first whole fraction and purple the second as their denominators are made equal. Each numerator stays above its own fraction bar.

Why this method?

Find values that would make a denominator zero. Then build equivalent fractions with the same denominator before combining them.

$\frac{4}{x+2} - \frac{2}{x+4}$	**Step 1** Use $(x+2)(x+4)$ as the common denominator. Original restrictions : $x \neq -2, -4$.
$\frac{4(x+4)}{(x+2)(x+4)} - \frac{2(x+2)}{(x+2)(x+4)}$	**Step 2** Multiply top and bottom of each fraction by its missing factor.
$\frac{4x+16}{(x+2)(x+4)} - \frac{2x+4}{(x+2)(x+4)}$	**Step 3** Expand the numerators. Keep each denominator in place.
$\frac{4x+16-(2x+4)}{(x+2)(x+4)}$	**Step 4** Subtract the entire second numerator, so put it in brackets.
$\frac{4x+16-2x-4}{(x+2)(x+4)}$	**Step 5** The minus sign changes both signs in the second bracket.
$\frac{2x+12}{(x+2)(x+4)}$	**Step 6** Collect like terms. Keep $x \neq -2, -4$.

Think about it: Before moving on: why must the whole second numerator be subtracted, and not just its first term?

Example 05 · Solve with variable denominators

Solve $\frac{1}{x} + \frac{1}{x+1} = \frac{3}{2}$.

$x \neq 0, -1$	**Step 1** Exclude values that make a denominator zero.

$2(x+1)+2x=3x(x+1)$	**Step 2** Multiply every term by the LCD, $2x(x+1)$.
$3x^2-x-2=0$	**Step 3** Expand and collect on one side.
$(3x+2)(x-1)=0$	**Step 4** Factorise.
$x=1$ or $x=-\frac{2}{3}$	**Final step** Both satisfy the denominator restrictions and the original equation.

Common mistake

Cancelling a term that is joined by addition. $\frac{x+3}{x}$ cannot become 3. Only common factors cancel: $\frac{x(x+3)}{x}=x+3$ when $x\neq 0$.

Exam technique

Keep the original restrictions even if a factor cancels.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Write as a single fraction : $\frac{3}{(x+5)}+\frac{2}{(x+1)}$.

A. $\frac{5}{((x+5)(x+1))}$

B. $\frac{(5x+13)}{(x+6)}$

C. $\frac{(5x+13)}{((x+5)(x+1))}$

D. $\frac{(5x+17)}{((x+5)(x+1))}$

Question 2

Write as a single fraction : $\frac{3}{(x+2)}-\frac{2}{(x+5)}$.

A. $\frac{((1)x+11)}{((x+2)(x+5))}$

B. $\frac{(1)}{((x+2)(x+5))}$

C. $\frac{((5)x+19)}{((x+2)(x+5))}$

D. $\frac{((1)x-4)}{((x+2)(x+5))}$

Watch an explanation

Cognito — Simplifying Algebraic Fractions

Corbettmaths — Algebraic fractions: addition

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

$$\frac{(5x+13)}{((x+5)(x+1))}$$

Correct : $3(x+1)+2(x+5)=5x+13$.

Question 2

$$\frac{((1)x+11)}{((x+2)(x+5))}$$

Correct : $3(x+5)-2(x+2)=x+11$.

Open the full chapter online

Solving linear equations

Leaving Certificate · Ordinary Level

Understand the idea

How to start

Solve means find a value that makes the equation true. The equals sign means both sides have the same value.

1. Keep the equation balanced: do the same operation to both sides.
2. Undo the operation on the variable, writing the complete new equation each time.
3. Put your answer into the original equation to check both sides agree.

Key terms & definitions

Solution
A value that makes the original equation true.

Inverse operation
An operation that undoes another, such as subtracting to undo addition.

Identity
An equation true for every permitted value.

Key ideas

• An equation is a balance. Do the same operation to both sides to preserve equality.

• Undo addition or subtraction, then multiplication or division. Choose an order that simplifies the equation.

• Expand brackets and collect like terms before isolating the variable.

• For fractions, multiply every term on both sides by a common denominator.

• Check the solution by substituting it into the original equation. A fractional solution can be fully correct.

Quick reminder

• Goal: leave the variable on its own.

• Do the same operation to both sides.

• Check in the original equation.

Worked examples

Example 01 · Equality as a relationship

$14 + 9 = 18 + \square$

Why this method?

Our goal is to leave the variable on its own. Use inverse operations on both sides so that the equation stays balanced.

$14 + 9 = 18 + \square$	**Step 1** Write the complete equation
$23 = 18 + \square$	**Step 2** Evaluate the left side
$\square = 5$	**Step 3** Find the missing value
$14 + 9 = 23$ and $18 + 5 = 23$	**Step 4** Check both sides

Example 02 · One-step additive equations

$x + 8 = 21$

Purple marks the same subtraction on both sides of the equation.

Why this method?

Our goal is to leave the variable on its own. Use inverse operations on both sides so that the equation stays balanced.

$x + 8 = 21$	**Step 1** The x has 8 added to it. Undo this by subtracting 8 from both sides.

$x + 8 \ -8 \ = 21 \ -8$	**Step 2** Both sides must change by the same amount.
$x = 13$	**Step 3** The $+8$ and -8 cancel. Calculate $21 - 8$.
$13 + 8 = 21$	**Step 4** Check the result in the original equation.

Example 03 · One-step multiplicative equations

$6x = 42$

Purple marks division on both sides. Each whole side is divided by 6.

Why this method?

Our goal is to leave the variable on its own. Use inverse operations on both sides so that the equation stays balanced.

$6x = 42$	**Step 1** Six x means 6 multiplied by x.
$\frac{6x}{6} = \frac{42}{6}$	**Step 2** Undo multiplication by 6 by dividing both sides by 6.
$x = 7$	**Step 3** Six divided by six is one, leaving one x.
$6(7) = 42$	**Step 4** Check the answer in the original equation.

Example 04 · Two-step equations

$3x + 5 = 20$

Purple marks the matching operation on both sides: first subtract 5, then divide by 3.

Why this method?

Our goal is to leave the variable on its own. Use inverse operations on both sides so that the equation stays balanced.

$3x + 5 = 20$	**Step 1** The goal is x on its own.

$3x + 5 - 5 = 20 - 5$	**Step 2** Undo the added 5 by subtracting 5 from both sides.
$3x = 15$	**Step 3** Simplify both sides.
$\frac{3x}{3} = \frac{15}{3}$	**Step 4** Undo multiplication by 3 by dividing both sides by 3.
$x = 5$	**Step 5** Calculate the divisions.
$3(5) + 5 = 20$	**Step 6** Check: the original left-hand side is $15 + 5 = 20$.

Example 05 · Equations involving brackets

$3(x + 4) = 27$

Why this method?

Our goal is to leave the variable on its own. Use inverse operations on both sides so that the equation stays balanced.

$3(x + 4) = 27$	**Step 1** Write the original equation
$3x + 12 = 27$	**Step 2** Expand: multiply each term in the bracket by 3
$3x = 15$	**Step 3** Subtract 12 from both sides to remove that term while keeping the equation balanced.
$x = 5$	**Step 4** Divide every term on both sides by 3 so that the coefficient of the variable becomes 1.
$3(5 + 4) = 3 \times 9 = 27$ □	**Step 5** Put the answer back into the original information and check that all the stated conditions are satisfied.

Example 06 · Variables on both sides

$5x + 3 = 2x + 18$

Why this method?

Our goal is to leave the variable on its own. Use inverse operations on both sides so that the equation stays balanced.

$5x + 3 = 2x + 18$	**Step 1** Write the original equation
$3x + 3 = 18$	**Step 2** Subtract $2x$ from both sides to remove that term while keeping the equation balanced.
$3x = 15$	**Step 3** Subtract 3 from both sides to remove that term while keeping the equation balanced.
$x = 5$	**Step 4** Divide every term on both sides by 3 so that the coefficient of the variable becomes 1.
$5(5) + 3 = 28$ and $2(5) + 18 = 28$ ▯	**Step 5** Put the answer back into the original information and check that all the stated conditions are satisfied.

Example 07 · Turn a short problem into an equation

A taxi charges a fixed €5 plus €2 per kilometre. The fare is €29. How far was the journey?

Why this method?

Call the unknown distance x. Write the fixed charge plus the distance charge before solving.

$5 + 2x = 29$	**Step 1** Two euro per kilometre gives $2x$ euro for x kilometres.
$2x = 24$	**Step 2** Subtract the fixed €5 from both sides.
$x = 12$	**Step 3** Divide both sides by 2.

$5 + 2(12) = 29$	**Step 4** Check the fare. The distance is 12 km.

Example 08 · Multiply both sides by the LCD

Solve $\frac{x+1}{3} - \frac{x}{2} = 1$.

$6\left(\frac{x+1}{3} - \frac{x}{2}\right) = 6(1)$	**Step 1** Multiply both sides by 6, applying it to each term.
$2(x+1) - 3x = 6$	**Step 2** Cancel the denominators.
$2 - x = 6$	**Step 3** Expand and collect.
$x = -4$	**Final step** Subtract 2 and divide by -1.

Common mistake

Dividing only one term when dividing an equation. From $2x + 6 = 10$, dividing both sides by 2 gives $x + 3 = 5$, not $x + 6 = 5$. Every term must be divided.

Exam technique

Keep an equals sign on each working line; show both sides changing together.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

$9 \times 3 = 20 + \square$. Find $\square$.

Question 2

$23 = k + 9$. Find k.

Before you move on

Try these without looking at a worked example. Revisit any part that is still difficult.

• Write an equation from a short word problem.

• Apply the same operation to both sides.

• Check the answer in the original equation.

Watch an explanation

Cognito — Algebraic Equations (2 Operations & 1 Subject)

Cognito — Algebraic Equations (2 Subjects)

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

7

Question 2

14

Open the full chapter online

Simultaneous linear equations

Leaving Certificate · Ordinary Level

Understand the idea

How to start

We need one x-value and one y-value that work in both equations at the same time.

1. Label equations A and B, and line up the x terms, y terms and right-hand sides.
2. Choose the variable to remove. For addition, make its terms equal in size and opposite in sign.
3. Solve for the remaining variable, substitute back and check both original equations.

Key terms & definitions

Elimination
Combining equations to remove a variable.

Substitution method
Replacing a variable using an equivalent expression from another equation.

Key ideas

• A simultaneous solution is a pair of values that makes both original equations true. We must find both values, then check both equations.

• Label the equations A and B. Write x terms under x terms, y terms under y terms and the constants under the constants on the right of the equals signs.

• To eliminate a variable by addition, make its two terms the same size but opposite in sign. Their sum is zero: for example, $+2x + (-2x) = 0$.

• If a multiplier is needed, multiply every term in that equation, including the right-hand side. Choose a multiplier that creates the opposite term you need.

• Add the aligned equations. One variable disappears, leaving a one-variable equation you can solve.

• Substitute that value into one original equation to find the second variable. State the pair and check it in both original equations.

• You can also subtract equations with equal coefficients. We use addition with opposite terms in the early examples so the cancellation is easy to see.

• On a graph, the solution is the intersection of two lines: the same x-value gives the same y-value on both. Only after this is clear do we check a third line.

Quick reminder

• Choose which variable to remove.

• For addition, make equal-sized opposite terms and add.

• Substitute back to find the second value; check both equations.

Worked examples

Example 01 · Opposite terms are already ready to cancel

Solve the simultaneous equations $x + y = 7$ and $x - y = 1$.

Blue follows x terms and purple follows y terms in the aligned equations. Crossed-out terms add to zero.

Why this method?

We will remove y. One equation has $+y$ and the other has $-y$, so adding them removes y without any multiplication.

$x \quad +y \quad = \quad 7$ A $x \quad -y \quad = \quad 1$ B	**Step 1** Label and align the original equations. The y terms already have opposite signs.
$x \quad \cancel{+y} \quad = \quad 7$ A $x \quad \cancel{-y} \quad = \quad 1$ B $2x \quad = \quad 8$ $A + B$	**Step 2** Add A and B. The crossed terms sum to zero $: +y - y = 0$. Add the x terms and the right-hand sides too.
$x = 4$	**Step 3** Divide both sides by 2 because $2x$ means two lots of x.
$4 + y = 7$	**Step 4** Substitute $x = 4$ into original equation A. Now the only unknown is y.
$y = 3$	**Step 5** Subtract 4 from both sides to find y.
$4 + 3 = 7$ and $4 - 3 = 1$. Therefore $(x, y) = (4, 3)$.	**Step 6** Check the pair in both original equations. Both equalities must be true.

Example 02 · Multiply one equation to remove x

Solve $2x + y = 8$ and $x + 2y = 7$.

Blue follows x terms and purple follows y terms in the aligned equations. Crossed-out terms add to zero.

Why this method?

We will remove x. A contains $+2x$. B contains $+x$, so multiplying B by -2 creates $-2x$. Then adding the equations makes the x terms cancel.

$$2x + y = 8 \quad \text{A}$$
$$x + 2y = 7 \quad \text{B}$$

Step 1
Label the equations. Compare $+2x$ in A with $+x$ in B: their sizes are not yet equal.

$$-2(x + 2y) = -2(7),$$
so $-2x - 4y = -14$.

Step 2
Keep A unchanged. Multiply every term of B by -2, including the 7 on the right, to make $-2x$.

$$\begin{array}{rrcll} \cancel{2x} & +y & = & 8 & \text{A unchanged} \\ \cancel{-2x} & -4y & = & -14 & B \times (-2) \\ \hline & -3y & = & -6 & \text{Add} \end{array}$$

Step 3
Add A and $B \times (-2)$. The x terms cancel because $+2x - 2x = 0$. For y, $+y - 4y = -3y$.

$$y = 2$$

Step 4
Divide both sides by -3. A negative divided by a negative is positive.

$$x + 2(2) = 7$$

Step 5
Substitute $y = 2$ into original equation B, which has just one x.

$$x + 4 = 7$$

Step 6
Simplify, then subtract 4 from both sides.

$$x = 3,$$
so $(x, y) = (3, 2)$.

Step 7
Find x and state the pair in x, y order.

$$2(3) + 2 = 8$$
and $3 + 2(2) = 7$.

Step 8
Check A and B using both values.

Example 03 · When substitution is simpler

Solve $x = y + 1$ and $2x + y = 11$.

Why this method?

A already tells us what x equals. Replace x in B with the whole expression $y + 1$. This gives one equation containing only y.

$x = y + 1$ A $2x + y = 11$ B	**Step 1** Label the equations so it is clear which expression we will use.
$2(y + 1) + y = 11$	**Step 2** Replace the x in B with a bracket containing $y + 1$. The 2 multiplies that whole bracket.
$2y + 2 + y = 11$	**Step 3** Expand the bracket and collect the y terms.
$3y = 9$	**Step 4** Subtract 2 from both sides.
$y = 3$	**Step 5** Divide both sides by 3 to find y.
$x = 3 + 1 = 4$	**Step 6** Use A to find x from the y-value we have just found.
$4 = 3 + 1$ and $2(4) + 3 = 11$. Therefore $(x, y) = (4, 3)$.	**Step 7** Check the original equations.

Example 04 · Multiply both equations to remove y

Solve $5x + 3y = 67$ and $2x + 4y = 52$.

Why this method?

We will remove y. The coefficients are 3 and 4; their lowest common multiple is 12. Multiply A by 4 to get $+12y$ and B by -3 to get $-12y$. Then add.

$5x + 3y = 67$ A $2x + 4y = 52$ B	**Step 1** Label the original equations and line up the like terms.
$20x + 12y = 268$	**Step 2** Multiply every term of A by 4. We choose 4 because $3y \times 4$ becomes $+12y$.
$-6x - 12y = -156$	**Step 3** Multiply every term of B by -3. We choose -3 because $4y \times (-3)$ becomes $-12y$, the opposite of $+12y$.
$20x + 12y = 268$ $A \times 4$ $-6x - 12y = -156$ $B \times (-3)$ $14x = 112$ Add	**Step 4** Add the two new equations. $+12y - 12y = 0$, so only x remains. On the right, $268 - 156 = 112$.
$x = 8$	**Step 5** Divide both sides by 14 to find one x.
$2(8) + 4y = 52$	**Step 6** Substitute $x = 8$ into original equation B. Using an original equation helps avoid carrying a multiplication error forward.
$16 + 4y = 52$	**Step 7** Calculate 2×8. The equation is now written with a numerical constant and one unknown.
$4y = 36$	**Step 8** Subtract 16 from both sides to remove the constant 16.
$y = 9$, so $(x, y) = (8, 9)$.	**Step 9** Divide both sides by 4 to find y. State both values; finding y completes the solution.

$5(8) + 3(9) = 67$
and $2(8) + 4(9) = 52$.

Step 10

Check the pair in both A and B.

Example 05 · A line and a quadratic together

Solve $y = x + 2$ and $y = x^2$.

$x^2 = x + 2$	**Step 1** Both expressions equal y.
$x^2 - x - 2 = (x - 2)(x + 1) = 0$	**Step 2** Put zero on one side and factor.
$x = 2$ or $x = -1$	**Step 3** Solve each factor.
$(2, 4)$ and $(-1, 1)$	**Final step** Substitute both x values into $y = x + 2$.

Common mistake

Stopping after finding x when the question asks for x and y. Substitute x into an original equation to find y, then check the pair in both original equations.

Exam technique

Check the pair in both original equations.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Solve $x + y = 9$ and $x - y = 3$.

Question 2

Solve $2x + y = 7$ and $x + y = 5$.

Watch an explanation

tecmath — Simultaneous equations: another explanation

Cognito — Solving Simultaneous Equations 2 - Elimination Method

Cognito — Solving Simultaneous Equations 3 - Substitution Method

Cognito — Solving Simultaneous Equations 1 - Using Graphs

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

$x = 6,\ y = 3$. Add the equations to get $2x = 12$, then substitute into $x + y = 9$.

Check : $6 + 3 = 9$ and $6 - 3 = 3$.

Question 2

$x = 2,\ y = 3$. Subtract the second equation from the first, then substitute.

Check both original equations using the two values.

Open the full chapter online

Quadratic equations and roots

Leaving Certificate · Ordinary Level

Understand the idea

How to start

A quadratic contains an x^2 term. A root is an x-value that makes the expression equal to zero.

1. Move all terms to one side so the other side is zero.

2. Identify a, b and c with their signs. Try guide-number factorising when suitable factors are easy to find.

3. If factorising is not straightforward, use the quadratic formula. Check the required answer format.

Key terms & definitions

Root
A value making the polynomial equal to zero.

Discriminant
The expression $b^2 - 4ac$ used to determine the number of real roots.

Repeated root
One value occurring twice as a quadratic root.

Key ideas

• Write a quadratic equation in the form $ax^2 + bx + c = 0$, with $a \neq 0$.

• Try the guide-number method first when suitable factors are easy to find. After factorising an equation equal to zero, set each bracket equal to zero.

• The quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ applies whenever $a \neq 0$. A negative discriminant means there are no real solutions.

• The discriminant $b^2 - 4ac$ is positive for two distinct real roots, zero for a repeated root and negative for no real roots.

• For integer roots p and q, an equation is $(x - p)(x - q) = 0$. Expand it if required.

• Answers requested in surd form or to two decimal places often point towards the quadratic formula. It is a reliable choice if factors are not straightforward. The wording alone does not force the formula: for $x^2 = 2$, taking square roots is quicker.

Quick reminder

• Put the equation equal to zero first.

• Factorise when possible; set each factor equal to zero.

• Otherwise use the quadratic formula and consider both signs.

Worked examples

Example 01 · Guide number, then solve each bracket

Solve $x^2 - 5x - 14 = 0$.

First identify a, b and c

$x^2 \quad -5x \quad -14 \quad = 0$ This question

$ax^2 \quad +bx \quad +c \quad = 0$ Match the form

$a = 1$ — the number multiplying x^2.

$b = -5$ — the number multiplying x, including its sign.

$c = -14$ — the number on its own, including its sign.

x^2 means $1x^2$, so $a = 1$.

Guide number : $1 \times (-14) = -14$. **Target sum** : -5.

Chosen pair : $(-7) \times (2) = -14;\ -7 + (2) = -5$.

Why this method?

Check for a common factor first. The guide number is $a \times c$. Find two signed numbers that multiply to $a \times c$ and add to b, then split the middle term and group.

$\text{GN} = 1 \times (-14) = -14$	**Step 1** The required product is -14 and the required sum is -5.
$(-7)(2) = -14, \quad -7 + 2 = -5$	**Step 2** The signs matter : -7 and $+2$ pass both tests.
$x^2 - 7x + 2x - 14 = 0$	**Step 3** Split $-5x$ into $-7x + 2x$.
$x(x - 7) + 2(x - 7) = 0$	**Step 4** Factor the first pair and the second pair.
$(x + 2)(x - 7) = 0$	**Step 5** Take out the shared bracket $(x - 7)$.
$x + 2 = 0 \quad \text{or} \quad x - 7 = 0$	**Step 6** A product is zero only if at least one factor is zero.

$x = -2 \quad \text{or} \quad x = 7$	**Step 7** Solve each linear equation.
$(-2)^2 - 5(-2) - 14 = 0, \quad 7^2 - 5(7) - 14 = 0$	**Step 8** Check both answers in the original equation.

Example 02 · Build an equation from roots

Find a monic quadratic equation with roots 3 and -2.

Why this method?

Put the equation equal to zero. Start with factorising when possible; otherwise choose a method that can find all real roots.

$(x - 3)(x + 2) = 0$	**Step 1** Build the factors
$x^2 - x - 6 = 0$	**Step 2** Multiply every term in one bracket by every term in the other, then collect like terms.
$9 - 3 - 6 = 0$ and $4 + 2 - 6 = 0$.	**Step 3** Check both roots

Example 03 · Quadratic equations with integer roots

Solve $x^2 - 8x + 12 = 0$ by factorising. What is the larger root?

First identify a, b and c

$x^2 \quad -8x \quad +12 \quad = 0$ This question

$ax^2 \quad +bx \quad +c \quad = 0$ Match the form

$a = 1$ — the number multiplying x^2.

$b = -8$ — the number multiplying x, including its sign.

$c = 12$ — the number on its own, including its sign.

x^2 means $1x^2$, so $a = 1$.

Why this method?

Put the equation equal to zero. Start with factorising when possible; otherwise choose a method that can find all real roots.

$\text{GN} = 1 \times 12 = 12$	**Step 1** The coefficient of x is -8, so the pair must add to -8.
$(-6)(-2) = 12, \quad -6 - 2 = -8$	**Step 2** Both numbers must be negative.
$x^2 - 6x - 2x + 12 = 0$	**Step 3** Split the middle term.
$x(x - 6) - 2(x - 6) = 0$	**Step 4** Factor each pair.
$(x - 2)(x - 6) = 0$	**Step 5** Factor the common bracket.
$x = 2 \quad \text{or} \quad x = 6$	**Step 6** Set each bracket equal to zero. The larger root is 6.

Think about it: Before moving on: why must each bracket be set to zero, not the whole expression to a value?

Example 04 · Quadratic equations with surd solutions $(x^2 = n)$

Solve for x, giving exact answers : $x^2 = 2$.

Why this method?

Put the equation equal to zero. Start with factorising when possible; otherwise choose a method that can find all real roots.

$x = \pm\sqrt{2}$	**Step 1** Take the square root of both sides, remembering the negative root
$x = \pm\sqrt{2}$	**Step 2** Leave the answer as a surd, because the root is not a whole number

Think about it: Before moving on: why are there two solutions to $x^2 = n$ when n is positive?

Example 05 · Using the quadratic formula, one line at a time

Solve $x^2 - 5x + 3 = 0$, giving exact answers.

First identify a, b and c

$x^2 \quad -5x \quad +3 \quad = 0$ This question

$ax^2 \quad +bx \quad +c \quad = 0$ Match the form

$a = 1$ — the number multiplying x^2.

$b = -5$ — the number multiplying x, including its sign.

$c = 3$ — the number on its own, including its sign.

x^2 means $1x^2$, so $a = 1$.

Why this method?

The guide number is 3. No integer pair multiplies to 3 and adds to -5. Use the quadratic formula. We will give the exact surd answers first, then decimal answers.

$a = 1,\ b = -5,\ c = 3.$	**Step 1** Compare with $ax^2 + bx + c = 0$. Include the sign of each coefficient.
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**Step 2** Write the formula before substituting, so the whole numerator and denominator are clear.
$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(3)}}{2(1)}$	**Step 3** Substitute with brackets around the negative value of b.
$(-5)^2 - 4(1)(3) = 25 - 12 = 13$	**Step 4** Calculate the discriminant inside the root first.
$x = \frac{5 \pm \sqrt{13}}{2}$	**Step 5** Simplify the signs and denominator. The ± means we must consider both choices.
$x = \frac{5 + \sqrt{13}}{2}$ or $x = \frac{5 - \sqrt{13}}{2}$.	**Step 6** Write both exact roots. Do not round the surd when exact answers are requested.
$x \approx 4.30 \quad \text{or} \quad x \approx 0.70$	**Step 7** For two decimal places, calculate from the exact roots and round only at the end.

Example 06 · Use the quadratic formula when factoring is not convenient

Solve $2x^2 + x - 4 = 0$.

$a = 2, \quad b = 1, \quad c = -4$	**Step 1** Keep the minus sign with c.
$b^2 - 4ac = 1 - 4(2)(-4) = 33$	**Step 2** Calculate the discriminant.
$x = \frac{-1 \pm \sqrt{33}}{4}$	**Final step** Use both signs and divide the whole numerator by 4.

Common mistake

Dividing only the square-root part by $2a$ in the quadratic formula. The whole numerator is divided: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Keep both terms above the same fraction bar.

Exam technique

The guide-number method can be quicker when suitable factors are easy to find. The quadratic formula is always available for a quadratic equation. Keep surds exact, or round only the final answers to the requested decimal places.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Solve $\left(\frac{2}{4}\right)x^2 + \left(-\frac{8}{4}\right)x + \left(-\frac{6}{4}\right) = 0$ over the real numbers. Give both exact values; the radical need not be simplified.

A. $x = \frac{(8 - \sqrt{16})}{4}$ or $x = \frac{(8 + \sqrt{16})}{4}$

B. $x = \frac{(8 - \sqrt{112})}{2}$ or $x = \frac{(8 + \sqrt{112})}{2}$

C. $x = \frac{(8 - \sqrt{112})}{4}$ or $x = \frac{(8 + \sqrt{112})}{4}$

D. $x = \frac{(-8 - \sqrt{112})}{4}$ or $x = \frac{(-8 + \sqrt{112})}{4}$

Question 2

Solve $\left(\frac{1}{6}\right)x^2 + \left(-\frac{11}{6}\right)x + \left(\frac{133}{24}\right) = 0$ over the real numbers. Give every distinct solution exactly.

A. $x = \frac{11}{2} - \sqrt{3}$ or $x = \frac{11}{2} + \sqrt{3}$

B. $x = \frac{11}{2}$

C. All real x.

D. No real solutions.

Watch an explanation

Corbettmaths — Quadratics: solving (factorising)

Cognito — The Quadratic Formula

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

$x = \frac{(8 - \sqrt{112})}{4}$ or $x = \frac{(8 + \sqrt{112})}{4}$

Correct.

Question 2

No real solutions.

Correct.

Open the full chapter online

Inequalities and solution sets

Leaving Certificate · Ordinary Level

Understand the idea

How to start

The sign $<$ means less than. The sign $\leq$ means less than or equal to.

1. Read whether the endpoint is included.
2. Use the same operations on both sides. Reverse the sign if you multiply or divide by a negative number.
3. Check whether the answer needs all real numbers or only whole numbers.

Key terms & definitions

Strict inequality
A comparison using $<$ or $>$, excluding equality.

Interval
A continuous set of numbers between boundaries.

Boundary
An endpoint at which the truth of an inequality can change.

Key ideas

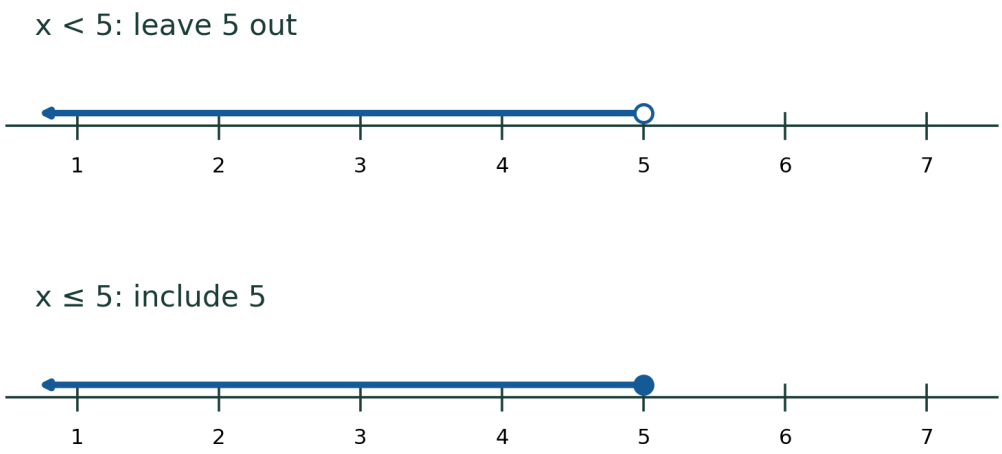
• Solve an inequality much like an equation, but reverse its sign when multiplying or dividing by a negative number.

• Use an open dot for a strict endpoint and a filled dot when the endpoint is included.

• State the domain. Integer solutions and real solutions are different sets.

• On a graph, $f(x) > g(x)$ where the graph of f is above the graph of g.

• For a strict inequality, crossings and touching points are excluded. For an inclusive inequality, include points where the graphs agree.



The arrow continues towards smaller numbers. An open endpoint excludes 5; a filled endpoint includes it. These sketches show part of the real number line.

Quick reminder

• Reverse the inequality when multiplying or dividing by a negative.

• Open endpoint: equality excluded. Filled endpoint: equality included.

• Check whether the domain is integers or real numbers.

Worked examples

Example 01 · Start here: one change to both sides

Solve $x + 3 < 8$.

Why this method?

Subtracting 3 undoes the addition. Subtract the same amount from both sides; subtraction does not reverse the inequality.

$x + 3 - 3 < 8 - 3$	**Step 1** Subtract 3 from both sides.
$x < 5$	**Step 2** Every real number below 5 works. The endpoint 5 is excluded.
$4 + 3 = 7 < 8$	**Step 3** Check an allowed value. At $x = 5$, the left side is 8, so the strict inequality would fail.

Example 02 · Linear inequalities: the largest whole solution

What is the largest whole number n that satisfies $7n + 1 < 85$?

Purple marks the same subtraction on both sides. Subtracting 1 leaves the inequality sign unchanged.

Why this method?

Isolate the variable or compare the heights of the graphs. Keep track of whether equality is included at the boundary.

$7n + 1 < 85$	**Step 1** We want n on its own. The sign means strictly less than.
$7n + 1 \color{#7c3aed}{-1} < 85 \color{#7c3aed}{-1}$	**Step 2** Subtract 1 from both sides.
$7n < 84$	**Step 3** Simplify the subtraction.
$n < 12$	**Step 4** Divide both sides by positive 7. The inequality sign stays the same.
$n = 11$	**Step 5** The question asks for the largest whole number below 12. Twelve itself is excluded.
$7(11) + 1 = 78 < 85$	**Step 6** Check 11. The next whole number gives $7(12) + 1 = 85$, which is not less than 85.

Think about it: Before moving on: why is the bound itself not a solution when the inequality is strict?

Example 03 · Divide by a positive number

Solve $2x + 1 \leq 9$.

Why this method?

Undo adding 1 first, then undo multiplying by 2. Dividing by positive 2 keeps the sign pointing the same way.

$2x \leq 8$	**Step 1** Subtract 1 from both sides.
$\frac{2x}{2} \leq \frac{8}{2}$	**Step 2** Divide both sides by positive 2.

$x \leq 4$	**Step 3** Include 4 because the original sign allows equality.

Example 04 · Why a negative divisor reverses the sign

Solve $-2x < 6$.

Why this method?

Division by a negative reverses the order of numbers. For example, $2 < 3$ becomes $-2 > -3$ after multiplication by -1.

$\frac{-2x}{-2} > \frac{6}{-2}$	**Step 1** Divide both sides by -2 and reverse $<$ to $>$.
$x > -3$	**Step 2** Every number greater than -3 works.
$-2(0) = 0 < 6$	**Step 3** Check $x = 0$. The boundary $x = -3$ gives $6 < 6$, which is false.

Example 05 · Reverse the inequality when dividing by a negative

Solve $7 - 3x \geq 1$.

$-3x \geq -6$	**Step 1** Subtract 7 from both sides.
$x \leq 2$	**Step 2** Divide by -3 and reverse the inequality.
Filled dot at 2; shade left.	**Final step** 2 is included, and all smaller real numbers work.

Common mistake

Forgetting to reverse the sign when multiplying or dividing both sides by a negative number. For example, $-2x < 6$ gives $x > -3$, not $x < -3$. Adding or subtracting does not reverse the sign.

Exam technique

Test one point in each region when reading a graphical inequality.

Try it yourself

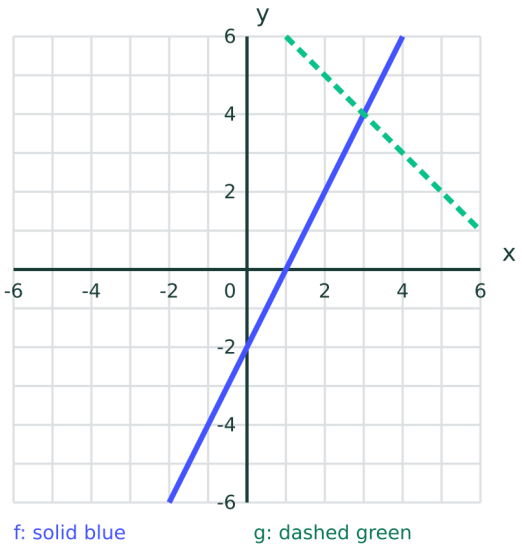
Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

What is the largest whole number n that satisfies $4n + 3 < 35$?

Question 2

The blue line is f(x) and the green line is g(x). Using the graph, find the solution set of $f(x) < g(x)$.



f: solid blue g: dashed green

Two straight lines drawn on axes marked from -6 to 6 on each side. The lines cross at one point.

A. $x < 3$

B. $y < 4$

C. $x \leq 3$

D. $x > 3$

Question 3

Solve $3x < 12$.

Question 4

Solve $-x \leq 5$.

Watch an explanation

Cognito — Algebra with Inequalities 1

Cognito — Introduction to Inequalities

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

7

Question 2

$x < 3$

Correct: the lines cross at $x = 3$, and the blue line is below the green line for $x < 3$.

Question 3

$x < 4$.

Divide both sides by positive 3; do not reverse the sign.

Question 4

$x \geq -5$.

Divide by -1 and reverse the inequality.

Open the full chapter online

Changing the subject of a formula

Leaving Certificate · Ordinary Level

Understand the idea

How to start

The subject is the letter on its own. Making t the subject means finishing with $t = \ldots$.

1. Circle the letter you need to isolate.
2. Undo operations in reverse order, doing the same to both sides.
3. Keep a whole sum in brackets or above the same fraction bar when dividing it.

Key terms & definitions

Subject
The variable written alone on one side of a formula.

Rearrange
Write an equivalent equation with a different structure.

Restriction
A condition needed for a division or other operation to be valid.

Key ideas

• The subject is the variable isolated on one side of a formula.

• Use inverse operations on both sides. Keep letters as quantities rather than trying to guess values.

• Clear denominators by multiplying every term. Use brackets when dividing a whole sum.

• If the required variable occurs more than once, collect those terms and factor it out.

• Check restrictions before dividing by an expression that could be zero.

Quick reminder

• Identify the variable that must be the subject.

• Undo addition / subtraction, then multiplication / division as needed.

• Keep a whole numerator together with a fraction bar or brackets.

Worked examples

Example 01 · Start here: undo one multiplication

Make w the subject of $A = lw$, where $l \neq 0$.

Why this method?

The letter w is multiplied by l. Divide both sides by l to leave w on its own.

$\frac{A}{l} = \frac{lw}{l}$	**Step 1** Divide the whole left side and the whole right side by l.
$w = \frac{A}{l}$	**Step 2** Write w on the left. We have not put numbers into the formula.

Example 02 · Start here: undo two operations

Make t the subject of $v = u + at$. Assume $a \neq 0$.

Why this method?

Work backwards through the operations around t: undo adding u, then undo multiplying by a.

$v - u = at$	**Step 1** Our goal is t on its own. Subtract u from both sides to remove the added term.
$\frac{v-u}{a} = t$	**Step 2** The remaining t is multiplied by a. Divide the whole left-hand side and the right-hand side by a.
$t = \frac{v-u}{a}$	**Step 3** Write the subject on the left if preferred. The numerator must stay together.

Example 03 · Keep the whole difference above the bar

Make x the subject of $y = 3x + 4$.

Why this method?

The last operation on x is adding 4, so undo that before dividing by 3.

$y - 4 = 3x$	**Step 1** Subtract 4 from both sides.
$\frac{y-4}{3} = x$	**Step 2** Divide the whole difference $y - 4$ by 3.
$x = \frac{y-4}{3}$	**Step 3** Keep both terms in the numerator. The 3 divides all of $y - 4$.

Example 04 · A formula with a square

Make r the subject of $A = \pi r^2$, where r is a non-negative radius.

Why this method?

First undo multiplication by π. Then undo the square by taking the positive square root because a radius cannot be negative.

$\frac{A}{\pi} = r^2$	**Step 1** Divide both sides by π.
$r = \sqrt{\frac{A}{\pi}}$	**Step 2** The square root applies to the whole fraction. Assume A is non-negative.

Example 05 · Make a denominator the subject

Make t the subject of $v = \frac{d}{t}$, with positive quantities.

$vt = d$	**Step 1** Multiply both sides by t.
$t = \frac{d}{v}$	**Final step** Divide both sides by the nonzero speed v.

Common mistake

Dividing only one part of a sum when changing the subject. From $y = 3x + 4$, subtract 4 first, then divide the whole difference: $x = \frac{y-4}{3}$, not $y - \frac{4}{3}$.

Exam technique

Check the rearranged formula using a simple numerical example.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

The formula is $y = a + bx$. Find x when $y = 31$, $a = 7$ and $b = 4$.

Question 2

The formula is $A = \frac{h(a+b)}{2}$. Find h when $A = 30$, $a = 4$ and $b = 8$.

Question 3

Make h the subject of $V = Ah$, with $A \neq 0$.

Question 4

Make x the subject of $y = 2x - 5$.

Before you move on

Try these without looking at a worked example. Revisit any part that is still difficult.

- Identify the letter that must be on its own.
- Undo operations in reverse order.
- Keep a whole sum or difference above the fraction bar.

Watch an explanation

Cognito — Rearranging Formulas (Subject Appears Once)

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

6

Question 2

5

Question 3

$h = \frac{V}{A}$.

Divide both sides by the area A.

Question 4

$x = \frac{y + 5}{2}$.

Add 5 to both sides, then divide the entire sum by 2.

Open the full chapter online

Patterns and nth terms

Leaving Certificate · Ordinary Level

Understand the idea

How to start

The term number tells you the position. The term value tells you the number at that position.

1. Write the position numbers above the terms, starting with 1 unless told otherwise.
2. Find the differences between neighbouring terms.
3. Test a proposed rule using more than one position.

Key terms & definitions

Sequence
An ordered list of terms.

First difference
The change from one term to the next.

Nth term
A rule for the term at position n.

Key ideas

• The position number n starts at 1 unless the question states otherwise.

• A constant first difference suggests a linear sequence. For first term a and difference d, $T_n = a + (n-1)d$.

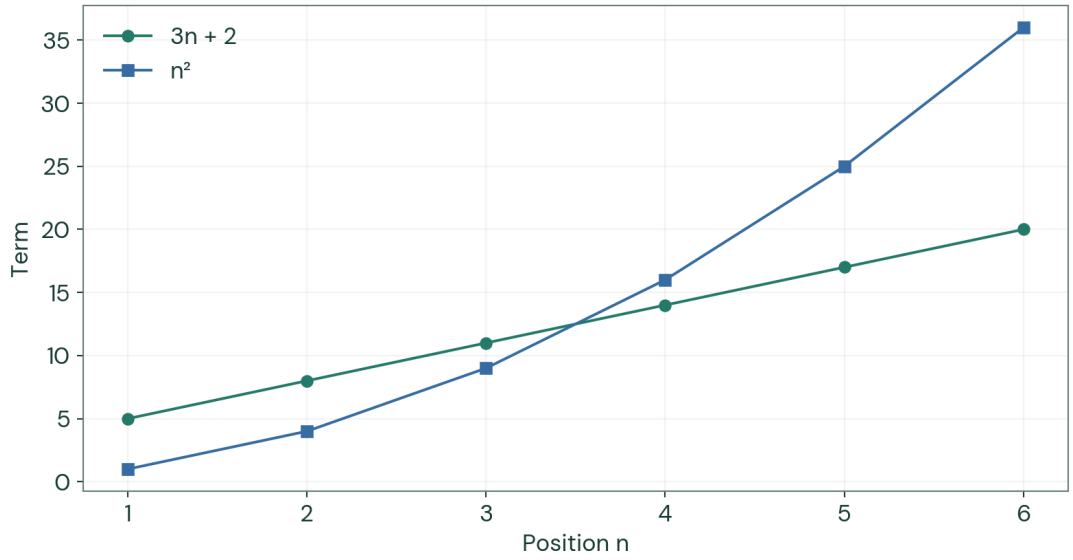
• A constant non-zero second difference suggests a quadratic sequence $T_n = an^2 + bn + c$; its second difference is $2a$.

• A constant ratio suggests a geometric pattern, such as repeated doubling or tripling.

• Check a proposed rule against several terms and the context. A short list alone does not prove a unique general rule.

• Arithmetic term: $T_n = a + (n-1)d$. Arithmetic sum: $S_n = \frac{n}{2}(2a + (n-1)d)$. Here a is the first term and d is the common difference.

• An arithmetic sequence adds the same amount each time. A geometric sequence multiplies by the same amount each time.



Linear pattern $3n + 2$ has constant first difference; quadratic pattern n^2 has constant second difference.

Quick reminder

• Write the term positions, starting at the stated n-value.

• Constant first difference → linear; constant non-zero second difference → quadratic.

• Check your rule against the original terms.

Worked examples

Example 01 · Continuing a numerical pattern

The pattern starts $3, 8, 13, 18, \ldots$ What is the next term?

Why this method?

Write the position numbers beside the terms. Check differences or ratios before choosing the form of the rule.

$8 - 3 = 5$	**Step 1** Find the change from the first term to the second.
$13 - 8 = 5, \quad 18 - 13 = 5$	**Step 2** Check the same change happens in the next two steps.

$18 + 5 = 23$	**Step 3** Continue the pattern by adding 5 to the last term shown.

Think about it: Before moving on: how can you check the common difference is right for every pair of neighbouring terms?

Example 02 · The term at any position

A pattern starts with 2 and goes up by 4 each time. What is term 10?

Position	1	2	3	4	5
Term	2	6	10	14	18

Why this method?

Write the position numbers beside the terms. Check differences or ratios before choosing the form of the rule.

$T_1 = 2$	**Step 1** The first term is 2. No increases have happened yet.
$10 - 1 = 9$	**Step 2** From position 1 to position 10 there are nine steps.
$9 \times 4 = 36$	**Step 3** Each step adds 4, so the total increase is 36.
$T_{10} = 2 + 36 = 38$	**Step 4** Add the increase to the first term.

Think about it: Before moving on: how many differences are added to reach term 10?

Example 03 · Finding a term of a linear pattern

The pattern starts $3, 9, 15, 21, \ldots$ What is term number 12?

Why this method?

Write the position numbers beside the terms. Check differences or ratios before choosing the form of the rule.

$9 - 3 = 6$	**Step 1** The pattern adds 6 each time.
$12 - 1 = 11$	**Step 2** Term 12 is eleven steps after term 1.

$11 \times 6 = 66$	**Step 3** Find the total increase across eleven steps.
$T_{12} = 3 + 66 = 69$	**Step 4** Start at the first term , 3, then add the increase.

Think about it: Before moving on: why is there one fewer difference than the term number?

Example 04 · Sum of an arithmetic series

Find the sum of the first 5 terms of $2, 5, 8, \ldots$

Why this method?

Look for a constant difference or ratio before choosing the matching rule.

$a = 2, \quad d = 3, \quad n = 5$	**Step 1** Identify the first term, difference and number of terms.
$S_5 = \frac{5}{2}(2(2) + (5 - 1)3)$	**Step 2** Substitute into the sum formula.
$S_5 = \frac{5}{2}(16) = 40$	**Step 3** Calculate. Check by adding $2 + 5 + 8 + 11 + 14$.

Example 05 · Find which term has a given value

The sequence is $5, 9, 13, 17, \ldots$. Which term is 81?

Why this method?

The term value and its position are different. First write a rule for the value, then solve for the position n.

$a = 5, \quad d = 4$	**Step 1** The first term is 5 and the sequence adds 4 each time.
$T_n = 5 + 4(n - 1) = 4n + 1$	**Step 2** There are $n - 1$ jumps from the first term to term n.
$4n + 1 = 81$	**Step 3** Set the term value equal to 81.

$4n = 80$	**Step 4** Subtract 1 from both sides.
$n = 20$	**Step 5** Divide by 4. Therefore 81 is the 20th term.

Example 06 · Find the sum, not the last term

Find the total of the first 12 terms of $8, 13, 18, \ldots$.

$T_{12} = 8 + (12 - 1)5 = 63$	**Step 1** Find the last term needed.
$S_{12} = \frac{12}{2}(8 + 63) = 426$	**Final step** Multiply the average of first and last terms by the number of terms.

Example 07 · Recognise a geometric sequence

Classify $81, 27, 9, 3, \ldots$ and find the next term.

$\frac{27}{81} = \frac{9}{27} = \frac{3}{9} = \frac{1}{3}$	**Step 1** Successive terms share a constant ratio.
$3 \times \frac{1}{3} = 1$	**Final step** It is geometric; multiply by one third again.

Common mistake

Starting at $n = 0$ when the question numbers the first term as 1. For $T_n = 3n + 2$, the first term is $3(1) + 2 = 5$, not 2.

Exam technique

Write a position row and a value row before finding a rule.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

The pattern starts $6, 11, 16, 21, \ldots$ What is the next term?

Question 2

A pattern starts with 8 and goes up by 3 each time. What is term 15?

Position	1	2	3	4
Term	8	11	14	17

Question 3 · Build confidence

For $7, 10, 13, \ldots$, write a term rule.

Question 4 · Apply or explain

Which term of $7, 10, 13, \ldots$ is 79?

Before you move on

Try these without looking at a worked example. Revisit any part that is still difficult.

- Identify the first term and common difference.
- Use n for the position of a term.
- Check a term rule against the first few terms.

Watch an explanation

Cognito — Finding the nth Term (Arithmetic Sequences)

Cognito — Finding the nth Term (Quadratic Sequences)

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

26

Question 2

50

Question 3

$T_n = 7 + 3(n-1) = 3n + 4.$

The first position is $n = 1$.

Question 4

$3n + 4 = 79$, so $n = 25$.

Solve for the position, rather than giving the term value again.

Open the full chapter online

Functions and graphs

Leaving Certificate · Ordinary Level

Understand the idea

How to start

$f(3)$ means the output of the rule f when the input is 3. It does not mean f multiplied by 3.

1. Decide whether you are finding an input, an output or a point on a graph.
2. For an output, replace x by the input and calculate.
3. On a graph, read the horizontal coordinate first, then the vertical coordinate.

Key terms & definitions

Domain
The set of allowed inputs.

Range
The set of outputs actually produced.

Intercept
A point at which a graph meets an axis.

Turning point
A point at which a curve changes from increasing to decreasing, or vice versa.

Key ideas

• A function assigns exactly one output to each input in its domain. $f(x)$ means the output for input x.

• The domain contains allowed inputs; the codomain is the stated set of possible outputs, and the range contains outputs actually reached.

• Make a table of values, label axes and choose consistent scales before plotting.

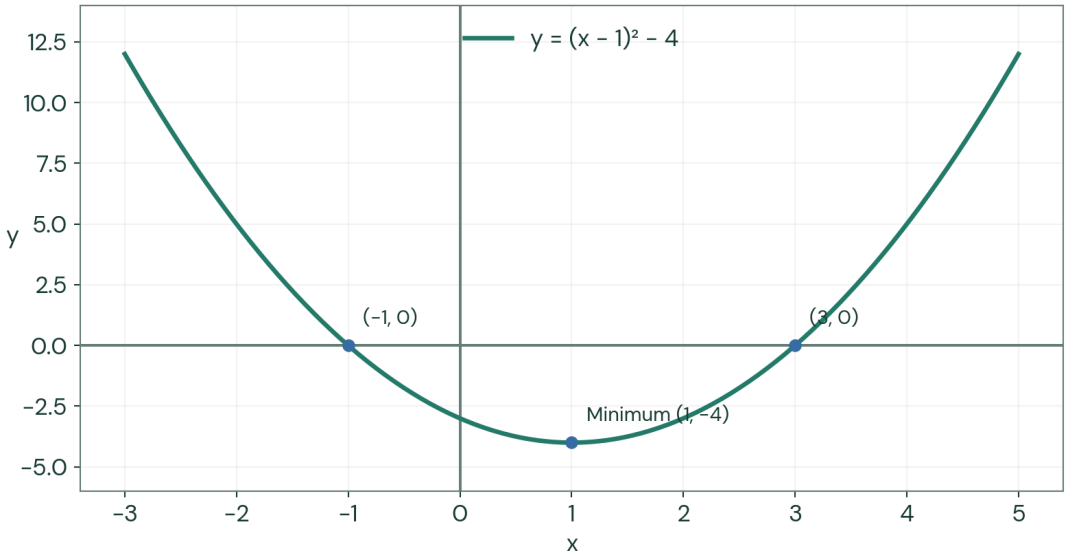
• A root is an x-value where the graph crosses or touches the x-axis. The y-intercept occurs when x is zero.

• An intersection solves $f(x) = g(x)$. A maximum or minimum describes an extreme output within the given domain.

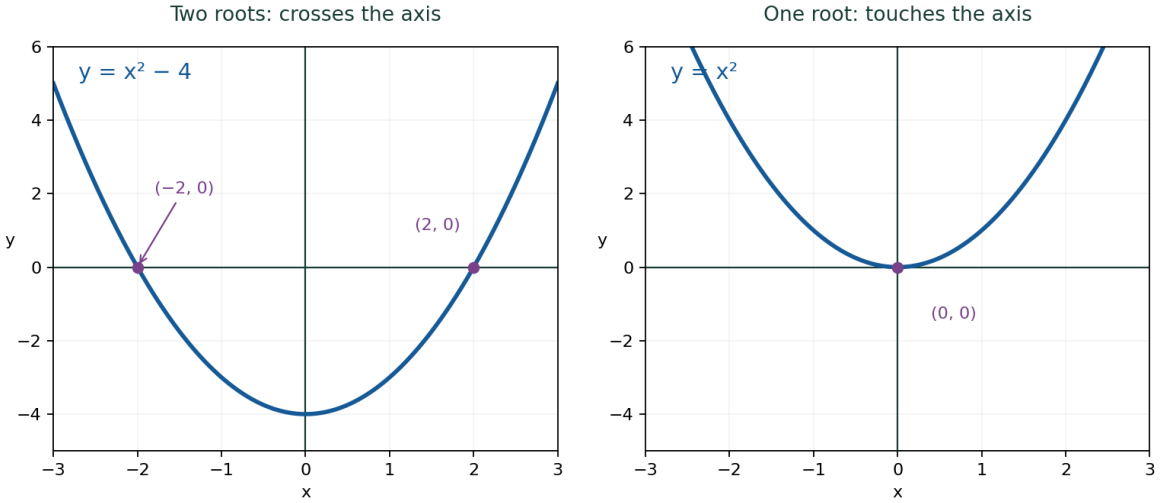
• For contextual graphs, interpret coordinates with units. A model may only make sense for non-negative or whole-number inputs.

• A composite function uses one output as the next input. $(f \circ g)(x) = f(g(x))$: do g first.

• For an informal limit, investigate the outputs as x approaches a value from both sides. The approached value need not equal the value of the function at that point.



The quadratic has roots -1 and 3 and a minimum at $(1, -4)$.



A root is an x-value for which y is zero. Crossing is not required: a graph can touch the x-axis at a root. The two panels use the same scales.

Quick reminder

• Input x first, output y second.

• Roots: where $y = 0$. Y-intercept: where $x = 0$.

• An intersection has the same coordinates on both graphs.

Worked examples

Example 01 · Evaluating a function

The function f is given by $f(x) = 3x + 4$. What is $f(3)$?

Purple follows the input 3 into the rule, then the multiplication result 9. The coefficient 3 stays in place.

Why this method?

Decide whether the input or the output is known. For graphs, label the axes and read the relevant coordinate in the right order.

$f(x) = 3x + 4$	**Step 1** The rule says multiply the input by 3, then add 4.
$f\ (3)\ = 3\ (3)\ + 4$	**Step 2** Replace x by the input 3. The 3 outside the brackets is the coefficient.
$f(3) = 9 + 4$	**Step 3** Multiply before adding.
$f(3) = 13$	**Step 4** Thirteen is the output when the input is 3.

Think about it: Before moving on: what does $f(3)$ mean, and how is it different from $3 \times f$?

Example 02 · Finding the input for a given output

The function f is given by $f(x) = 8x + 7$. For what value of x is $f(x) = 87$?

Why this method?

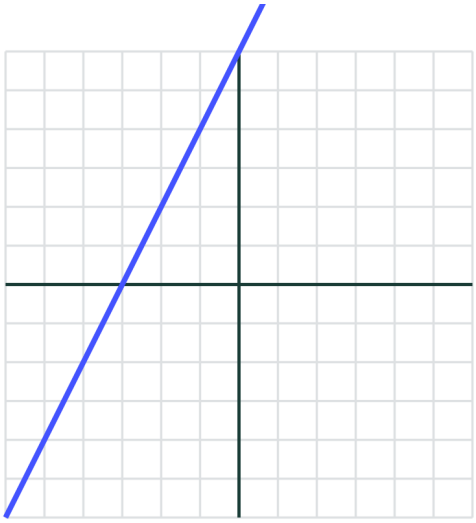
Decide whether the input or the output is known. For graphs, label the axes and read the relevant coordinate in the right order.

$8x + 7 = 87$	**Step 1** Write the equation
$8x = 87 - 7 = 80$	**Step 2** Take the constant away from both sides
$x = 80 \div 8 = 10$	**Step 3** Divide every term on both sides by the coefficient so that the coefficient of the variable becomes 1.

Think about it: Before moving on: how can you check your answer for x by evaluating the function?

Example 03 · Reading the y-intercept from a graph

Graph 1 : The straight line in the graph crosses the y-axis. What is the y-value where it crosses?



Straight line rising from left to right on axes marked from -6 to 6 on each side. The line is drawn across the whole grid.

Why this method?

Decide whether the input or the output is known. For graphs, label the axes and read the relevant coordinate in the right order.

The line crosses the y-axis at $y = 6$

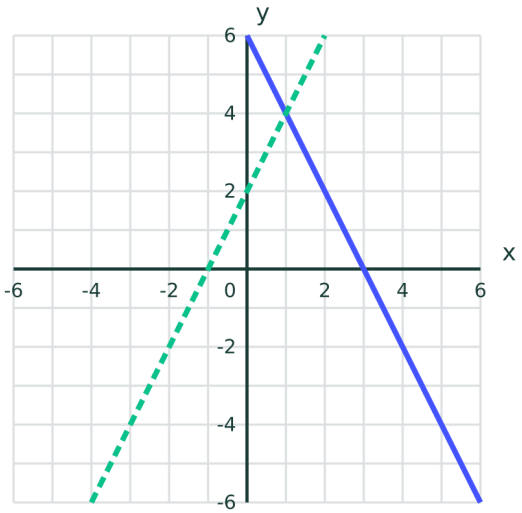
Step 1

The y-intercept is where the line crosses the y-axis, at $x = 0$

Think about it: Before moving on: which point on the graph gives the y-intercept, and why is it the only one you need?

Example 04 · Reading where two graphs cross

Graph 1 : The two straight lines in the graph cross at one point. What is the x-value of the point where they cross?



f: solid blue g: dashed green

Two straight lines drawn on axes marked from -6 to 6 on each side. The lines cross at one point.

Why this method?

Decide whether the input or the output is known. For graphs, label the axes and read the relevant coordinate in the right order.

The lines cross at $x = 1$	**Step 1** The lines cross where their y-values are equal. Read the x-value of that point from the horizontal axis

Think about it: Before moving on: why is the x-value where the lines cross the solution, and not the y-value?

Example 05 · Two functions in succession

$g(x) = x + 1$ and $f(x) = 2x$. Find $f(g(3))$.

Why this method?

Substitute carefully into the stated rule and keep brackets around a substituted expression.

$g(3) = 3 + 1 = 4$	**Step 1** Do the inside function first.
$f(4) = 2(4) = 8$	**Step 2** Put its output 4 into f.

Example 06 · A simple exponential table

Find values of $f(x) = 2^x$ at $x = 0, 1, 2$.

Why this method?

Substitute carefully into the stated rule and keep brackets around a substituted expression.

$f(0) = 2^0 = 1$	**Step 1** A non-zero number to the power zero is 1.
$f(1) = 2$	**Step 2** A power of 1 leaves the base unchanged.
$f(2) = 2(2) = 4$	**Step 3** Multiply two factors of 2. Plot $(0, 1), (1, 2), (2, 4)$.

Example 07 · A limit from nearby inputs

Investigate $f(x) = x + 1$ as x approaches 2.

Why this method?

Substitute carefully into the stated rule and keep brackets around a substituted expression.

$f(1.9) = 2.9, \quad f(1.99) = 2.99$	**Step 1** Approach 2 using slightly smaller inputs.
$f(2.1) = 3.1, \quad f(2.01) = 3.01$	**Step 2** Approach from larger inputs.
$\lim_{x \to 2}(x+1)=3$	**Step 3** The outputs approach 3 from both sides.

Example 08 · Build a table for a quadratic graph

Make a table for $y = x^2 - 2$ using $x = -2, -1, 0, 1, 2$.

x	-2	-1	0	1	2
y	2	-1	-2	-1	2

Why this method?

Choose an input, replace x, square it and then subtract 2. Repeat the same procedure for every input.

$x = -2: \quad y = (-2)^2 - 2 = 4 - 2 = 2$	**Step 1** Keep the negative input in brackets before squaring.
$x = -1: \quad y = (-1)^2 - 2 = 1 - 2 = -1$	**Step 2** The square of negative 1 is positive 1.
$x = 0: \quad y = 0^2 - 2 = -2$	**Step 3** This gives the point on the y-axis.
$x = 1: \quad y = 1^2 - 2 = -1$	**Step 4** The inputs 1 and -1 give the same output.
$x = 2: \quad y = 2^2 - 2 = 2$	**Step 5** The inputs 2 and -2 also give the same output.
$(-2, 2), (-1, -1), (0, -2), (1, -1), (2, 2)$	**Step 6** Plot x first and y second. Draw a smooth curve through the points, rather than straight segments.

Example 09 · A graph can touch the axis at a root

Compare the roots of $y = x^2 - 4$ and $y = x^2$.

Why this method?

A root is an input where the output is zero. The graph may cross the x-axis or just touch it.

$x^2 - 4 = 0 \quad \Rightarrow \quad x^2 = 4$	**Step 1** For the first graph, set y equal to zero.
$x = -2 \quad \text{or} \quad x = 2$	**Step 2** Both positive 2 and negative 2 square to 4. This graph crosses the axis twice.

$x^2 = 0 \quad \Rightarrow \quad x = 0$	**Step 3** For the second graph, zero is the only root. This graph touches the axis at $(0, 0)$.

Example 10 · Read a cost graph in context

A taxi fare is modelled by $C = 5 + 2d$, where C is in euro and d is in kilometres. Interpret the intercept and slope.

Why this method?

Read what each axis measures. The intercept gives the cost at distance zero; the slope gives the increase in cost for each kilometre.

$d = 0: \quad C = 5 + 2(0) = 5$	**Step 1** The vertical intercept is $(0, 5)$. The fixed charge is $€5$.
$\frac{\text{change in cost}}{\text{change in distance}} = \frac{2}{1} = 2$	**Step 2** Each extra kilometre adds $€2$. The slope is $€2$ per km.
$d = 3: \quad C = 5 + 2(3) = 11$	**Step 3** At 3 km the model predicts a fare of $€11$.

Example 11 · Compose in the right order

If $f(x) = 2x + 1$ and $g(x) = x^2$, find $g(f(3))$.

$f(3) = 2(3) + 1 = 7$	**Step 1** Work from the inside out.
$g(7) = 7^2 = 49$	**Final step** The first output is the next input.

Example 12 · Read an approaching value

For $f(x) = \frac{x^2 - 4}{x - 2}$, what does f(x) approach as x approaches 2?

$f(x) = x + 2, \quad x \neq 2$	**Step 1** Factor the numerator and cancel only away from 2.
The values approach 4.	**Final step** The limit is about nearby values; $f(2)$ itself is undefined.

Example 13 · Plot a cubic using a table

Sketch the graph of $f(x) = x^3 - 4x$ near its roots.

$f(-3) = -15,\ f(-1) = 3,\ f(1) = -3,\ f(3) = 15$	**Step 1** Substitute selected negative and positive inputs.
$f(x) = x(x-2)(x+2)$	**Step 2** Factor to locate the crossings exactly.
Plot the values and join with a smooth curve.	**Final step** The curve crosses the axis at $-2, 0$ and 2. Use closer inputs if a turning point is hard to see.

Common mistake

Swapping input and output coordinates. The point $(2, 5)$ means input 2 gives output 5. It does not mean that input 5 gives output 2.

Exam technique

Read the scale before estimating a crossing and label a graphical estimate as approximate.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

The function f is given by $f(x) = 6x + 11$. What is $f(7)$?

Question 2

The function f is given by $f(x) = 5x + 3$. For what value of x is $f(x) = 38$?

Question 3 · Build confidence

For $y = x^2 - 2$, find y when $x = -3$.

Question 4 · Apply or explain

Find the root of $y = (x-2)^2$. Does the graph cross or touch the x-axis?

Question 5 · Apply or explain

The cost model is $C = 8 + 3d$, in euro, for distance d in kilometres. Interpret both numbers.

Before you move on

Try these without looking at a worked example. Revisit any part that is still difficult.

- Make a table by substituting each input.
- Plot coordinates in the order (x,y).
- Find roots where the graph crosses or touches the axis.
- Interpret slope and intercept using the axis units.

Watch an explanation

Corbettmaths — Function Machines

Cognito — What are Quadratic Graphs

Tulla Maths — Calculator: make a table of values

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

53

Question 2

7

Question 3

$y = (-3)^2 - 2 = 7.$

Square the negative input in brackets.

Question 4

The root is $x = 2$. The graph touches the axis at $(2, 0)$.

A square is never negative, so the graph stays on or above the axis.

Question 5

€8 fixed charge and €3 for each kilometre.

The fixed charge remains when d is zero; 3 is the cost increase per kilometre.

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Differentiation: the first steps

Leaving Certificate · Ordinary Level

Understand the idea

How to start

Differentiation finds how quickly a quantity changes. On a graph it gives the slope at a point.

1. Identify the power on each x term.
2. Multiply by that power, then reduce the power by 1.
3. A constant becomes zero. Combine the resulting terms.

Key terms & definitions

Derivative
A rate of change; the slope of a tangent on a graph.

Second derivative
The derivative of the first derivative.

Key ideas

• For a power, multiply by the old power and reduce the power by 1 : $\frac{d}{dx}(x^n) = nx^{n-1}$.

• Differentiate each term separately. Keep its coefficient.

• A constant differentiates to zero because it does not change.

• $f'(x)$ and $\frac{dy}{dx}$ are ways of writing the derivative.

• Differentiate again to find the second derivative.

• For this Ordinary Level course, practise linear, quadratic and cubic polynomial functions by rule.

Quick reminder

• For a power, multiply by the old power and reduce the power by 1 : $\frac{d}{dx}(x^n) = nx^{n-1}$.

• Differentiate each term separately. Keep its coefficient.

Worked examples

Example 01 · Start here: a straight line and a constant

Differentiate $y = 4x + 7$.

Why this method?

A straight line has a constant slope. Each increase of 1 in x increases $4x + 7$ by 4; adding 7 does not change the slope.

$\frac{d}{dx}(4x) = 4$	**Step 1** The derivative of x is 1, so $4x$ differentiates to 4.
$\frac{d}{dx}(7) = 0$	**Step 2** The constant 7 does not change with x.
$\frac{dy}{dx} = 4 + 0 = 4$	**Step 3** Combine the two derivatives.

Example 02 · Differentiate x squared

Find the derivative of $y = x^2$.

Why this method?

Differentiate one term at a time using its power; a constant contributes zero.

$\frac{dy}{dx} = 2x^{2-1}$	**Step 1** Bring the power 2 in front. Reduce the power by 1.
$\frac{dy}{dx} = 2x$	**Step 2** A power of 1 is normally left unwritten.

Example 03 · Keep the coefficient

Differentiate $y = 3x^2 + 4x - 5$.

Why this method?

Differentiate one term at a time using its power; a constant contributes zero.

$\frac{dy}{dx} = 3(2)x + 4 - 0$	**Step 1** The derivative of x^2 is $2x$. The derivative of $4x$ is 4. The constant disappears.

$\frac{dy}{dx} = 6x + 4$	**Step 2** Multiply 3 by 2.

Example 04 · **A cubic and a second derivative**

Find both derivatives of $f(x) = 2x^3 - 3x^2 + 5$.

Why this method?

Differentiate one term at a time using its power; a constant contributes zero.

$f'(x) = 2(3)x^2 - 3(2)x$	**Step 1** Differentiate each non-constant term.
$f'(x) = 6x^2 - 6x$	**Step 2** Simplify the coefficients.
$f''(x) = 12x - 6$	**Step 3** Differentiate the first derivative once more.

Example 05 · **First and second derivatives**

Find both derivatives of $f(x) = 2x^3 - 3x^2 + 4$.

$f'(x) = 6x^2 - 6x$	**Step 1** Multiply by each power and reduce the power by one. The constant disappears.
$f''(x) = 12x - 6$	**Final step** Differentiate the first derivative again.

Common mistake

Reducing the power without multiplying by the original power. The derivative of x^3 is $3x^2$, not x^2.

Exam technique

Write the derivative before putting in a particular x-value.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Differentiate $y = 4x^2 + 3x + 7$.

Question 2

Find the second derivative of x^3.

Question 3

Differentiate $y = 5x + 2$.

Watch an explanation

Corbettmaths — Differentiation: using the power rule

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

$8x + 3$

Question 2

$6x$

Question 3

$\frac{dy}{dx} = 5.$

The constant 2 differentiates to zero.

Open the full chapter online

Slopes, tangents and turning points

Leaving Certificate · Ordinary Level

Understand the idea

How to start

A derivative is a rule for the slope. Put an x-value into it to find the slope at that point.

1. Find the derivative before substituting any x-value.
2. For a slope, substitute x. For a stationary point, set the derivative equal to zero.
3. Use the original function for y and check maximum or minimum separately.

Key terms & definitions

Stationary point
A point where the first derivative is zero.

Tangent
A line having the same slope as the curve at the point of contact.

Key ideas

• A stationary point has $f'(x) = 0$. Solve this equation for x, then use the original function to find y.

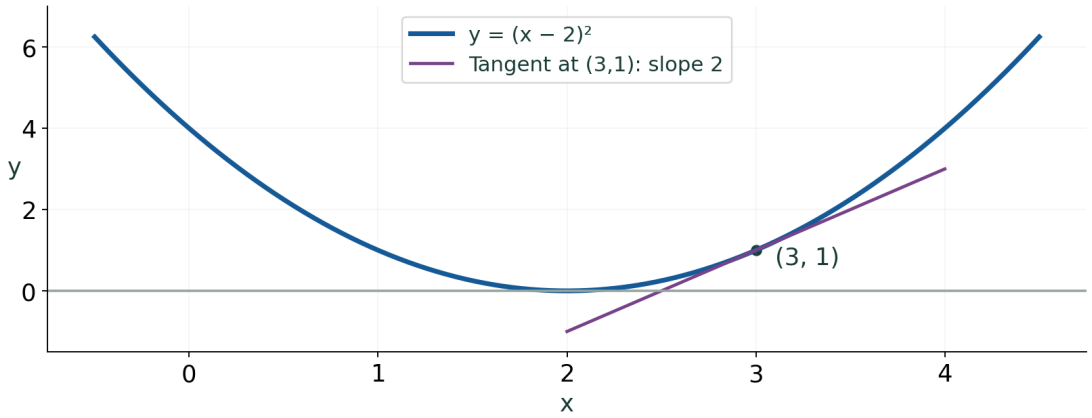
• At a stationary point, $f''(x) > 0$ gives a minimum and $f''(x) < 0$ gives a maximum. A zero second derivative needs another check.

• A tangent through (x_1, y_1) with slope m has equation $y - y_1 = m(x - x_1)$.

• A positive first derivative means the function is increasing locally; a negative one means decreasing.

• In a context, use the allowed domain and compare endpoints where necessary.

A derivative gives the tangent slope



For $y = (x - 2)^2$ the derivative is $2x - 4$. At $x = 3$ the slope is 2, matching the drawn tangent $y = 2x - 5$.

Quick reminder

• A stationary point has $f'(x) = 0$. Solve this equation for x, then use the original function to find y.

• At a stationary point, $f''(x) > 0$ gives a minimum and $f''(x) < 0$ gives a maximum. A zero second derivative needs another check.

Worked examples

Example 01 · Start here: what a slope value means

The distance of a trolley is modelled by $s(t) = t^2$, in metres, with t in seconds. Find its instantaneous speed at $t = 3$.

Why this method?

The derivative of distance with respect to time is speed in this increasing-distance model. Find the derivative first, then put in the time.

$s'(t) = 2t$	**Step 1** Differentiate t squared.
$s'(3) = 2(3) = 6$	**Step 2** Only now replace t with 3.
Speed $= 6$ m/s	**Step 3** State what the number means and include units.

Example 02 · Slope at one point

Find the slope of $y = x^2$ at $x = 3$.

Why this method?

The derivative gives slope; the original function gives the height of a point.

$\frac{dy}{dx} = 2x$	**Step 1** Differentiate the function first.
$m = 2(3) = 6$	**Step 2** Put $x = 3$ into the derivative. The slope is 6.

Example 03 · Find a minimum

Find the minimum point of $f(x) = x^2 - 4x + 7$.

Why this method?

The derivative gives slope; the original function gives the height of a point.

$f'(x) = 2x - 4$	**Step 1** Differentiate.
$2x - 4 = 0$	**Step 2** The slope at the minimum is zero.
$x = 2$	**Step 3** Add 4 and divide by 2.
$f(2) = 4 - 8 + 7 = 3$	**Step 4** Use the original function to find the height.
$f''(x) = 2 > 0$	**Step 5** The positive second derivative confirms a minimum at $(2, 3)$.

Example 04 · Equation of a tangent

Find the tangent to $y = x^2$ at $(2, 4)$.

Why this method?

The derivative gives slope; the original function gives the height of a point.

$m = 2(2) = 4$	**Step 1** The derivative $2x$ gives slope 4.

$y - 4 = 4(x - 2)$	**Step 2** Use the given point and slope.
$y = 4x - 4$	**Step 3** Expand and add 4 to both sides.

Example 05 · Find a maximum step by step

Find the maximum point of $f(x) = -x^2 + 6x + 1$.

Why this method?

At a turning point the tangent is horizontal, so its slope is zero. Solve $f'(x) = 0$, then find the height using the original function.

$f'(x) = -2x + 6$	**Step 1** Differentiate each term.
$-2x + 6 = 0$	**Step 2** Set the first derivative equal to zero.
$-2x = -6$	**Step 3** Subtract 6 from both sides.
$x = 3$	**Step 4** Divide both sides by -2.
$f(3) = -(3)^2 + 6(3) + 1 = -9 + 18 + 1 = 10$	**Step 5** Use the original function to find the y-coordinate.
$f''(x) = -2 < 0$	**Step 6** The negative second derivative confirms a maximum.
$\text{Maximum point} = (3, 10)$	**Step 7** A point needs both coordinates.

Example 06 · Show a stationary point is a minimum

Find and classify the stationary point of $f(x) = x^2 - 6x + 5$.

$f'(x) = 2x - 6$	**Step 1** Differentiate to get the slope.
$2x - 6 = 0 \quad \Rightarrow \quad x = 3$	**Step 2** A stationary point has zero slope.

$f''(3) = 2 > 0$	**Step 3** The curve bends upwards.
$f(3) = -4$	**Final step** The minimum point is $(3, -4)$.

Common mistake

Using the derivative to find a point's y-coordinate. For $f(x) = x^2 + 1$ at $x = 2$, the slope is $f'(2) = 4$, but the height is $f(2) = 5$. Use the original function for y.

Exam technique

State both coordinates when asked for a turning point.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Find the slope of $x^2 + 2x$ at $x = 1$.

Question 2

Find the minimum point of $x^2 - 6x + 10$.

Question 3

Find the slope of $y = x^2 + 1$ at $x = 2$.

Watch an explanation

Corbettmaths — Stationary points: maximum and minimum

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

4

Question 2

$(3, 1)$

Question 3

$\frac{dy}{dx} = 2x$, so the slope is 4.

Differentiate first, then substitute 2.

Open the full chapter online

Units, time and scale

Leaving Certificate · Ordinary Level

Understand the idea

How to start

A measurement needs a number and a unit. One metre is the same length as 100 centimetres.

1. Write the starting unit and the unit you need.
2. Use a known conversion such as $1m = 100$ cm.
3. Check the size: the same length has more centimetres than metres.

Key terms & definitions

Scale
The ratio of a drawing length to the corresponding actual length.

Capacity
The volume a container can hold.

Duration
The amount of time between two events.

Key ideas

• Convert all lengths to compatible units before calculating.

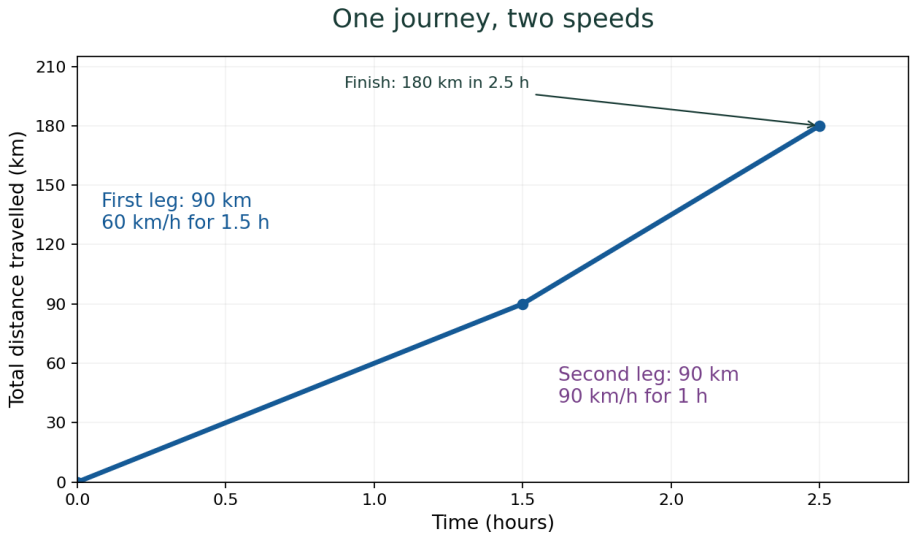
• Area conversions square the length scale factor: $1\text{m}^2 = 10000\text{cm}^2$. Volume conversions cube it.

• $1\text{litre} = 1000\text{cm}^3$ and $1\text{m}^3 = 1000\text{litres}$.

• A decimal part of an hour is not minutes. Multiply the fractional hour by 60.

• A scale of $1 : 500$ means one unit on the diagram represents 500 of the same units in reality.

• Average speed is total distance divided by total time. Convert every part of the journey to the same time unit before adding.



The steeper second segment shows the faster leg. Overall average speed uses the finish totals : 180 km divided by 2.5 hours, giving 72 km$/h$.

Quick reminder

• Use compatible units before calculating.

• Area scale factors are squared; volume scale factors are cubed.

• Multiply a fractional hour by 60 to convert it to minutes.

Worked examples

Example 01 · Converting units of length

Convert 4 cm to mm.

Why this method?

Identify the starting unit and the required unit. Use the conversion factor in the direction that gives the required unit.

4 cm $= 40$ mm

Step 1

Use the ratio between the two units

Think about it: Before moving on: why does a number of centimetres become a bigger number of millimetres?

Example 02 · Converting units of mass and capacity

Convert 5 kg to g.

Why this method?

Identify the starting unit and the required unit. Use the conversion factor in the direction that gives the required unit.

$1\text{kg} = 1000\text{g}$	**Step 1** One kilogram contains one thousand grams.
$5 \times 1000 = 5000$	**Step 2** There are 5 lots of 1000 grams.
$5\text{kg} = 5000\text{g}$	**Step 3** The unit is smaller, so the number of units is larger.

Think about it: Before moving on: when you convert millilitres to litres, why is the answer a smaller number?

Example 03 · Hours, minutes and days

How many minutes are there in 3 hours and 25 minutes?

Why this method?

Identify the starting unit and the required unit. Use the conversion factor in the direction that gives the required unit.

$3 \times 60 = 180$ minutes	**Step 1** There are 60 minutes in each hour
$180 + 25 = 205$ minutes	**Step 2** Add the extra minutes

Think about it: Before moving on: why is 2 hours and 30 minutes not 230 minutes?

Example 04 · A scale plan

A room is $4.8m$ long. Draw its length at a scale of $1:50$.

Why this method?

Identify the starting unit and the required unit. Use the conversion factor in the direction that gives the required unit.

$4.8m = 480$ cm.	**Step 1** Use the same units

$480 \div 50 = 9.6$	**Step 2** Divide by the scale factor
The plan length is 9.6 cm.	**Step 3** State the drawing length

Example 05 · Average speed: one journey

A car travels 150 km in 2 hours 30 minutes. Find its average speed in km/h.

Why this method?

Divide total distance by total time. Convert the minutes to hours before dividing.

$30 \text{ min} = \frac{30}{60} \text{ h} = 0.5 \text{ h}$	**Step 1** There are 60 minutes in one hour.
$\text{Total time} = 2 + 0.5 = 2.5 \text{ h}$	**Step 2** 2 hours 30 minutes is 2.5 hours, not 2.30 hours.
$\text{Average speed} = \frac{150}{2.5} = 60 \text{ km/h}$	**Step 3** Use distance in kilometres and time in hours.

Example 06 · Average speed: combine the whole journey

A car travels 90 km at 60 km/h, then another 90 km at 90 km/h. Find its average speed for the entire journey.

Why this method?

The car spends different amounts of time at the two speeds. Find each time, then divide the total distance by the total time.

$t_1 = \frac{90}{60} = 1.5 \text{ h}$	**Step 1** Time equals distance divided by speed. The slower leg takes longer.
$t_2 = \frac{90}{90} = 1 \text{ h}$	**Step 2** Calculate the time for the faster leg separately.
$\text{Total distance} = 90 + 90 = 180 \text{ km}$	**Step 3** Add the distances.
$\text{Total time} = 1.5 + 1 = 2.5 \text{ h}$	**Step 4** Add the times, not the speeds.

Average speed $= \frac{180}{2.5} = 72$ km/h	**Step 5** The answer is not 75 km$/h$: *a* simple mean of these speeds would ignore their different durations.

Example 07 · Convert area units

A floor covers $2.7m^2$. Express this in cm^2 .

$1\ \text{m}^2 = 100^2\ \text{cm}^2$	**Step 1** Convert each of the two lengths.
$2.7 \times 10,000 = 27,000\ \text{cm}^2$	**Final step** Use the square of the linear conversion factor.

Common mistake

Using the length conversion factor for area. Since $1m = 100$ cm, $1\text{m}^2 = 100 \times 100 = 10000\text{cm}^2$, not 100cm^2.

Exam technique

Write units on every intermediate quantity and check the final unit matches the question.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Convert $13m$ to cm.

Question 2

Convert $15000g$ to kg.

Question 3 · Build confidence

Convert 45 minutes to hours.

Question 4 · Apply or explain

A cyclist travels 24 km uphill at 16 km$/h$ and 24 km back at 24 km$/h$. Find the overall average speed.

Before you move on

Try these without looking at a worked example. Revisit any part that is still difficult.

- Convert minutes to hours.
- Find total time for a journey with several stages.
- Calculate average speed from total distance and total time.

Watch an explanation

Corbettmaths — Units: Metric units (length)

Corbettmaths — Time: calculations

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

1300

Question 2

15

Question 3

$\frac{45}{60} = 0.75$ hours.

Divide minutes by 60.

Question 4

$t_1 = 1.5$ h; $t_2 = 1$ h; average speed $= \frac{48}{2.5} = 19.2$ km/h.

Use the total distance and total time, rather than the mean of the speeds.

Open the full chapter online

Perimeter and area

Leaving Certificate · Ordinary Level

Understand the idea

How to start

Perimeter measures the distance around a shape. Area measures the space inside it.

1. Choose a formula for the shape and name each measurement.
2. Use a perpendicular height, measured at a right angle to the base.
3. Use square units for area, such as cm^2. Use length units for perimeter.

Key terms & definitions

Perimeter
The total length around a shape.

Perpendicular height
A distance measured at a right angle to the base.

Radius
The distance from the centre of a circle to its circumference.

Key ideas

• Perimeter is the distance around a boundary; area measures the surface enclosed.

• Rectangle: $A = lw$. Triangle: $A = \frac{1}{2}bh$ with perpendicular height h.

• Parallelogram: $A = bh$. Trapezium: $A = \frac{1}{2}(a + b)h$ where a and b are parallel sides.

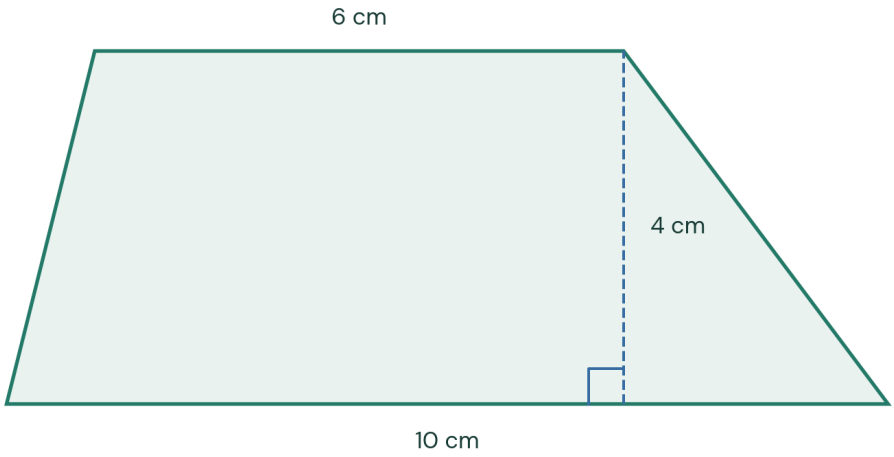
• Circle: $C = 2\pi r$ and $A = \pi r^2$. A diameter is twice the radius.

• Split a composite shape into familiar shapes or subtract a missing part. Count only the outside boundary for its perimeter.

• An arc is part of a circumference: $L = \frac{\theta}{360}2\pi r$ when θ is in degrees.

• A sector is part of a disc: $A = \frac{\theta}{360}\pi r^2$.

• For equally spaced ordinates, the trapezoidal rule estimates area:
$A \approx \frac{h}{2}[y_0 + y_n + 2(\text{sumofinteriorordinates})]$.



Trapezium: average the parallel sides 6 cm and 10 cm, then multiply by the perpendicular height 4 cm.

Quick reminder

• Perimeter: distance around. Area: surface inside.

• Use perpendicular height, not automatically a sloping side.

• Circle formulas use radius, not diameter.

Worked examples

Example 01 · Area and circumference

A circle has diameter 10 cm. Find its exact area and circumference.

Why this method?

Sketch and label the shape. Choose a formula using the perpendicular height or split the shape into familiar pieces.

$r = \frac{10}{2} = 5\text{cm}$	**Step 1** The circle formulas use the radius, which is half the given diameter.
$A = \pi(5)^2 = 25\pi\text{cm}^2$	**Step 2** Substitute the radius into the area formula. Squaring the radius gives square units for the area.

$C = 2\pi(5) = 10\pi\text{cm}$	**Step 3** Use twice the radius in the circumference formula. The answer is a length, so use ordinary length units.

Example 02 · A trapezium

A trapezium has parallel sides 6 cm and 10 cm and perpendicular height 4 cm. Find its area.

Why this method?

Sketch and label the shape. Choose a formula using the perpendicular height or split the shape into familiar pieces.

$A = \frac{1}{2}(a + b)h$	**Step 1** Choose the formula
$A = \frac{1}{2}(6 + 10)(4)$	**Step 2** Substitute
$A = 32\text{cm}^2$	**Step 3** The parallel sides add to 16. Half of 16×4 is 32, with square centimetres for area.

Example 03 · A composite L-shape

$A10$ cm by 8 cm rectangle has $a4$ cm by 3 cm corner removed. Find the remaining area.

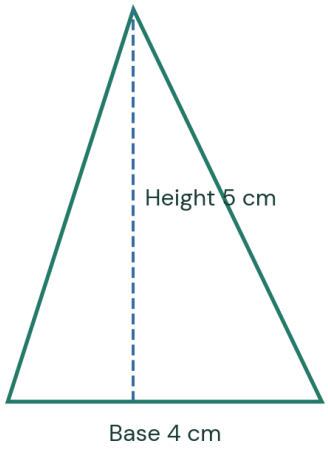
Why this method?

Sketch and label the shape. Choose a formula using the perpendicular height or split the shape into familiar pieces.

$10 \times 8 = 80\text{cm}^2$	**Step 1** Find the whole rectangle
$4 \times 3 = 12\text{cm}^2$	**Step 2** Find the removed part
$80 - 12 = 68\text{cm}^2$	**Step 3** Subtract the removed area from the area of the whole shape. This leaves the area that is still present.

Example 04 · Area of a triangle and of an L-shape

A triangle has a base of 4 cm and a perpendicular height of 5 cm. What is its area?



Triangle with base 4 cm and perpendicular height 5 cm.

Why this method?

Sketch and label the shape. Choose a formula using the perpendicular height or split the shape into familiar pieces.

$\frac{1}{2} \times 4 \times 5$	**Step 1** Area of a triangle $= \frac{1}{2} \times$ base $\times$ height
$= 10 \text{ cm}^2$	**Step 2** Four times five is 20. A triangle is half the corresponding rectangle, so divide 20 by 2.

Think about it: Before moving on: why is the area of a triangle half of base $\times$ height?

Example 05 · A quarter-circle sector

Find the sector area and arc length for radius 4 cm and angle 90°.

Why this method?

Choose the shape formula or approximation first, then substitute measurements with consistent units.

$\frac{90}{360} = \frac{1}{4}$	**Step 1** The angle is one quarter of a full turn.
$A = \frac{1}{4}\pi(4)^2 = 4\pi \text{cm}^2$	**Step 2** Take one quarter of the full disc area.

$L = \frac{1}{4}(2\pi)(4) = 2\pi\text{cm}$	**Step 3** Take one quarter of the full circumference.

Example 06 · Trapezoidal rule: two strips

Three heights are $2m, 4m$ and $3m$, measured $5m$ apart. Estimate the area under the boundary.

Why this method?

Choose the shape formula or approximation first, then substitute measurements with consistent units.

$A \approx \frac{5}{2}[2 + 3 + 2(4)]$	**Step 1** There are two strips, each $5m$ wide. The middle height belongs to both trapezia, so count it twice.
$A \approx \frac{5}{2}(13) = 32.5\text{m}^2$	**Step 2** Add the heights inside the bracket, then multiply. The curved boundary makes this an estimate.

Example 07 · Use the trapezoidal rule

Widths at equal $4m$ intervals are $5, 7, 8$ and $6m$. Estimate the area.

$A \approx \frac{4}{2}[5 + 2(7 + 8) + 6]$	**Step 1** Count the two endpoint widths once and each interior width twice.
$A \approx 82\ \text{m}^2$	**Final step** Multiply the bracket by half the interval width.

Common mistake

Using a sloping side as the height. In $A = \frac{1}{2}bh$, h must meet the chosen base at a right angle. A sloping side can be used as height only if it is perpendicular to that base.

Exam technique

Sketch and mark missing lengths before choosing an area formula.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

A circle has diameter 12 cm. Find its exact area and circumference.

Question 2

A trapezium has parallel sides 8 cm and 14 cm and perpendicular height 5 cm. Find its area.

Watch an explanation

Cognito — Area Formulas - Rectangle, Parallelogram, Triangle & Trapezium

Cognito — Circles - Area & Circumference

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

$r = 6\text{cm};\ A = 36\pi\text{cm}^2;\ C = 12\pi\text{cm}.$

Halve the diameter first. Area uses the square of the radius; circumference uses the radius once.

Question 2

$A = \frac{1}{2}(8 + 14)(5) = 55\text{cm}^2.$

Average the two parallel lengths, then multiply by the perpendicular height.

Open the full chapter online

Volume, surface area and nets

Leaving Certificate · Ordinary Level

Understand the idea

How to start

Volume is the space inside a solid. Surface area is the area of its outside faces.

1. Read whether the question asks for volume or surface area.
2. For a prism, find the cross-section area, then multiply it by the prism length.
3. Use cubic units for volume and square units for surface area.

Key terms & definitions

Prism
A solid with a constant polygonal cross-section along its length.

Net
A flat arrangement of the surfaces that fold into a solid.

Exposed surface
A face or curved surface visible on the outside.

Key ideas

• Volume measures space inside a solid. For a prism, $V = A_{\text{cross}-\text{section}} l$.

• Cuboid: $V = lwh$. Cylinder: $V = \pi r^2 h$. Sphere: $V = \frac{4}{3}\pi r^3$.

• Surface area is the total exposed area. A net helps identify every face.

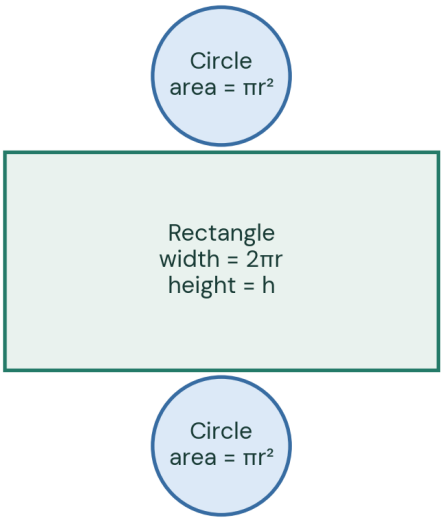
• Cylinder curved surface area is $2\pi rh$; add $2\pi r^2$ for two closed circular ends. Sphere surface area is $4\pi r^2$.

• In a composite solid, add or subtract volumes as required. For exposed surface area, exclude joined internal faces.

• A cylinder net has a rectangle of length $2\pi r$ and height h, plus two circles when both ends are closed.

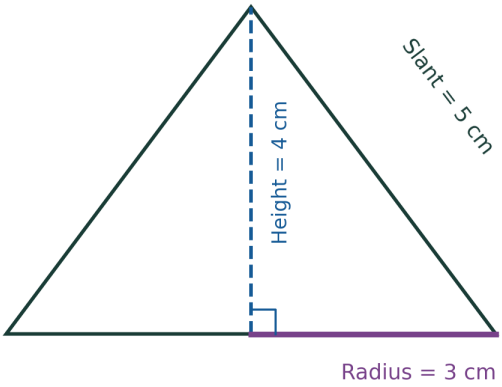
• Cone volume: $V = \frac{1}{3}\pi r^2 h$. Here h is the perpendicular height.

• Hemisphere volume: $V = \frac{2}{3}\pi r^3$. Total surface area of a closed hemisphere is $3\pi r^2$, including its flat circular base.



Closed cylinder net: one rectangle and two circular ends. This schematic shows the surfaces separately.

Cone: use the perpendicular height



A cross-section through the centre of the cone. For volume, use the perpendicular height of 4 cm, not the slant height of 5 cm. The radius is half the base width.

Quick reminder

• Volume uses cubic units; surface area uses square units.

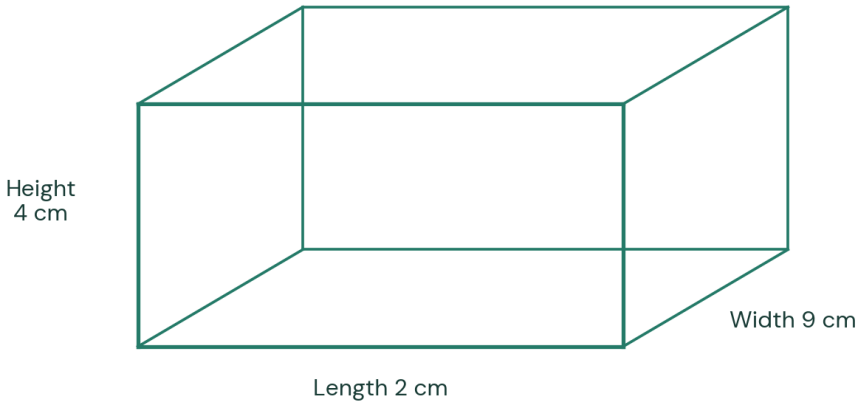
• Prism volume $=$ cross-section area $\times$ length.

• Count exposed surfaces only; do not count joined internal faces.

Worked examples

Example 01 · Volume of a cuboid

A cuboid has length 2 cm, width 9 cm and height 4 cm. What is its volume?



Cuboid with length 2 cm, width 9 cm and height 4 cm. Schematic; use stated dimensions.

Why this method?

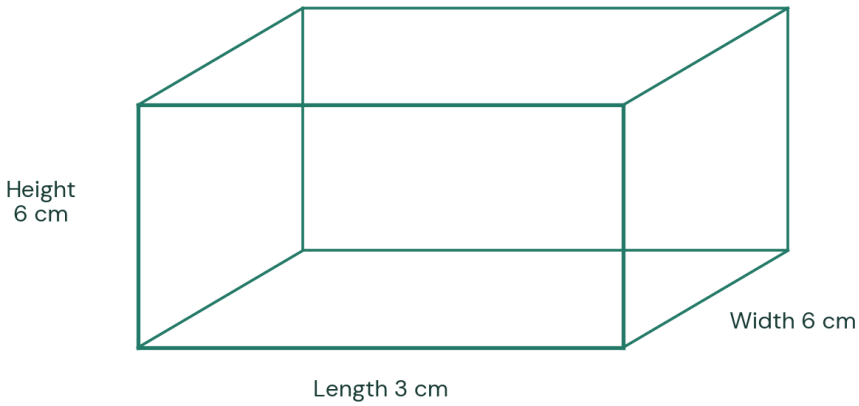
A cuboid has rectangular layers. Its volume is the area of one layer multiplied by the height.

$V = lwh$	**Step 1** V means volume. Here $l = 2$ cm, $w = 9$ cm and $h = 4$ cm.
$V = 2 \times 9 \times 4$	**Step 2** Put each length into the matching place in the formula.
$V = 18 \times 4 = 72$	**Step 3** Multiply the lengths.
$V = 72\text{cm}^3$	**Step 4** Three lengths were multiplied, so the result is in cubic centimetres.

Think about it: Before moving on: why does a volume need all three dimensions?

Example 02 · Total surface area of a cuboid

A cuboid has length 3 cm, width 6 cm and height 6 cm. What is its total surface area?



Cuboid with length 3 cm, width 6 cm and height 6 cm. Schematic; use stated dimensions.

Why this method?

Decide whether the question asks for space inside or area outside. List the required volumes or exposed surfaces before calculating.

$3 \times 6, 3 \times 6, 6 \times 6$	**Step 1** A cuboid has three pairs of equal faces
$2 \times (18 + 18 + 36) = 144 \text{ cm}^2$	**Step 2** Add the three face areas and double

Think about it: Before moving on: why are the faces counted in pairs?

Example 03 · Volume of a cylinder

A cylinder has radius 2 cm and height 4 cm. Find its volume, giving your answer in terms of π.

Why this method?

Decide whether the question asks for space inside or area outside. List the required volumes or exposed surfaces before calculating.

$V = \pi \times 2^2 \times 4$	**Step 1** Volume of a cylinder $= \pi \times \text{radius}^2 \times$ height
$V = 16\pi \text{ cm}^3$	**Step 2** Work out the square and the product

Think about it: Before moving on: why do we square the radius, and why is the answer given in terms of π?

Example 04 · A cone: identify perpendicular height

Find the volume of a cone with radius 3 cm and perpendicular height 4 cm.

Why this method?

A cone occupies one third of the volume of a cylinder with the same base and perpendicular height.

$V = \frac{1}{3}\pi r^2 h$	**Step 1** Choose the cone formula, not the cylinder formula.
$V = \frac{1}{3}\pi(3)^2(4)$	**Step 2** The height here is perpendicular to the base; it is not the sloping length.
$V = \frac{1}{3}\pi(9)(4) = 12\pi \text{cm}^3$	**Step 3** Square the radius and multiply. This is an exact answer.
$V \approx 37.7 \text{cm}^3$	**Step 4** Round only if a decimal answer is required.

Example 05 · A hemisphere is half a sphere

Find the volume of a hemisphere with radius 3 cm.

Why this method?

A hemisphere is half a sphere. Apply the one-half factor to the entire sphere-volume formula.

$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$	**Step 1** Begin with one half of the sphere formula.
$V = \frac{2}{3}\pi(3)^3$	**Step 2** Simplify the numerical factor and put in the radius.
$V = \frac{2}{3}\pi(27) = 18\pi \text{cm}^3$	**Step 3** Cube 3 to get 27, then multiply.

Example 06 · Recasting: conserve the volume

A metal cylinder of radius 3 cm and height 8 cm is melted to make small spheres of radius 1 cm. No metal is lost. How many complete spheres can be made?

Why this method?

Melting changes the shape but preserves the volume when no metal is lost. Divide the starting volume by the volume of one sphere.

$V_{\text{cylinder}} = \pi(3)^2(8) = 72\pi \text{ cm}^3$	**Step 1** Calculate the total volume of metal.
$V_{\text{sphere}} = \frac{4}{3}\pi(1)^3 = \frac{4\pi}{3} \text{ cm}^3$	**Step 2** Calculate the volume needed for just one small sphere.
$\text{Number} = \frac{72\pi}{4\pi/3} = 72\pi \times \frac{3}{4\pi} = 54$	**Step 3** Divide by a fraction by multiplying by its reciprocal. The π factors cancel.
$\text{Number of complete spheres} = 54$	**Step 4** A number of objects must be a whole number. Here there is exactly enough metal for 54.

Example 07 · Surface area of a hemisphere

A solid hemisphere has radius 3 cm. Find its total surface area.

$\frac{1}{2}(4\pi r^2) = 18\pi$	**Step 1** The curved surface is half that of a sphere.
$18\pi + \pi(3^2) = 27\pi \text{ cm}^2$	**Final step** Add the flat circular base.

Common mistake

Giving surface area in cubic units or volume in square units. Surface area measures covering and uses units such as cm^2; volume measures space and uses cm^3.

Exam technique

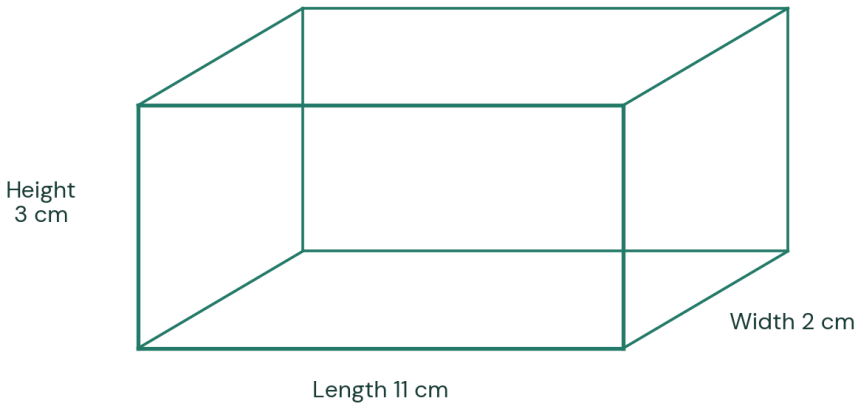
Make a face list and cross off hidden joined surfaces before adding areas.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

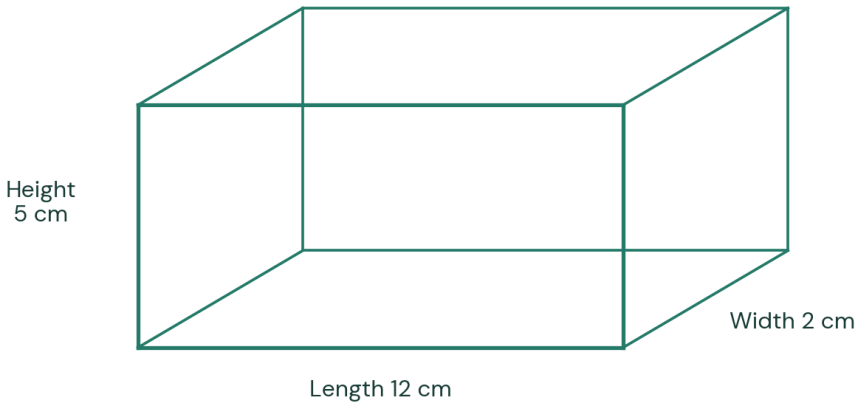
A cuboid has length 11 cm, width 2 cm and height 3 cm. What is its volume?



Cuboid with length 11 cm, width 2 cm and height 3 cm. Schematic; use stated dimensions.

Question 2

A cuboid has length 12 cm, width 2 cm and height 5 cm. What is its total surface area?



Cuboid with length 12 cm, width 2 cm and height 5 cm. Schematic; use stated dimensions.

Question 3

Find the volume of a cone with radius 2 cm and perpendicular height 3 cm.

Question 4 · Build confidence

A cylinder of radius 2 cm and height 6 cm is recast into spheres of radius 1 cm. No metal is lost. How many spheres?

Before you move on

Try these without looking at a worked example. Revisit any part that is still difficult.

- Choose the correct volume formula.
- Use perpendicular height for a cone.
- Use equal volumes when material is recast without loss.

Watch an explanation

Cognito — Volumes of Cylinders & Prisms

Corbettmaths — Surface area: cylinders

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

66

Question 2

188

Question 3

$4\pi \text{cm}^3$.

Use one third of π times radius squared times height.

Question 4

$\frac{24\pi}{4\pi/3} = 18$ spheres.

Volume is conserved; divide by the volume of one sphere.

Open the full chapter online

Angles and geometrical reasoning

Leaving Certificate · Ordinary Level

Understand the idea

How to start

An angle measures a turn. A straight angle is 180° and a full turn is 360°.

1. Mark the angle you need and the angles you know.
2. Name the angle fact that links them.
3. Write an equation or subtraction using that fact, then calculate.

Key terms & definitions

Axiom
An accepted starting statement in a mathematical system.

Theorem
A statement established by reasoning from accepted facts.

Converse
A statement made by reversing the condition and conclusion.

Corollary
A result that follows from a theorem.

Key ideas

• Angles on a straight line total 180°; angles around a point total 360°. Vertically opposite angles are equal.

• The interior angles of a triangle total 180°. An exterior angle equals the sum of the two opposite interior angles.

• With parallel lines, corresponding angles are equal, alternate angles are equal, and interior angles on the same side total 180°.

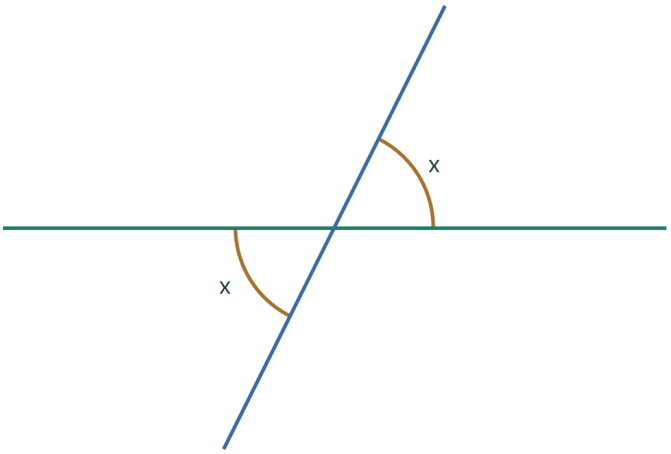
• An isosceles triangle has equal base angles; the converse also holds. An equilateral triangle has three equal sides and three 60° angles.

• An axiom is accepted as a starting point. A theorem is justified from earlier results. A converse reverses a statement and needs its own justification.

• Give a reason for each angle step. A diagram looking equal or parallel is not enough evidence.

• The five starting axioms concern the unique line through two distinct points, measurement of length, measurement of angles, triangle congruence and the unique parallel through a point off a line. Use them as agreed starting facts.

• Understand and explain reasoning, including what each step relies on. Formal theorem proofs are not examined as memorised reproductions.



Vertically opposite angles are equal. Each is supplementary to the same adjacent angle.

Quick reminder

• Name the angle fact before calculating.

• Line : 180°. Point : 360°. Triangle : 180°.

• Parallel-line rules require parallel lines.

Worked examples

Example 01 · Angles on a line, at a point and vertically opposite

Two angles lie on a straight line. One of them is 33°. What is the other angle?

Why this method?

Identify the angle fact and check its conditions. Use it to write an equation for the unknown angle.

$x + 33^\circ = 180^\circ$

Step 1

Let x be the missing angle. Angles making a straight line add to 180°.

$x = 180^\circ - 33^\circ$	**Step 2** Subtract the known angle from the straight angle.
$x = 147^\circ$	**Step 3** Calculate the missing angle.
$147^\circ + 33^\circ = 180^\circ$	**Step 4** Check the two angles make a straight angle.

Think about it: Before moving on: why do vertically opposite angles have to be equal, and not just add to 180°?

Example 02 · Angles in a triangle

A triangle has angles of 61° and 24°. What is the third angle?

Why this method?

Identify the angle fact and check its conditions. Use it to write an equation for the unknown angle.

$180^\circ - 61^\circ - 24^\circ = 95^\circ$	**Step 1** The angles of a triangle add up to 180°

Think about it: Why do we subtract both known angles from 180° to find the third angle?

Example 03 · Angles with parallel lines

Two parallel lines are cut by a transversal. One of the angles formed is 42°. What is the alternate angle to it?

Why this method?

Identify the angle fact and check its conditions. Use it to write an equation for the unknown angle.

$42^\circ = 42^\circ$	**Step 1** Alternate and corresponding angles are equal for parallel lines

Think about it: Before moving on: how can you tell whether two angles are alternate, corresponding or co-interior?

Example 04 · Use the exterior angle theorem

An exterior angle is 126°. One remote interior angle is 58°. Find the other.

$58^\circ + x = 126^\circ$	**Step 1** The exterior angle equals the sum of the two remote interior angles.
$x = 68^\circ$	**Final step** Subtract 58°.

Common mistake

Assuming angles are equal just because two lines look parallel. Corresponding and alternate-angle equalities require parallel lines. Use the given parallel marks or establish that the lines are parallel first.

Exam technique

Write the angle fact beside the calculation rather than giving an unexplained number.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Two angles lie on a straight line. One of them is 125°. What is the other angle?

Question 2

A triangle has angles of 115° and 20°. What is the third angle?

Watch an explanation

Cognito — 5 Simple Angle Rules

Cognito — Alternate, Corresponding & Allied Angles

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

55

Question 2

45

Open the full chapter online

Congruence, similarity and triangle results

Leaving Certificate · Ordinary Level

Understand the idea

How to start

Congruent triangles match exactly in size and shape. Similar triangles have the same shape but may differ in size.

1. Match the corresponding vertices and sides before calculating.
2. Check the stated conditions, such as parallel lines or equal angles.
3. Name the geometrical fact you use and say what it tells you here.

Key terms & definitions

Congruent
The same shape and size.

Similar
The same shape, with a common scale factor between corresponding lengths.

Corresponding
Matching positions in two figures.

Key ideas

• Congruent triangles have corresponding sides and angles equal. Useful criteria are SSS, SAS and ASA, and RHS for right triangles.

• Similar triangles have equal corresponding angles and proportional corresponding sides. Match the vertices in the same order.

• The midpoint theorem links midpoints of two triangle sides: the joining segment is parallel to the third side and half its length.

• Parallel cuts create equal intercepts or proportional segments under the relevant conditions.

• In a parallelogram, opposite sides are equal and diagonals bisect each other. A longer triangle side is opposite a larger angle.

• Explain why a result applies, including its conditions; memorising theorem numbers alone is not enough.

• A diagonal divides a parallelogram into congruent triangles. The converse is not generally valid: a kite can have this diagonal property without being a parallelogram.

• Triangle inequality: any two side lengths must add to more than the third. This is useful when checking whether an SSS construction is possible.

Quick reminder

• Congruent: same size and shape. Similar: same shape.

• Match corresponding vertices before using ratios.

• Check the conditions of the theorem or congruence test.

Worked examples

Example 01 · Match sides before naming congruence

Two triangles each have side lengths 3 cm, 4 cm and 5 cm. Are they congruent?

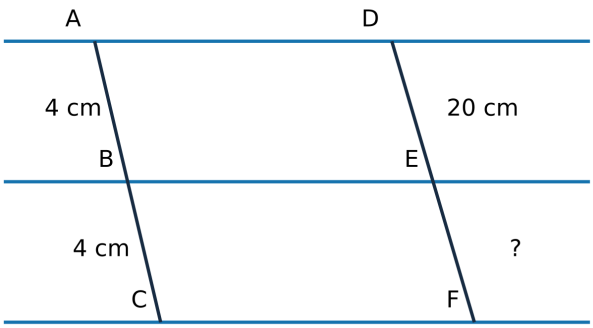
Why this method?

Match the corresponding sides and angles before applying a congruence or similarity rule.

$3 = 3, \quad 4 = 4, \quad 5 = 5$	**Step 1** Match each side with the side of the same length.
Congruent by SSS.	**Step 2** All three corresponding sides match. The triangles have the same size and shape.

Example 02 · Equal gaps made by parallel lines

Three parallel lines meet one transversal at A, B, C in that order and another at D, E, F in that order. AB $=$ BC $=4$ cm and DE $=20$ cm. Find EF.



Three blue lines are parallel.

Equal intercepts on two transversals. Diagram not to scale.

Why this method?

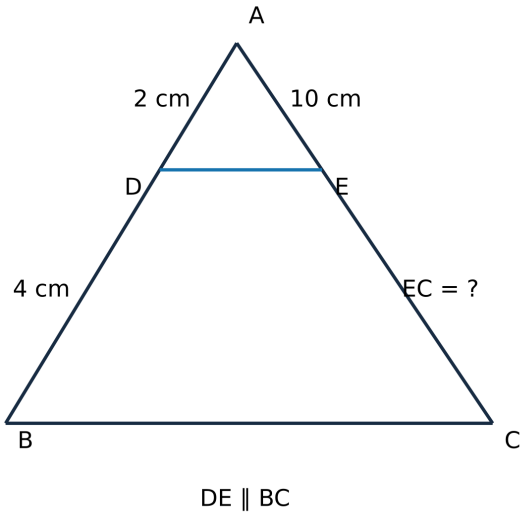
Mark the matching sides or angles. Check the conditions of the geometry result before using it.

The three lines are parallel. AB $=$ BC $=4$ cm.	**Step 1** A transversal is a line crossing the parallel lines. The first transversal has two equal gaps.
$DE = EF$	**Step 2** Three parallel lines that cut equal gaps on one transversal also cut equal gaps on another transversal.
$20\text{cm} = EF$	**Step 3** DE is given as 20 cm. Replace DE by that known length.
$EF = 20\text{cm}$	**Step 4** State the length asked for. The gaps on the two different transversals need not have the same size.

Think about it: Why do DE and EF have to match each other, but do not have to equal 4 cm?

Example 03 · A parallel line divides two sides in the same ratio

In triangle ABC, D is between A and B, E is between A and C, and DE is parallel to BC. AD $= 2$ cm, DB $= 4$ cm and AE $= 10$ cm. Find EC.



Triangle with a parallel cross-section. Diagram not to scale.

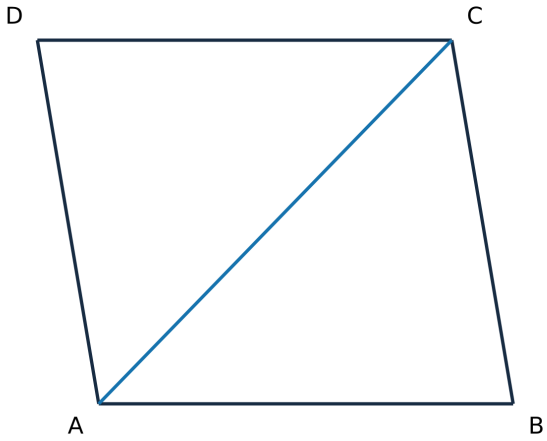
Why this method?

Mark the matching sides or angles. Check the conditions of the geometry result before using it.

$\frac{AD}{DB} = \frac{AE}{EC}$	**Step 1** DE is parallel to BC, so it divides the two sides in the same ratio. Keep upper segment over lower segment on both sides.
$\frac{2}{4} = \frac{10}{EC}$	**Step 2** Replace AD, DB and AE by their given lengths.
$2 \times EC = 4 \times 10$	**Step 3** Multiply both sides by $4 \times$ EC to clear both denominators.
$2EC = 40$	**Step 4** Calculate the right-hand side.
$EC = 20\text{cm}$	**Step 5** Divide both sides by 2. Check : $\frac{2}{4}$ and $\frac{10}{20}$ are both one half.

Example 04 · Corollaries 1, 2 and 5

ABCD is a parallelogram with diagonal AC. AB $= 9$ cm, BC $= 8$ cm and AC $= 11$ cm. Find the perimeter of triangle CDA.



Parallelogram divided by a diagonal. Diagram not to scale.

Why this method?

Mark the matching sides or angles. Check the conditions of the geometry result before using it.

Triangles ABC and CDA are congruent.	**Step 1** Use the diagonal congruence corollary
CD $=$ AB $= 9$ cm; DA $=$ BC $= 8$ cm; CA $= 11$ cm.	**Step 2** Match the corresponding sides
$9 + 8 + 11 = 28$ cm.	**Step 3** Add the three sides of triangle CDA

Think about it: Which sides in the two triangles correspond, and why does the shared diagonal have the same length in both?

Example 05 · A parallel cut gives proportional sides

DE is parallel to BC in triangle ABC. AD $= 4$, DB $= 6$, AE $= 3$. Find EC.

$\frac{AD}{DB} = \frac{AE}{EC}$	**Step 1** The parallel cut divides the two sides in the same ratio.
$\frac{4}{6} = \frac{3}{EC}$	**Step 2** Match top segments and lower segments.

$EC = 4.5$	**Final step** Cross-multiply and divide by 4.

Common mistake

Using equal angles to claim two triangles are congruent. Equal corresponding angles show similarity, so the shape is the same; the triangles may still be different sizes.

Exam technique

Mark corresponding sides before writing a ratio.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Three parallel lines meet one transversal at A, B, C in that order and another at D, E, F in that order. AB $=$ BC $=4$ cm and DE $=6$ cm. Find EF.

A. 4 cm

B. 12 cm

C. 6 cm

D. 3 cm

Question 2

ABCD is a parallelogram with diagonal AC. AB $=7$ cm, BC $=10$ cm and AC $=9$ cm. Find the perimeter of triangle CDA.

A. 17 cm

B. 26 cm

C. 35 cm

D. 34 cm

Watch an explanation

Cognito — Proving Congruence in Triangles (SSS, SAS, AAS & RHS Rules)

Cognito — Similar Shapes

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

6 cm

Correct.

Question 2

26 cm

Correct.

Open the full chapter online

Pythagoras and its converse

Leaving Certificate · Ordinary Level

Understand the idea

How to start

Pythagoras applies to a right-angled triangle. The hypotenuse is the side opposite the right angle.

1. Label the hypotenuse c and the shorter sides a and b.
2. Write $a^2 + b^2 = c^2$, then put the known lengths in their places.
3. After finding the missing square, take the positive square root to find a length.

Key terms & definitions

Hypotenuse
The side opposite the right angle in a right-angled triangle.

Converse of Pythagoras
If the square of the longest side equals the sum of the other two squares, the triangle is right-angled.

Key ideas

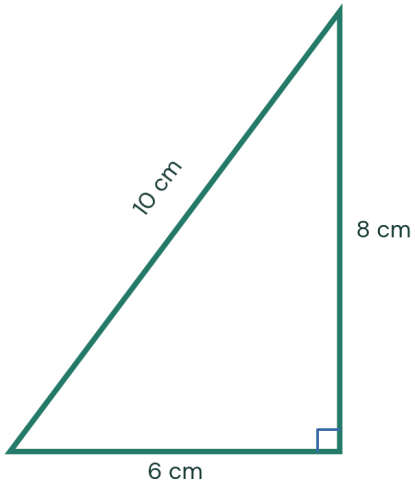
• In a right-angled triangle, $a^2 + b^2 = c^2$, where c is the hypotenuse.

• The hypotenuse is opposite the right angle and is the longest side.

• To find a shorter side, subtract the known shorter side squared from the hypotenuse squared.

• The converse checks whether a triangle is right-angled: square the longest side and compare it with the sum of the other squares.

• Keep exact roots if requested; round only after finding the square root.



Right triangle : $6^2 + 8^2 = 10^2$. The 10 cm side is opposite the right angle.

Quick reminder

• Right triangle only: $a^2 + b^2 = c^2$.

• c is the hypotenuse, opposite the right angle.

• Square first; take the positive square root for a length.

Worked examples

Example 01 · Find the hypotenuse

A right triangle has shorter sides 6 cm and 8 cm. Find the hypotenuse.

Blue follows the first shorter side and purple the second. The hypotenuse remains c.

Why this method?

Identify the right angle and the longest side. The longest side has its square alone on one side of the equation.

$a = 6$, $b = 8$	**Step 1** The given sides are the shorter sides. The missing hypotenuse is c.
$a^2 + b^2 = c^2$	**Step 2** The triangle is right-angled, so Pythagoras applies.
$6^2 + 8^2 = c^2$	**Step 3** Substitute the two known lengths.

$36 + 64 = c^2$	**Step 4** Square each length separately.
$100 = c^2$	**Step 5** Add the two squares. This is c squared, not c.
$c = \sqrt{100} = 10\text{cm}$	**Step 6** Take the positive square root. Ten is longer than either shorter side.

Example 02 · Find a shorter side

A right triangle has hypotenuse 13 cm and one shorter side 5 cm. Find the other side.

Why this method?

Identify the right angle and the longest side. The longest side has its square alone on one side of the equation.

$a^2 + 5^2 = 13^2$	**Step 1** Write the equation
$a^2 = 169 - 25 = 144$	**Step 2** Subtract the square of the known shorter side from the square of the hypotenuse.
$a = 12\text{cm}$	**Step 3** Take the square root to undo squaring. Use the positive root because this value is a length.

Example 03 · Test the converse

Does a triangle with sides 7 cm, 9 cm and 12 cm have a right angle?

Why this method?

Identify the right angle and the longest side. The longest side has its square alone on one side of the equation.

$12^2 = 144$	**Step 1** Square the longest side
$7^2 + 9^2 = 49 + 81 = 130$	**Step 2** Add the other squares

144 is not 130, so the triangle is not right-angled.	**Step 3** Compare the two calculated quantities. Equality or inequality determines which conclusion is justified.

Example 04 · Find the missing shorter side

A right triangle has hypotenuse 13 cm and one shorter side 5 cm. Find the other.

$x^2 + 5^2 = 13^2$	**Step 1** Put the hypotenuse by itself.
$x^2 = 169 - 25 = 144$	**Step 2** Subtract the known square.
$x = 12$ cm	**Final step** Take the positive square root for a length.

Common mistake

Using $a^2 + b^2 = c^2$ for a triangle that is not right-angled. It applies to right-angled triangles, and c must be the hypotenuse: the side opposite the right angle.

Exam technique

Mark the right angle and identify c before substituting.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

A right triangle has shorter sides 9 cm and 12 cm. Find the hypotenuse.

Question 2

A right triangle has hypotenuse 17 cm and one shorter side 8 cm. Find the other side.

Watch an explanation

Cognito — Pythagoras' Theorem

Corbettmaths — Pythagoras: show triangle is right angle

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

$c^2 = 9^2 + 12^2 = 225$, so $c = 15\text{cm}$.

The hypotenuse is opposite the right angle and must be longer than 12 cm.

Question 2

$a^2 = 17^2 - 8^2 = 225$, so $a = 15\text{cm}$.

Subtract the square of the known shorter side. The unknown shorter side must be less than 17 cm.

Open the full chapter online

Right-angled trigonometry

Leaving Certificate · Ordinary Level

Understand the idea

How to start

These sine, cosine and tangent ratios apply to right-angled triangles. The side names depend on the angle you choose.

1. Label opposite, adjacent and hypotenuse from the chosen angle.
2. Choose the ratio containing the known side and the side you need.
3. Write the equation, rearrange it and calculate in degree mode.

Key terms & definitions

Opposite
The side across from the chosen angle.

Adjacent
The side beside the chosen angle other than the hypotenuse.

Inverse trigonometric function
A calculator function used to recover an angle from a ratio.

Key ideas

• Relative to the chosen angle, label the opposite side, adjacent side and hypotenuse.

• $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$, $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$, $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$.

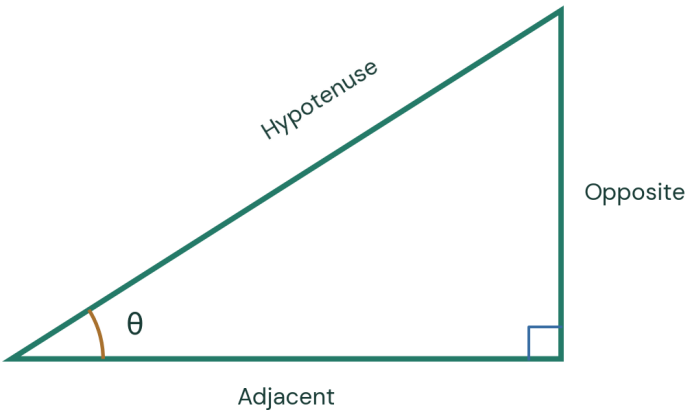
• Choose the ratio containing the known side and the side you need. Write the equation before rearranging.

• For an angle, use the inverse ratio on your calculator, such as $\theta = \sin^{-1}\left(\frac{\text{opposite}}{\text{hypotenuse}}\right)$.

• Use degree mode. Keep extra decimal places during working, including decimal angles.

• On the unit circle, a point at angle θ has coordinates $(\cos\theta, \sin\theta)$. The signs of the coordinates tell you the signs of cosine and sine.

• For angles from 90° to 180°, sine is positive and cosine is negative; from 180° to 270° both are negative; from 270° to 360° cosine is positive and sine is negative.



Label sides relative to θ: opposite across from θ, adjacent beside θ, hypotenuse opposite the right angle.

Quick reminder

• Label opposite, adjacent and hypotenuse relative to the angle.

• Choose the ratio containing the known and unknown sides.

• Use degree mode; use an inverse ratio to find an angle.

Worked examples

Example 01 · Start here: choose the ratio before calculating

A right triangle has hypotenuse 10 cm and angle 30°. Find the side opposite this angle.

Why this method?

Label the triangle relative to the marked angle. Choose sine because the question uses opposite and hypotenuse; do not choose a ratio just because it is familiar.

$\sin 30^\circ = \frac{o}{10}$	**Step 1** The known side is the hypotenuse and the unknown is opposite. Sine connects exactly these two sides.
$o = 10\sin 30^\circ$	**Step 2** Multiply both sides by 10 to undo the division by 10.
$o = 10(0.5) = 5$	**Step 3** In degree mode, $\sin 30^\circ$ is 0.5.

The opposite side is 5cm.	**Step 4** State the length and its unit.

Example 02 · Find an angle

A right triangle has opposite side 7 cm and hypotenuse 12 cm relative to angle θ. Find θ to one decimal place.

Why this method?

Label opposite, adjacent and hypotenuse relative to the chosen angle. Choose the ratio that contains the known and required sides.

$\sin\theta = \frac{7}{12}$	**Step 1** Choose sine
$\theta = \sin^{-1}(\frac{7}{12}) \approx 35.685^\circ$	**Step 2** Use inverse sine in degree mode
$\theta \approx 35.7^\circ$	**Step 3** Round at the end

Example 03 · Find a denominator side

A ladder makes an angle of 65° with level ground and reaches $4m$ up a vertical wall. Find its length.

Why this method?

Label opposite, adjacent and hypotenuse relative to the chosen angle. Choose the ratio that contains the known and required sides.

The ladder is the hypotenuse; the height is opposite the 65° angle.	**Step 1** Label the right triangle
$\sin 65^\circ = \frac{4}{L}$, so $L = \frac{4}{\sin 65^\circ}$.	**Step 2** Choose and rearrange sine
$L \approx 4.41\text{m}$	**Step 3** Divide 4 by $\sin 65^\circ$ in degree mode, then round the length to two decimal places.

Example 04 · An angle beyond 90 degrees

Find $\sin 150^\circ$ and $\cos 150^\circ$.

Why this method?

Read the requested quantity, choose the matching rule and follow the reason beside each line.

$180^\circ - 150^\circ = 30^\circ$	**Step 1** The reference angle is 30°. The point is in the upper-left quarter of the unit circle.
$\sin 150^\circ = \sin 30^\circ = \frac{1}{2}$	**Step 2** The vertical coordinate is positive.
$\cos 150^\circ = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$	**Step 3** The horizontal coordinate is negative.

Example 05 · Use an exact ratio outside the first quadrant

Find $\sin 150^\circ$ and $\cos 150^\circ$ exactly.

$180^\circ - 150^\circ = 30^\circ$	**Step 1** Find the reference angle.
$\sin 150^\circ = \frac{1}{2}$	**Step 2** In quadrant II, y is positive.
$\cos 150^\circ = -\frac{\sqrt{3}}{2}$	**Final step** In quadrant II, x is negative.

Common mistake

Treating the same side as “opposite” whichever angle is used. Identify the chosen angle first. The opposite side faces that angle; the adjacent side touches it and is not the hypotenuse.

Exam technique

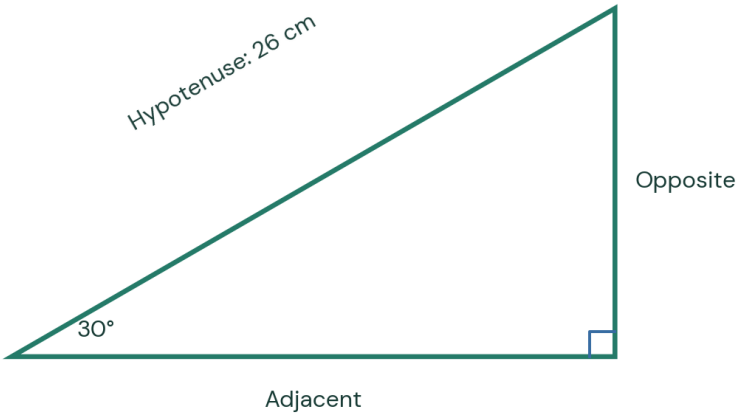
An angle of elevation is measured upwards from a horizontal line.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

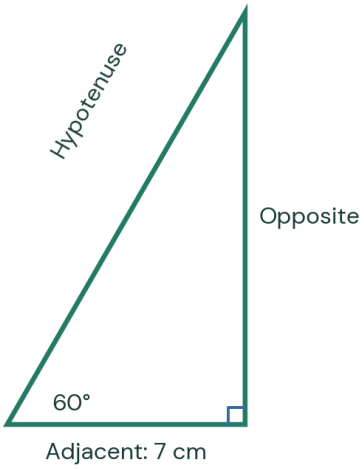
In a right-angled triangle, the angle θ is 30°. The hypotenuse is 26 cm. Find the side opposite θ.



Right triangle for this question. Angle 30 degrees; known hypotenuse is 26 cm. Other sides are not labelled with answers.

Question 2

In a right-angled triangle, the angle θ is 60°. The side adjacent to θ is 7 cm. Find the hypotenuse.



Right triangle for this question. Angle 60 degrees; known adjacent to θ is 7 cm. Other sides are not labelled with answers.

Watch an explanation

Corbettmaths — Trigonometry introduction:

Corbettmaths — Trigonometry missing sides:

Corbettmaths — Trigonometry missing angles:

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

13

Question 2

14

Open the full chapter online

Sine rule, cosine rule and triangle area

Leaving Certificate · Ordinary Level

Understand the idea

How to start

For a triangle without a right angle, choose a rule from the information given. Side a is opposite angle A.

1. Match every side with the angle opposite it.
2. Choose sine rule for opposite pairs, cosine rule for three sides or two sides with the included angle.
3. Write the formula, substitute the values and use degree mode.

Key terms & definitions

Included angle
The angle between two named sides.

Opposite pair
A side and the angle facing it.

Key ideas

• Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B}$. Match each side with its opposite angle.

• Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$. Use two sides and the angle between them, or all three sides.

• Triangle area: $A = \frac{1}{2}bc\sin A$ when A is the angle between the known sides.

• Use degree mode and keep calculator digits until the final rounding.

• The sine rule can sometimes give two possible triangles. Check the given diagram and angle conditions.

Quick reminder

• Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B}$. Match each side with its opposite angle.

• Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$. Use two sides and the angle between them, or all three sides.

Worked examples

Example 01 · Sine rule with clear opposite pairs

$A = 30^\circ$, $a = 5$ cm and $B = 60^\circ$. Find b.

Why this method?

Identify the known sides and angles first, then match them to the rule used below.

$\frac{b}{\sin 60^\circ} = \frac{5}{\sin 30^\circ}$	**Step 1** Pair b with 60° and 5 with 30°.
$b = \frac{5\sin 60^\circ}{\sin 30^\circ}$	**Step 2** Multiply both sides by $\sin 60^\circ$.
$b \approx 8.66\text{cm}$	**Step 3** Calculate in degree mode, then round.

Example 02 · Cosine rule to find a side

Two sides are 3 cm and 4 cm, with included angle 60°. Find the opposite side a.

Why this method?

Identify the known sides and angles first, then match them to the rule used below.

$a^2 = 3^2 + 4^2 - 2(3)(4)\cos 60^\circ$	**Step 1** Use the two sides and the angle between them.
$a^2 = 25 - 12 = 13$	**Step 2** Cos 60° is one half.
$a = \sqrt{13} \approx 3.61\text{cm}$	**Step 3** Take the positive square root.

Example 03 · Area using an included angle

Find the area with sides 4 cm, 6 cm and included angle 30°.

Why this method?

Identify the known sides and angles first, then match them to the rule used below.

$A = \frac{1}{2}(4)(6)\sin 30^\circ$	**Step 1** The given angle is between the known sides.
$A = 12(0.5) = 6\text{cm}^2$	**Step 2** Use $\sin 30^\circ = 0.5$ and include square units.

Example 04 · Find an angle with the cosine rule

A triangle has sides 3 cm, 4 cm and 5 cm. Find the angle A opposite the 5 cm side.

Why this method?

All three sides are known. Use the cosine rule and rearrange it before pressing inverse cosine.

$a = 5, \quad b = 3, \quad c = 4$	**Step 1** a must be the side opposite the angle A that we want.
$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$	**Step 2** Write the angle version of the cosine rule.
$\cos A = \frac{3^2 + 4^2 - 5^2}{2(3)(4)}$	**Step 3** Keep the entire numerator above the fraction bar.
$\cos A = \frac{9 + 16 - 25}{24} = 0$	**Step 4** Square the side lengths, then simplify.
$A = \cos^{-1}(0) = 90^\circ$	**Step 5** Use inverse cosine in degree mode.

Example 05 · Cosine rule with two sides and the included angle

Two sides are 5 cm and 7 cm, with included angle 60°. Find the opposite side.

$a^2 = 5^2 + 7^2 - 2(5)(7)\cos 60^\circ$	**Step 1** Use the included angle in the cosine rule.
$a^2 = 25 + 49 - 35 = 39$	**Step 2** Evaluate in degree mode.

$a = \sqrt{39} \approx 6.24$ cm	**Final step** Take the positive square root.

Common mistake

Pairing a side with an angle beside it in the sine rule. In $\frac{a}{\sin A} = \frac{b}{\sin B}$, side a must be opposite angle A, and side b opposite angle B.

Exam technique

Label a, b, c and their opposite angles before substituting.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Find the area with sides 5 cm and 8 cm and included angle 90°.

Question 2

Find the area of a triangle with sides 6 cm and 8 cm enclosing 30°.

Question 3

In a triangle, $A = 30^\circ$, $a = 4$ cm and $B = 90^\circ$. Find b.

Self-check answers

Question 1

20cm^2

Question 2

$A = \frac{1}{2}(6)(8)\sin 30^\circ = 12\text{cm}^2.$

The angle must be between the two known sides.

Question 3

$b = \frac{4\sin 90^\circ}{\sin 30^\circ} = 8\text{cm}.$

Match a with its opposite angle A and b with B.

Open the full chapter online

Coordinate geometry

Leaving Certificate · Ordinary Level

Understand the idea

How to start

In $(3, 2)$, move 3 units horizontally first and 2 units vertically second.

1. Write the coordinates of both points clearly.
2. For midpoint, average matching coordinates. For slope or distance, find the changes between the points.
3. Keep the same point order in both subtractions.

Key terms & definitions

Coordinate
A number locating a point relative to axes.

Midpoint
The point halfway between two endpoints.

Slope
Vertical change divided by horizontal change.

Key ideas

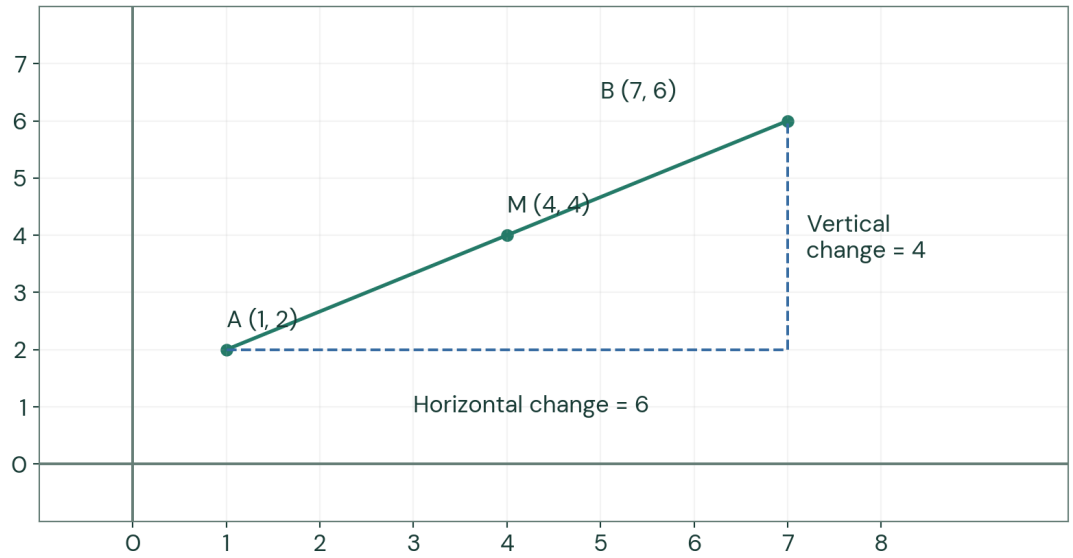
• An ordered pair (x, y) gives horizontal movement first and vertical movement second.

• The midpoint is $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$.

• Distance is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, from Pythagoras.

• Slope is $m = \frac{y_2 - y_1}{x_2 - x_1}$ when the denominator is non-zero.

• A horizontal line has slope zero. A vertical line has undefined slope and equation $x = k$.



From A to B: slope is $\frac{4}{6} = \frac{2}{3}$; the midpoint averages the coordinates to give $(4, 4)$.

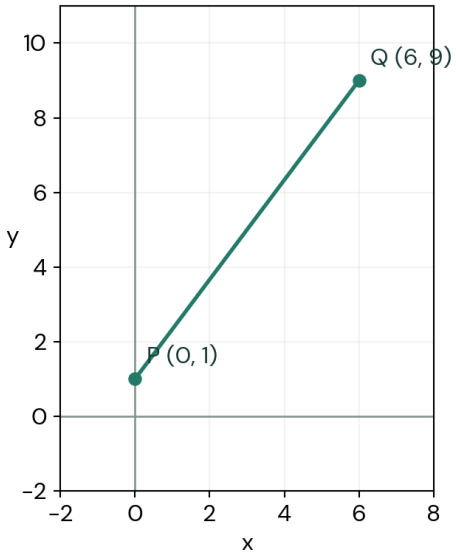
Quick reminder

- Keep the same point order in each subtraction.
- Midpoint: average the x-values and the y-values.
- Slope: vertical change ÷ horizontal change.

Worked examples

Example 01 · Distance between two points

$P(0, 1)$ and $Q(6, 9)$ are two points. Find the distance between them.



Coordinates matching this question : $P(0, 1), Q(6, 9)$.

Blue follows the horizontal change. Purple follows the vertical change. They become the two shorter sides used in Pythagoras.

Why this method?

Write the coordinates in the same point order. Decide whether we need a distance, a midpoint or a slope before substituting.

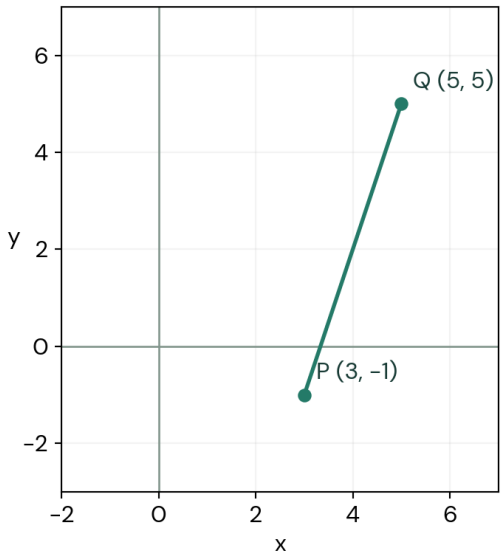
$6 - 0 = 6$	**Step 1** Find the horizontal change from P to Q. The x-coordinate changes from 0 to 6.
$9 - 1 = 8$	**Step 2** Find the vertical change in the same point order. The y-coordinate changes from 1 to 9.
$d^2 = 6^2 + 8^2$	**Step 3** These changes are the perpendicular sides of a right triangle. The straight-line distance is its hypotenuse.
$d^2 = 36 + 64 = 100$	**Step 4** Square each change and add.

$d = \sqrt{100} = 10$	**Step 5** Take the positive square root because a distance cannot be negative.

Think about it: Before moving on: why do the horizontal and vertical distances not simply add up to the distance?

Example 02 · Midpoint of a line segment

$P(3, -1)$ and $Q(5, 5)$ are two points. What is the x-coordinate of the midpoint of PQ?



Coordinates matching this question : $P(3, -1), Q(5, 5)$.

Why this method?

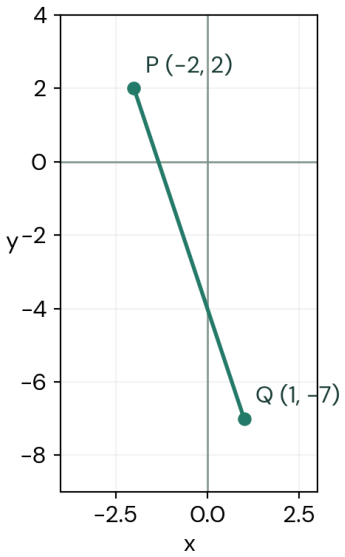
Write the coordinates in the same point order. Decide whether we need a distance, a midpoint or a slope before substituting.

$x_M = \frac{x_1 + x_2}{2}$	**Step 1** The midpoint x-coordinate is halfway between the two x-coordinates.
$x_M = \frac{3 + 5}{2}$	**Step 2** Use 3 and 5, the x-coordinates of P and Q. The whole sum stays above the fraction bar.
$x_M = \frac{8}{2} = 4$	**Step 3** Add first, then divide by 2.
The x-coordinate is 4.	**Step 4** It lies halfway between 3 and 5. The question does not ask for the y-coordinate.

Think about it: Before moving on: why must the sum of the coordinates be halved?

Example 03 · Slope of a line through two points

$P(-2, 2)$ and $Q(1, -7)$ lie on a straight line. What is its slope?



Coordinates matching this question : $P(-2, 2), Q(1, -7)$.

Why this method?

Write the coordinates in the same point order. Decide whether we need a distance, a midpoint or a slope before substituting.

$(x_1, y_1) = (-2, 2)$ and $(x_2, y_2) = (1, -7)$.	**Step 1** Label the first point and second point. Keep the same order for both subtractions.
$m = \frac{-7-2}{1-(-2)}$	**Step 2** Slope is vertical change divided by horizontal change. Put the whole differences above and below the fraction bar.
$m = \frac{-9}{3}$	**Step 3** Subtracting -2 is adding 2, so the horizontal change is $+3$.
$m = -3$	**Step 4** Divide. A negative slope means the line falls as x increases.

Think about it: Before moving on: why does a falling line have a negative slope?

Example 04 · Area from three coordinate points

Find the area of the triangle with vertices $(1, 2), (7, 2)$ and $(4, 6)$.

Base $= 7 - 1 = 6$	**Step 1** The first two points have the same y-coordinate.
Height $= 6 - 2 = 4$	**Step 2** Use perpendicular distance above the horizontal base.
$A = \frac{1}{2}(6)(4) = 12$ square units	**Final step** Use half base times height.

Common mistake

Reversing the point order in only half of the slope formula. For $(1, 2)$ and $(3, 6)$, use $\frac{6-2}{3-1} = 2$, not $\frac{6-2}{1-3} = -2$. Keep the same point order in both differences.

Exam technique

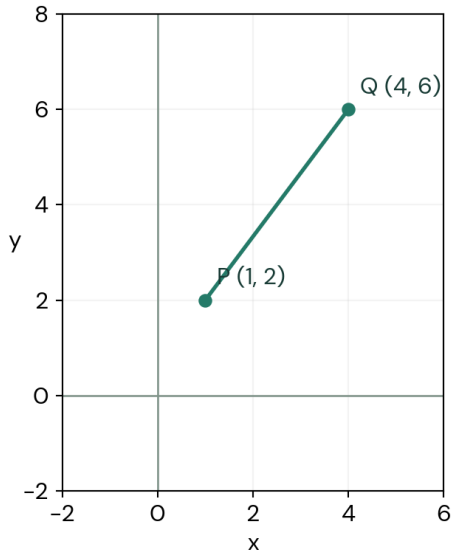
Sketch the two points to check the signs and approximate distance.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

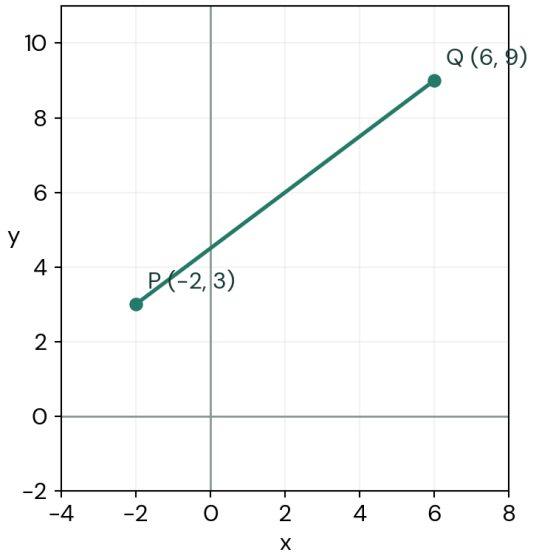
$P(1, 2)$ and $Q(4, 6)$ are two points. Find the distance between them.



Coordinates matching this question : $P(1, 2), Q(4, 6)$.

Question 2

$P(-2, 3)$ and $Q(6, 9)$ are two points. What is the x-coordinate of the midpoint of PQ?



Coordinates matching this question : $P(-2, 3), Q(6, 9)$.

Watch an explanation

Corbettmaths — Linear graphs: length of a line

Cognito — Finding the Gradient Using Coordinates

Cognito — Finding the Midpoint of a Line

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

5

Question 2

2

Open the full chapter online

Straight lines and their equations

Leaving Certificate · Ordinary Level

Understand the idea

How to start

In $y = mx + c$, m tells you the slope and c tells you where the line meets the y-axis.

1. Identify the slope and a known point, or read them from the graph.
2. Put the point coordinates into the line equation to find a missing constant.
3. Check that the completed equation gives the correct y for the known x.

Key terms & definitions

Y-intercept
The y-value when x is zero.

Point-slope form
An equation using one known point and the slope.

General form
A line written as $ax + by + c = 0$ with a and b not both zero.

Key ideas

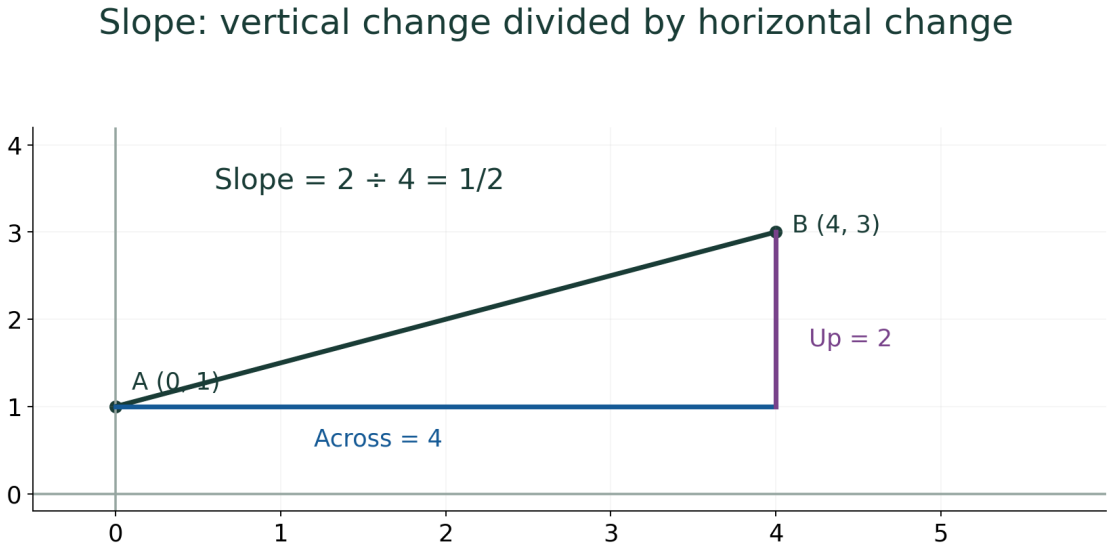
• In $y = mx + c$, m is the slope and c is the y-intercept.

• Given a point (x_1, y_1) and slope m, use $y - y_1 = m(x - x_1)$.

• Parallel non-vertical lines have equal slopes. Perpendicular lines with defined non-zero slopes satisfy $m_1 m_2 = -1$.

• For $ax + by + c = 0$ with $b \neq 0$, the slope is $-\frac{a}{b}$. If b is zero, the line is vertical.

• Find an intersection by solving both equations together. Interpret the point in the context.



From $A(0, 1)$ to $B(4, 3)$, move 4 across and 2 up. The slope is vertical change over horizontal change : $\frac{2}{4} = \frac{1}{2}$.

Quick reminder

- $y = mx + c$: m is slope; c is y-intercept.
- Parallel lines have equal slopes; check vertical-line exceptions.
- At an intersection, set the two expressions for y equal.

Worked examples

Example 01 · A parallel line through a point

A line passes through $P(-2, -3)$ and is parallel to $y = -3x + 2$. What is the y-intercept of this line?

Why this method?

Identify the slope and any known point. Use these to form an equation, and check it using the original information.

$y = -3x + c$	**Step 1** The given line has slope -3. A parallel line has the same slope, but may have a different intercept c.
$-3 = -3(-2) + c$	**Step 2** The new line passes through $(-2, -3)$, so put $x = -2$ and $y = -3$ into its equation.

$-3 = 6 + c$	**Step 3** Calculate $-3 \times (-2) = +6$.
$-3 - 6 = c$	**Step 4** Subtract 6 from both sides to leave c on its own.
$c = -9$	**Step 5** The y-intercept value is -9, at the point $(0, -9)$.
$-3 = -3(-2) - 9$	**Step 6** Check the known point: the right side is $6 - 9 = -3$.

Think about it: Before moving on: why does a parallel line have the same slope as the line it is parallel to?

Example 02 · Where two lines meet

The lines $y = 1x + 6$ and $y = 2x + 3$ meet at one point. What is the x-coordinate of that point?

Why this method?

Identify the slope and any known point. Use these to form an equation, and check it using the original information.

$x + 6 = 2x + 3$	**Step 1** At the intersection both expressions have the same y-value. Set them equal.
$6 = x + 3$	**Step 2** Subtract x from both sides to leave one x on the right.
$x = 3$	**Step 3** Subtract 3 from both sides.
$y = 3 + 6 = 9$	**Step 4** The question asks only for x. To see the complete point, substitute $x = 3$ into either line.
$y = 2(3) + 3 = 9$. The intersection is $(3, 9)$.	**Step 5** Check the other line; it must give the same y.

Think about it: Before moving on: why do the two lines have the same y-value at the point where they meet?

Example 03 · Rate of change with a starting amount

A tank holds 55 litres and loses 6 litres each minute. How many litres are in the tank after 4 minutes?

Why this method?

Identify the slope and any known point. Use these to form an equation, and check it using the original information.

$6 \times 4 = 24$	**Step 1** Find the total lost : 6 litres each minute for 4 minutes is 24 litres.
$55 - 24 = 31$	**Step 2** Subtract the loss from the starting 55 litres.
There are 31 litres left.	**Step 3** The result is smaller than 55 litres, as expected for a leak.

Think about it: Before moving on: why is the starting amount needed as well as the rate?

Example 04 · The equation of a line from its slope and a point

A straight line has slope -2 and passes through the point $(2, -5)$. Find its equation in the form $y = mx + c$.

Why this method?

Identify the slope and any known point. Use these to form an equation, and check it using the original information.

$c = -5 - (-2 \times 2) = -1$	**Step 1** Substitute the slope m and the point (x_1, y_1) into $c = y_1 - mx_1$
$y = -2x - 1$	**Step 2** Write the line with that intercept

Think about it: Before moving on: why does the intercept c come from subtracting mx_1 from y_1?

Example 05 · Find a perpendicular equation

Find the line through $(2, 3)$ perpendicular to $y = 2x - 1$.

$m = -\frac{1}{2}$	**Step 1** Take the negative reciprocal of 2.

$y-3=-\frac{1}{2}(x-2)$	**Step 2** Use the point–slope form.
$y=-\frac{1}{2}x+4$	**Final step** Expand and rearrange.

Common mistake

Using a negative slope to mean "perpendicular." A line of slope 2 is perpendicular to one of slope $-\frac{1}{2}$, not -2. For two finite slopes, the product must be -1. A horizontal line is perpendicular to a vertical line, whose slope is undefined.

Exam technique

Substitute the given point into your final equation to check it.

Try it yourself

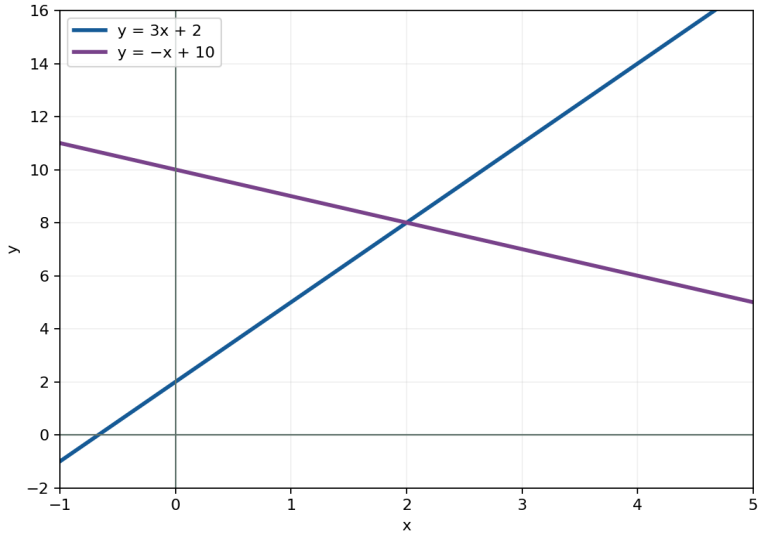
Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

A line passes through $P(4, 9)$ and is parallel to $y = 2x - 3$. What is the y-intercept of this line?

Question 2

The lines $y = 3x + 2$ and $y = -1x + 10$ meet at one point. What is the x-coordinate of that point?



Two lines $y = 3x + 2$ and $y = -x + 10$ on numbered axes. Use their intersection to read the x-coordinate.

Watch an explanation

Cognito — What is 'y = mx + c'

Cognito — Finding the Equation of a Line from 2 Coordinates

Cognito — Perpendicular Lines

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

1

Question 2

2

Open the full chapter online

Equations of circles

Leaving Certificate · Ordinary Level

Understand the idea

How to start

A circle consists of points at a fixed distance from its centre. That fixed distance is the radius.

1. Compare the equation with the standard circle form.
2. Read the centre carefully; a bracket $(x - a)$ uses centre coordinate a.
3. Take the positive square root of the right-hand number to find the radius.

Key terms & definitions

Centre
The fixed point in the middle of the circle.

Radius
Distance from the centre to the circle.

Key ideas

• Centre (h, k) and radius r give $(x - h)^2 + (y - k)^2 = r^2$.

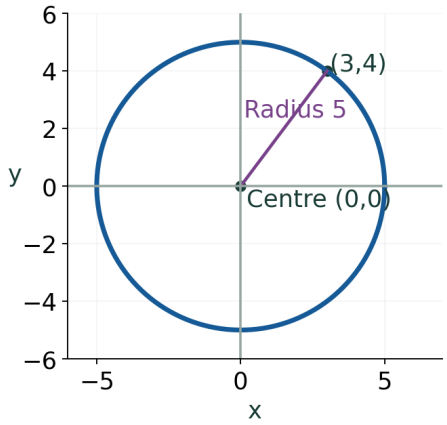
• The signs inside the brackets are opposite to the centre coordinates.

• A circle centred at the origin has equation $x^2 + y^2 = r^2$.

• To test a point, substitute both coordinates. It lies on the circle only if the equation is true.

• To find line–circle intersections, substitute the line relationship into the circle and solve.

A circle: every point is one radius from the centre



The circle $x^2 + y^2 = 25$ has centre $(0, 0)$ and radius 5. The point $(3, 4)$ lies on it because $3^2 + 4^2 = 25$.

Quick reminder

• Centre (h, k) and radius r give $(x - h)^2 + (y - k)^2 = r^2$.

• The signs inside the brackets are opposite to the centre coordinates.

Worked examples

Example 01 · Start here: read a radius from the equation

Find the centre and radius of $x^2 + y^2 = 36$.

Why this method?

Compare with $x^2 + y^2 = r^2$. The number on the right is the radius squared, not the radius.

$r^2 = 36$	**Step 1** The centre is $(0, 0)$ because neither x nor y has a shift inside a bracket.
$r = \sqrt{36} = 6$	**Step 2** Take the positive square root.
Centre $= (0, 0)$, $\quad r = 6$	**Step 3** Give both pieces of information.

Example 02 · Circle at the origin

Write the equation of a circle with centre $(0, 0)$ and radius 5.

Why this method?

Compare the equation with the standard circle form before calculating.

$x^2 + y^2 = 5^2$	**Step 1** The centre is the origin, so no shifts are needed.
$x^2 + y^2 = 25$	**Step 2** Square the radius.

Example 03 · Read centre and radius

Find the centre and radius of $(x - 2)^2 + (y + 3)^2 = 16$.

Why this method?

Compare the equation with the standard circle form before calculating.

$h = 2, \quad k = -3$	**Step 1** Compare with $x - h$ and $y - k$. A plus 3 means k is -3.
$r = \sqrt{16} = 4$	**Step 2** The right side is radius squared, so take a square root.

Example 04 · A line meets a circle

Find where $y = 3$ meets $x^2 + y^2 = 25$.

Why this method?

Compare the equation with the standard circle form before calculating.

$x^2 + 3^2 = 25$	**Step 1** Substitute $y = 3$ into the circle.
$x^2 = 16$	**Step 2** Subtract 9 from both sides.
$x = 4 \quad \text{or} \quad x = -4$	**Step 3** Both signs square to 16.

$(4, 3), \quad (-4, 3)$	**Step 4** Use $y = 3$ for both points.

Example 05 · Find the points where a line crosses a circle

Find the intersections of $x = 3$ with $x^2 + y^2 = 25$.

$3^2 + y^2 = 25$	**Step 1** Put the stated x value into the circle equation.
$y^2 = 16$	**Step 2** Subtract 9.
$(3, 4)$ and $(3, -4)$	**Final step** Both signs work; write both coordinate pairs.

Common mistake

Reading the number on the right as the radius. In $x^2 + y^2 = 25$, 25 is r^2, so the radius is $\sqrt{25} = 5$, not 25.

Exam technique

Check each intersection point in both original equations.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Write the circle with centre $(1, 2)$, radius 3.

Question 2

Find the centre and radius of $(x - 2)^2 + (y + 1)^2 = 9$.

Question 3

Does $(0, 5)$ lie on $x^2 + y^2 = 25$?

Self-check answers

Question 1

$(x-1)^2+(y-2)^2=9$

Question 2

Centre $(2,\ -1)$ and radius 3.

The signs inside the brackets are opposite to the centre coordinates.

Question 3

Yes: $0^2+5^2=25$.

Substitute both coordinates into the equation.

Open the full chapter online

Circle angles and cyclic quadrilaterals

Leaving Certificate · Ordinary Level

Understand the idea

How to start

A radius joins the centre to the circle. A diameter passes through the centre and has length two radii.

1. Mark the centre, diameter or chord named in the question.
2. Check which arc or chord the angles stand on before choosing a fact.
3. Write the angle relationship and explain why it applies to these angles.

Key terms & definitions

Chord
A segment joining two points on a circle.

Arc
A part of the circumference.

Cyclic quadrilateral
A quadrilateral with all four vertices on one circle.

Key ideas

• The angle at the centre is twice the angle at the circumference standing on the same arc.

• Angles standing on the same chord and in the same segment are equal. The side-of-chord condition matters.

• An angle in a semicircle is a right angle. Conversely, a chord subtending a right angle at the circumference is a diameter.

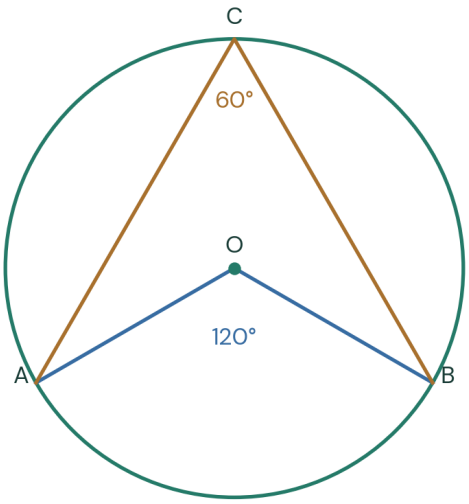
• A cyclic quadrilateral has all four vertices on a circle. Its opposite angles total 180°.

• Explain a circle angle by naming the relevant arc or diameter; do not assume a line through a sketch is a diameter unless given.

• A tangent is perpendicular to the radius at the point of contact.

• The angle between a tangent and a chord equals the angle in the alternate segment.

• The perpendicular from the centre to a chord bisects the chord.



On the same lower arc AB, angle AOB at the centre is 120° and angle ACB at the circumference is 60°.

Quick reminder

• Use the same arc when comparing centre and circumference angles.

• An angle in a semicircle is 90°.

• Opposite angles in a cyclic quadrilateral total 180°.

Worked examples

Example 01 · Centre and circumference

The angle at the centre standing on an arc is 124°. Find the angle at the circumference on the same arc.

Why this method?

Identify the arc, chord or diameter named in the question. Use the circle result whose conditions match that information.

$2\theta = 124^\circ$	**Step 1** Let θ be the angle at the circumference. The angle at the centre standing on the same arc is twice θ.
$\theta = \frac{124^\circ}{2}$	**Step 2** Divide both sides by 2 because we want one θ.

$\theta = 62^\circ$	**Step 3** The circumference angle is half the given centre angle.
$2(62^\circ) = 124^\circ$	**Step 4** Check the angle relationship.

Example 02 · Cyclic quadrilateral

ABCD is cyclic. Angle A is 108° and angle B is 76°. Find C and D.

Why this method?

Identify the arc, chord or diameter named in the question. Use the circle result whose conditions match that information.

$C = 180^\circ - 108^\circ = 72^\circ$	**Step 1** Opposite angles are supplementary
$D = 180^\circ - 76^\circ = 104^\circ$	**Step 2** Use the other opposite pair
$108 + 76 + 72 + 104 = 360$	**Step 3** Add the four interior angles. A quadrilateral should have a total of 360°.

Example 03 · An angle in a semicircle

AB is a diameter and C is on the circle. Angle CAB is 34°. Find angle ABC.

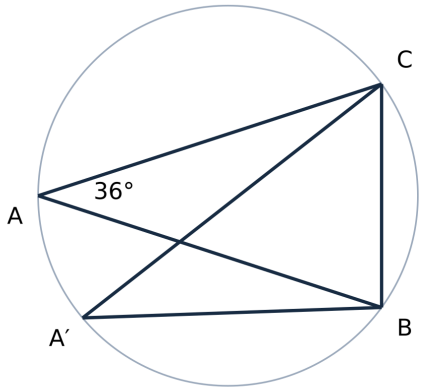
Why this method?

Identify the arc, chord or diameter named in the question. Use the circle result whose conditions match that information.

$\angle ACB = 90^\circ.$	**Step 1** The stated diameter makes the angle at the circumference a right angle. We can then use the triangle angle sum.
$\angle ABC = 180^\circ - 90^\circ - 34^\circ = 56^\circ$	**Step 2** Use the triangle angle sum

Example 04 · Corollaries 1, 2 and 5 — another example

A, A□, B and C lie on a circle. A and A□ are on the same side of line BC. Angle BAC $= 36^\circ$. Find angle BA□C.



A and A′ are on the same side of BC.

Angles standing on the same arc. Diagram not to scale.

Why this method?

Identify the arc, chord or diameter named in the question. Use the circle result whose conditions match that information.

A and A□ are on the same side of BC; all four points are distinct.	**Step 1** Check the same-arc condition
angle BAC = angle BA□C.	**Step 2** Apply corollary 2
angle BA□$C = 36^\circ$.	**Step 3** Substitute

Example 05 · Find arc length and sector area

$A72^\circ$ sector has radius 10 cm. Find its arc length and area.

$\frac{72}{360} = \frac{1}{5}$	**Step 1** The sector is one fifth of the full circle.
$L = \frac{1}{5}(2\pi \cdot 10) = 4\pi$ cm	**Step 2** Take one fifth of the circumference.

$A = \frac{1}{5}(\pi \cdot 10^2) = 20\pi \text{ cm}^2$	**Final step** Take one fifth of the area.

Common mistake

Adding adjacent angles of a cyclic quadrilateral to get 180°. It is the opposite angles that add to 180°: if one angle is 110°, its opposite angle is 70°.

Exam technique

Draw the radii to see which central angle stands on the same arc.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

The angle at the centre standing on an arc is 136°. Find the angle at the circumference on the same arc.

Question 2

ABCD is cyclic. Angle A is 112° and angle B is 82°. Find C and D.

Self-check answers

Question 1

$136^\circ \div 2 = 68^\circ$.

The centre angle is twice the circumference angle on the same arc.

Question 2

$C = 68^\circ$ and $D = 98^\circ$.

Subtract each given angle from 180°. Pair opposite angles: A with C, and B with D.

Open the full chapter online

Enlargements and scale factors

Leaving Certificate · Ordinary Level

Understand the idea

How to start

An enlargement multiplies every distance from a stated centre by the same scale factor.

1. Write down the scale factor k.
2. Multiply a length by k, an area by k squared or a volume by k cubed.
3. Check the corresponding units.

Key terms & definitions

Scale factor
New corresponding length divided by original length.

Centre of enlargement
The point from which all distances scale.

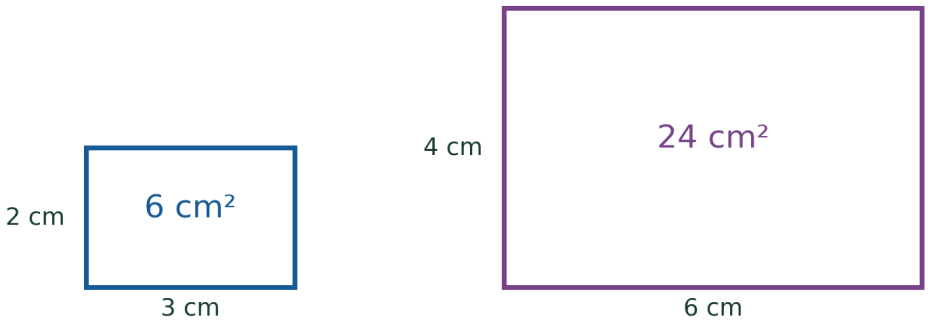
Key ideas

• A scale factor of 2 doubles every length. A factor of one half halves every length.

• Lengths scale by k; areas by k^2 and volumes by k^3 for similar solids.

• Corresponding angles stay the same.

• Use matching lengths to find the scale factor, then use the correct power for area or volume.



A 3 cm by 2 cm rectangle has area 6 cm^2. Doubling both lengths makes a 6 cm by 4 cm rectangle with area 24 cm^2: four times as large.

Quick reminder

- A scale factor of 2 doubles every length. A factor of one half halves every length.
- Lengths scale by k; areas by k^2 and volumes by k^3 for similar solids.

Worked examples

Example 01 · Start here: a length changes by the scale factor

A drawing enlarges a 4 cm length by scale factor 3. Find its new length.

Why this method?

A length is multiplied by the scale factor once. Area and volume use different powers.

$\text{New length} = 3(4)$	**Step 1** Multiply the original length by 3.
$\text{New length} = 12\text{cm}$	**Step 2** The answer is a length, so use cm.

Example 02 · Scale a length

A side is 3 cm. Enlarge it by scale factor 2.

Why this method?

Identify whether the question asks for length, area or volume before choosing a power of the scale factor.

$3(2) = 6\text{cm}$	**Step 1** Multiply the old length by 2.

Example 03 · Scale an area

A similar shape has length scale factor 2. Its original area is 6cm^2. Find the new area.

Why this method?

Identify whether the question asks for length, area or volume before choosing a power of the scale factor.

$k^2 = 2^2 = 4$	**Step 1** Area uses the square of the length scale factor.
$A = 6(4) = 24\text{cm}^2$	**Step 2** Multiply the old area by 4.

Example 04 · Enlarge a point from the origin

Enlarge $(2, 3)$ by scale factor 3 with centre $(0, 0)$.

Why this method?

Identify whether the question asks for length, area or volume before choosing a power of the scale factor.

$(x', y') = (3(2), 3(3))$	**Step 1** With centre at the origin, multiply both coordinates by 3.
$(6, 9)$	**Step 2** State the new point.

Example 05 · Shrink an area

An enlargement has scale factor 0.6. The original area is $50\ \text{cm}^2$. Find the image area.

$0.6^2 = 0.36$	**Step 1** Square the linear scale factor for area.
$50(0.36) = 18\ \text{cm}^2$	**Final step** Multiply the original area by the area factor.

Common mistake

Multiplying area by the length scale factor only once. If all lengths double, the area multiplies by $2^2 = 4$ and the volume by $2^3 = 8$.

Exam technique

Name the centre as well as the scale factor when describing an enlargement.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

A length is 10 cm. Reduce it with scale factor 0.4.

Question 2

$A5$ cm length is enlarged by scale factor 4. Find its new length.

Question 3

A solid of volume 10cm^3 is enlarged by scale factor 2. Find its new volume.

Self-check answers

Question 1

4cm

Question 2

20cm.

Lengths use the factor once.

Question 3

80cm^3.

Volume multiplies by 2 cubed, which is 8.

Open the full chapter online

Constructions and clear geometry reasoning

Leaving Certificate · Ordinary Level

Understand the idea

How to start

A construction uses compass arcs and a straightedge to locate points. Leave the arcs visible.

1. Read what you are asked to find.
2. Write down the known numbers and choose the matching rule.
3. Do one step at a time and check the result.

Key terms & definitions

Median
A segment joining a vertex to the midpoint of the opposite side.

Circumcentre
The intersection of perpendicular bisectors of triangle sides.

Incentre
The intersection of internal angle bisectors.

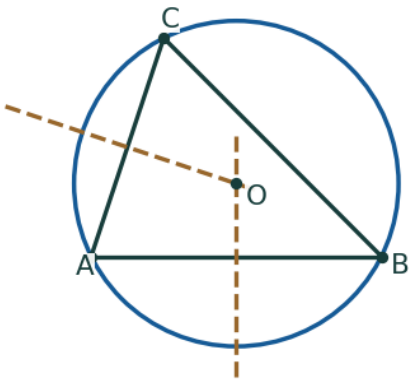
Key ideas

• State the given conditions when explaining a geometric result. A sketch alone does not establish equal lengths or parallel lines.

Construction methods and reasoning

16 · Circumcentre and circumcircle

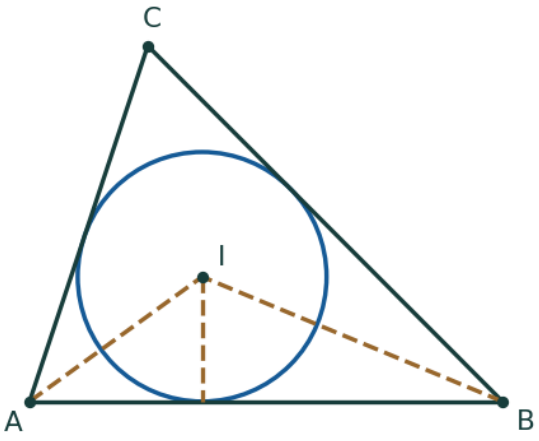
With compass and straightedge, construct perpendicular bisectors of two sides. Their intersection O is the centre. Draw the circle centred at O through any vertex.



Circumcentre and circumcircle — completed construction with guide lines

17 · Incentre and incircle

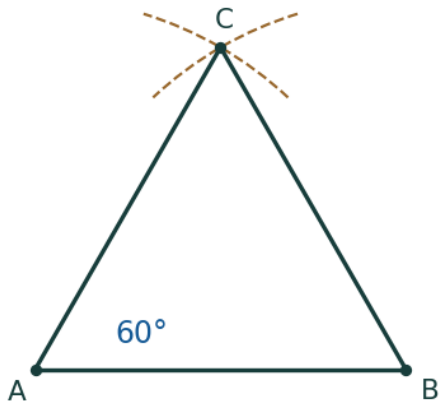
With compass and straightedge, bisect two internal angles. Their intersection I is the centre. Drop a perpendicular from I to a side to get the radius; draw the circle.



Incentre and incircle — completed construction with guide lines

18 · *A*60° **angle**

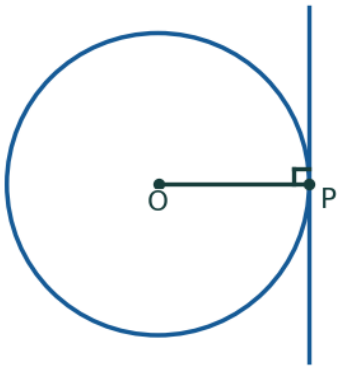
Without a protractor or set square, choose B on ray AB. Use the same radius AB to draw arcs centred at A and B. Join A to their intersection C. Triangle ABC is equilateral.



*A*60° angle — completed construction with guide lines

19 · **Tangent at a point on a circle**

Join centre O to the given point P. Construct the perpendicular to OP at P; this is the tangent.



Tangent at a point on a circle — completed construction with guide lines

20 · Parallelogram from sides and angle

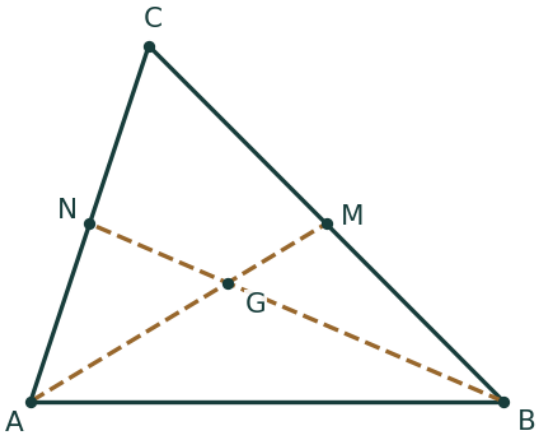
Draw AB to length. Construct the given angle at A and mark D at the second side length. Draw parallels to AD through B and to AB through D. Their intersection is C.



Parallelogram from sides and angle — completed construction with guide lines

21 · Centroid

Construct the midpoints of two sides. Join each midpoint to the opposite vertex. The two medians meet at the centroid.



Centroid — completed construction with guide lines

Quick reminder

• 16 : Circumcircle. Construct perpendicular bisectors of two triangle sides. Their intersection is the circumcentre. Draw a circle from it through a vertex.

• 17 : Incircle. Construct two internal angle bisectors. Their intersection is the incentre. Construct its perpendicular distance to a side as the radius.

Worked examples

Example 01 · Tangent at a known point

Construct a tangent at P on a circle with centre O.

Why this method?

Use the defining property of the required construction; keep the construction lines visible.

OP	**Step 1** Join the centre O to P.
ConstructaperpendiculartoOPatP.	**Step 2** The radius is perpendicular to the tangent at the point of contact.

Example 02 · Locate a circumcentre

Find the centre of the circle through three non-collinear points A, B, C.

Why this method?

Use the defining property of the required construction; keep the construction lines visible.

BisectABperpendicularly.	**Step 1** Every point on that bisector is equally distant from A and B.
BisectBCperpendicularly.	**Step 2** The intersection O is equally distant from A, B and C.
DrawthecirclewithcentreOandradiusOA.	**Step 3** The circle passes through all three points.

Example 03 · Locate a centroid

Construct the centroid of triangle ABC.

Why this method?

Use the defining property of the required construction; keep the construction lines visible.

ConstructthemidpointMofBC.	**Step 1** A median joins a vertex to the opposite side midpoint.
ConstructthemidpointNofAC.	**Step 2** We need two medians to locate their intersection.
JoinAtoMandBtoN.	**Step 3** Their intersection is the centroid.

Example 04 · Find an incentre and draw the incircle

Construct the incircle of a given triangle.

Bisect two internal angles.	**Step 1** Their crossing I is equally far from all three sides.
Draw the perpendicular from I to one side.	**Step 2** Its length is the radius; do not measure to a vertex.
Draw the circle centred at I.	**Final step** Set the compass to that perpendicular distance.

Common mistake

Erasing the compass arcs after finishing a construction. The arcs show how the required point or line was found. Leave the useful construction marks visible; do not replace the method with a measured guess.

Exam technique

Leave compass arcs and construction lines visible.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Where does the centre of an incircle lie?

Watch an explanation

Corbettmaths — Constructions: perpendicular bisector

Corbettmaths — Constructions: angle bisector

Corbettmaths — Constructions: triangles SSS

Corbettmaths — Constructions: perpendicular to point

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

At the intersection of the internal angle bisectors.

Open the full chapter online

Planning a statistical investigation

Leaving Certificate · Ordinary Level

Understand the idea

How to start

A population is the whole group you want to learn about. A sample is the smaller group you collect data from.

1. State a question that data can help answer.
2. Choose a sample that can represent the population fairly.
3. Describe what the results show and what the sample cannot tell you.

Key terms & definitions

Population
The full group the investigation is about.

Sample
The part of the population actually studied.

Bias
A systematic tendency to favour some outcomes or groups.

Discrete data
Counted values, usually separate whole numbers.

Key ideas

• Begin with a question that needs data and expects variation, rather than one with a single known answer.

• Define the population, decide how to sample fairly and choose what to measure.

• Distinguish categorical data from numerical data; numerical data may be discrete or continuous.

• Use neutral questions, consistent units and sensible recording methods. Do not collect unnecessary personal information.

• Consider sampling bias, sample size, non-response and measurement error before generalising.

• Describe conclusions in context and state limits. An association does not establish that one variable causes the other.

Quick reminder

• Define the population and question.

• Choose a sample that avoids systematic bias.

• State what the data can and cannot show.

Worked examples

Example 01 · Improve a sample

A student asks only their basketball team how much exercise first years do. Is the sample suitable?

Why this method?

Decide what the question is trying to find out. Then check whether the data and method can support that conclusion.

Population: all first-year students.	**Step 1** The question is about the whole year group, so identify that group first.
Sample used: the basketball team.	**Step 2** This is only one part of the year group.
The sample may overestimate exercise.	**Step 3** Team members are likely to do more organised exercise than many other students.
Choose students randomly across the first-year classes.	**Step 4** This gives students outside the team a chance of selection. Use the same question and time period for everyone.
A sample still has uncertainty.	**Step 5** Students may estimate their exercise inaccurately, and another random sample could give a different result.

Example 02 · Classify variables

Classify favourite sport, number of siblings and journey time to school.

Why this method?

Decide what the question is trying to find out. Then check whether the data and method can support that conclusion.

Categorical: values are categories.	**Step 1** Favourite sport
Numerical and discrete: it is a count.	**Step 2** Number of siblings
Numerical and continuous: in principle it can take any value in a range, even if recorded to whole minutes.	**Step 3** Journey time

Example 03 · A statistical question

Improve: “Do you agree our brilliant new lunch menu is healthier?”

Why this method?

Decide what the question is trying to find out. Then check whether the data and method can support that conclusion.

Use neutral wording rather than brilliant or healthier.	**Step 1** Remove leading language
For example: “How many days in the last school week did you buy a school lunch?”	**Step 2** Define what is measured
Survey a suitable sample and report the distribution. This measures lunch purchases, not health itself.	**Step 3** Collect and interpret

Example 04 · Choose a simple random sample

A school has 240 students. Select 30 for a survey about journey time.

Number the students 1 to 240.	**Step 1** Start with a complete list of the population.

Draw 30 distinct numbers randomly.	**Step 2** Select without replacement so each student has the same chance.
State the limit of the conclusion.	**Final step** A sample from one school does not represent every school.

Common mistake

Assuming a larger sample automatically removes bias. Asking many members of one sports club about favourite sports can still misrepresent the whole school. Use a sampling method suited to the population.

Exam technique

Explain how your sampling method reaches different groups, not just your friends.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

A student surveys only the school choir about how much music all students listen to. Explain one problem and suggest an improvement.

Question 2

Classify eye colour, number of pets and measured arm span as categorical, discrete numerical or continuous numerical.

Watch an explanation

Corbettmaths — Sampling: random

Corbettmaths — Questionnaires

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

The sample may over-represent students interested in music. Select students from across the school using a suitable random sampling method.

A large sample from one club can still be biased. Match the sampling method to the population.

Question 2

Eye colour: categorical. Number of pets: discrete numerical. Arm span: continuous numerical.

Counts take separate whole-number values. A measured length can take any value within a range.

Open the full chapter online

Tables, charts and stem-and-leaf plots

Leaving Certificate · Ordinary Level

Understand the idea

How to start

Frequency means how often a value or category occurs. A chart turns these counts into a display.

1. Read the title, key, units and axis scales first.
2. Translate the marks or bars into actual values or counts.
3. Use the display that matches the kind of data and label it clearly.

Key terms & definitions

Frequency
The number of observations in a category or interval.

Histogram
A display of grouped numerical data with touching bars.

Key
An explanation of how to read the stem and leaf units.

Key ideas

• A frequency table records how often each category or value occurs. Check the frequencies add to the sample size.

• Bar charts suit categories; bars are separated. A histogram groups numerical data; equal-width intervals use adjoining bars with frequency as height.

• For a pie chart, sector angle is $\frac{\text{frequency}}{\text{total}} \times 360^\circ$.

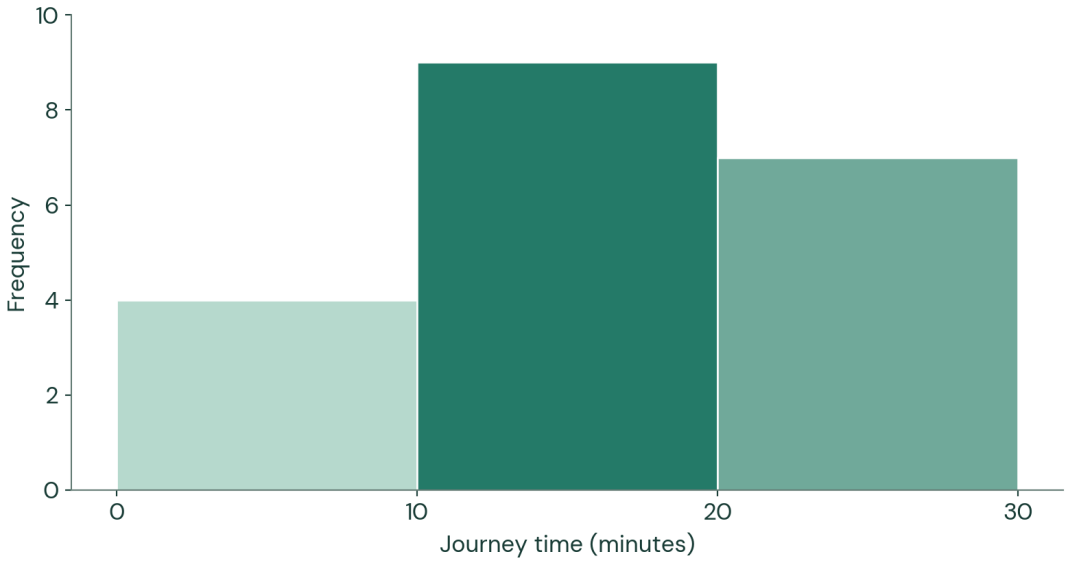
• A line graph can show change over time. Label both axes, units and scales.

• In a stem-and-leaf plot, order the leaves and provide a key. A back-to-back plot compares two groups using shared stems.

• Check whether an axis is truncated, intervals are inconsistent or area pictures exaggerate a difference.

• A scatter plot shows paired numerical data. An upward pattern suggests positive correlation; a downward pattern suggests negative correlation. Correlation does not establish cause.

• Ordinary Level: interpret scatterplots and match them to a supplied correlation coefficient. Drawing a line of best fit or calculating correlation is Higher Level; the fitted-line example is optional enrichment.



Equal-width histogram: frequencies are 4, 9 and 7 for adjoining 10 −minute intervals.

Quick reminder

• Read the key and scale first.

• Order stem-and-leaf entries and give a key.

• Compare both a typical value and spread when comparing groups.

Worked examples

Example 01 · Pie chart sector

In a survey of 40 students , 12 walk to school. Find the walking sector angle.

Why this method?

Read the key, units and scales before using a display. Translate the displayed information back into data values.

$$\text{walkingfraction} = \frac{12}{40}$$

Step 1

Twelve of the 40 students walk. Put the part over the total.

$\text{angle} = \frac{12}{40} \times 360^\circ$	**Step 2** The whole pie chart is 360°, so take the same fraction of a full turn.
$\text{angle} = 0.3 \times 360^\circ = 108^\circ$	**Step 3** Calculate the fraction first, then multiply.
$108^\circ < 180^\circ$	**Step 4** A check: fewer than half the students walk, so the sector must be less than half a circle.

Example 02 · Equal-width histogram

Journey times are grouped as $0 - < 10, 10 - < 20$ and $20 - < 30$ minutes, with frequencies $4, 9$ and 7. Describe the histogram.

Why this method?

Read the key, units and scales before using a display. Translate the displayed information back into data values.

Horizontal axis: time in minutes. Vertical axis: frequency.	**Step 1** Set the axes
Use intervals of width 10 and bar heights $4, 9$ and 7.	**Step 2** Draw equal-width adjoining bars
The modal class is $10 - < 20$ minutes. There are 20 students in total; the exact individual times are unknown.	**Step 3** Read the bar heights as frequencies, then interpret the most frequent interval in the context of the question.

Example 03 · Median from a back-to-back stem and leaf plot

Plot 1 : The back-to-back stem and leaf plot shows two groups of seven values. What is the median of Group B?

```
Group A | Stem | Group B
       7 5 1 | 1 | 1 4 7
             | 2 | 3
           8 | 3 | 0
         7 5 | 4 | 2
           5 | 5 | 9
Key: stems = tens; leaves = units.
```

Group A | Stem | Group B
751 | 1 | 147
| 2 | 3
8 | 3 | 0
75 | 4 | 2
5 | 5 | 9
Key: stems = tens; leaves = units.

Why this method?

Read the key, units and scales before using a display. Translate the displayed information back into data values.

11, 14, 17, 23, 30, 42, 59	**Step 1** Order the values of Group B from smallest to largest
23	**Step 2** The median is the 4th of the 7 values

Think about it: Before moving on: why do the leaves of Group A run to the left of the stems, and those of Group B to the right?

Example 04 · Interpret correlation without claiming causation

A scatterplot of hours of practice and score has correlation 0.82.

0.82 is positive and fairly close to 1.	**Step 1** The plot shows a strong positive linear association.

Association is not proof of cause.	**Final step** Prior knowledge, teaching or other variables could also affect scores.

Optional extension · not required for Ordinary Level

Example 05 · Use a supplied fitted line

A fitted model is $y = 2x + 5$. Estimate y when $x = 3$, within the observed data range.

Why this method?

Read the requested quantity, choose the matching rule and follow the reason beside each line.

$y = 2(3) + 5$	**Step 1** Replace x with 3. This is a prediction from a model.
$y = 11$	**Step 2** Multiply first, then add 5. A fitted line does not predict every observed value exactly.

Common mistake

Reading left-hand leaves in a back-to-back stem-and-leaf plot as though it were a right-hand plot. With key $2|3 = 23$, the row $8\ 5\ 1\ |\ 2$ on the left represents 28, 25 and 21; read outwards from the stem to list them in increasing order.

Exam technique

Compare two groups using both a typical value and spread, with the units.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

The back-to-back stem and leaf plot shows two groups of seven values. What is the median of Group B?

```
Group A | Stem | Group B
         5 2 | 1 | 3 8
         7 2 | 2 | 4 6
         8 1 | 3 | 3 6
           4 | 4 | 9
             | 5 |
Key: stems = tens; leaves = units.
```

Group A | Stem | Group B
52 | 1 | 38
72 | 2 | 46
81 | 3 | 36
4 | 4 | 9
| 5 |
Key: stems = tens; leaves = units.

A. 27

B. 28

C. 26

D. 2

Question 2

Data A : 7.1, 7.1, 7.5, 6.1, 8.2. Data B : 7.3, 7.7, 6.8, 8.7, 8.8, 8.3. Key : 6 | 1 means 6.1. Compare the centre and spread of the two groups using the plot. Which statement about median and range is supported?

A	Stem	B
1	6	8
5 1 1	7	3 7
2	8	3 7 8

Key: 2 | 7 means 2.7. Repeated leaves are retained.

Ordered back-to-back plot for the two data lists in the question. Left A, shared stems, right B. Key : 2 | 7 means 2.7.

A. Higher median: B. Larger range: A.

B. Higher median: A. Larger range: A.

C. Higher median: B. Larger range: B.

D. Higher median: A. Larger range: B.

Watch an explanation

Cognito — Pie Charts

Corbettmaths — Graphs: stem and leaf (draw)

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

26

Correct: ordered, Group B is $13, 18, 24, 26, 33, 36, 49$, and the 4th value is 26.

Question 2

Higher median: B. Larger range: A.

Correct.

Open the full chapter online

Averages, spread and grouped data

Leaving Certificate · Ordinary Level

Understand the idea

How to start

Mean is a shared-out total. Median is the middle of an ordered list. Mode is the most common value.

1. Read which measure the question asks for.
2. For the mean, add the values and divide by the number of values.
3. For the median, order the list first. Range is largest minus smallest.

Key terms & definitions

Mean
The total shared equally across all observations.

Median
The middle of the ordered data.

Mode
The most frequent value.

Range
The largest value minus the smallest value.

Key ideas

• The mean is $\frac{\text{sumofvalues}}{\text{numberofvalues}}$. For frequencies, use $\frac{\sum fx}{\sum f}$.

• Find the median after ordering the data. With an even number of values, average the two middle values.

• The mode is the most frequent value; data can have more than one mode or none. The range is maximum minus minimum.

• An outlier can change the mean strongly. The median is often more useful for very skewed data.

• For grouped data, use class midpoints to estimate the mean. It is an estimate because the individual values are not known.

• Quartiles split ordered data into four parts. The interquartile range is the upper quartile minus the lower quartile. State your convention if asked to calculate quartiles.

Quick reminder

• Mean: add and divide by the number of values.

• Median: order first, then find the middle.

• Grouped mean uses midpoints and is an estimate.

Worked examples

Example 01 · Mean, median, mode and range

The data set is $5, 18, 6, 8, 18$. Find the mean.

Why this method?

For a mean, add and count. Ordering is needed for the median, but it is not needed to calculate this mean.

$5 + 18 + 6 + 8 + 18 = 55$	**Step 1** Add every value, including the repeated 18.
Number of values $= 5$.	**Step 2** Count observations, including repeats. There are five values, not four different values.
$\text{mean} = \frac{55}{5}$	**Step 3** The mean shares the total equally among the five observations.
$\text{mean} = 11$	**Step 4** Divide 55 by 5. The mean lies between the smallest and largest values.

Think about it: Before moving on: which of these four measures is not an average, and why?

Example 02 · Reading a stem-and-leaf plot

Using the stem-and-leaf plot, what is the range of the data?

Stem	Leaves
1	117
2	146
3	33578

Why this method?

Identify the measure requested. Order values for a median; add and count for a mean; use the extremes for a range.

11 and 38	**Step 1** Find the smallest and largest values
$38 - 11 = 27$	**Step 2** Subtract the smallest from the largest

Think about it: Before moving on: which value is the smallest, and how do you read it from the plot?

Example 03 · Mean from grouped data

Intervals $0 - <10, 10 - <20$ and $20 - <30$ have frequencies $2, 5$ and 3. Estimate the mean.

Why this method?

Identify the measure requested. Order values for a median; add and count for a mean; use the extremes for a range.

$5, 15$ and 25.	**Step 1** The individual values are unknown. Use the middle of each class as a representative value, so the final mean will be an estimate.
$\sum fx = 2(5) + 5(15) + 3(25) = 160$	**Step 2** Multiply each representative value by its frequency because it stands for that many observations.
$\bar{x} \approx \frac{160}{10} = 16$	**Step 3** Divide by the number of observations, not the number of classes, to share the total equally.

This is an estimate because each class is represented by its midpoint.	**Step 4** State why this is an estimate: we used a midpoint to represent all the unknown individual values in each class.

Example 04 · Quartiles and interquartile range

Find the quartiles and IQR of $1, 2, 3, 4, 5, 6, 7, 8$.

Why this method?

Read the requested quantity, choose the matching rule and follow the reason beside each line.

$Q_1 = \frac{2+3}{2} = 2.5$	**Step 1** The lower half is $1, 2, 3, 4$. Its median is halfway between 2 and 3.
$Q_2 = \frac{4+5}{2} = 4.5$	**Step 2** The overall median is halfway between the two middle values.
$Q_3 = \frac{6+7}{2} = 6.5$	**Step 3** The upper half is $5, 6, 7, 8$. Find its median.
$\text{IQR} = 6.5 - 2.5 = 4$	**Step 4** Subtract the lower quartile from the upper quartile.

Example 05 · Compare spread using the IQR

Find the interquartile range of $2, 3, 5, 7, 8, 11, 13, 19$.

$Q_1 = \frac{3+5}{2} = 4$	**Step 1** Find the median of the lower four values.
$Q_3 = \frac{11+13}{2} = 12$	**Step 2** Find the median of the upper four values.
$\text{IQR} = 12 - 4 = 8$	**Final step** The IQR describes the width of the middle half.

Common mistake

Finding the middle value before sorting the data. In $8, 2, 5$, the median is 5 after ordering as $2, 5, 8$; it is not the middle entry 2 in the original list.

Exam technique

State whether a calculated mean is exact or estimated from grouped data.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

The data set is $2, 5, 5, 9, 14$. Find the median.

Question 2

Using the same stem-and-leaf plot, how many values are 25 or more?

Stem	Leaves
1	117
2	146
3	33578

Watch an explanation

Cognito — Averages - Mean, Median, Mode & Range

Cognito — Grouped Frequency Tables

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

5

Question 2

6

Open the full chapter online

Probability and relative frequency

Leaving Certificate · Ordinary Level

Understand the idea

How to start

Probability describes how likely an event is. It runs from 0, impossible, to 1, certain.

1. Describe the event you want.
2. Count all possible outcomes and the successful ones. Check that the outcomes are equally likely.
3. Write successful outcomes over total outcomes, then simplify if possible.

Key terms & definitions

Sample space
The set of all possible outcomes.

Event
A set of outcomes of interest.

Equally likely
Having the same chance of occurring.

Relative frequency
The proportion of trials in which an event occurs.

Key ideas

• Probability lies between 0 and 1. Zero means impossible and 1 means certain.

• For equally likely outcomes, $P(E) = \frac{\text{favourable outcomes}}{\text{all outcomes}}$.

• List a sample space systematically so outcomes are not omitted or counted twice.

• The complement rule is $P(\text{not } E) = 1 - P(E)$.

• Relative frequency is $\frac{\text{observed successes}}{\text{trials}}$. It estimates probability; more trials often improve the estimate but do not guarantee a particular result.

• $n!$ counts arrangements of n distinct objects. For r positions chosen without replacement, multiply the number of remaining choices.

• Expected value is the sum of each outcome multiplied by its probability. Subtract any cost when asked for expected net gain.

Quick reminder

• Use equally likely outcomes when counting probabilities.

• Probability $=$ favourable outcomes $\div$ all outcomes.

• Check that the result lies between 0 and 1.

Worked examples

Example 01 · Probability on the $0-1$ scale

A bag has 3 red and 7 blue balls. Each ball is equally likely to be drawn. What is the probability of drawing a red ball? Give it as a decimal.

Why this method?

Identify equally likely outcomes, then count the whole sample space and the outcomes that match the event.

$3+7=10$	**Step 1** Count all the balls that could be selected.
$P(\text{red})=\frac{3}{10}$	**Step 2** Three balls are red out of ten equally likely balls.
$P(\text{red})=0.3$	**Step 3** Divide 3 by 10 because the question asks for a decimal.

Think about it: Before moving on: where would a probability of 0.5 sit on the scale, and why?

Example 02 · Relative frequency as an estimate of probability

In an experiment, a coin lands on heads in 12 of 50 trials. Use the relative frequency to estimate the probability that it lands on heads in one trial. Give your answer as a decimal.

Why this method?

Identify equally likely outcomes, then count the whole sample space and the outcomes that match the event.

$12 \div 50 = 0.24$	**Step 1** Relative frequency $=$ number of times it happened $\div$ number of trials
Estimate : 0.24	**Step 2** With more trials, the estimate is usually more reliable

Think about it: Before moving on: why would more trials usually give a better estimate?

Example 03 · Sample spaces and equally likely outcomes

When two fair dice are rolled, how many of the 36 equally likely outcomes give a total of 4?

Why this method?

Identify equally likely outcomes, then count the whole sample space and the outcomes that match the event.

$(1, 3), (2, 2), (3, 1)$	**Step 1** Write out the outcomes with that total
3 of 36	**Step 2** Count them, out of the 36 equally likely outcomes

Think about it: Before moving on: why are there 36 outcomes for two dice, not 11?

Example 04 · The fundamental counting principle

A cafe offers 3 sandwiches and 4 drinks. A meal is one sandwich and one drink. How many different meals are possible?

Why this method?

Identify equally likely outcomes, then count the whole sample space and the outcomes that match the event.

3 sandwiches $\times 4$ drinks	**Step 1** Count the choices for each part

12 meals	**Step 2** Multiply, because each sandwich can go with every drink

Think about it: Before moving on: why multiply, not add, for two separate choices?

Example 05 · Order three different objects

How many orders can A, B and C have?

Why this method?

List the possible outcomes or choices systematically so none is missed or counted twice.

$3 \times 2 \times 1 = 6$	**Step 1** There are 3 choices for first, 2 remaining for second and 1 for last. This is $3!$.

Example 06 · Expected return

A fair coin gives a €4 prize for heads and €0 for tails. Find the expected prize.

Why this method?

List the possible outcomes or choices systematically so none is missed or counted twice.

$E = 4(0.5) + 0(0.5)$	**Step 1** Multiply each prize by its probability.
$E = 2$	**Step 2** The expected prize is €2 per play over many plays. A fee of €2 would make the expected net gain zero.

Example 07 · Check whether a game is fair

A game pays €6 on a six and nothing otherwise. It costs €1.20 to play. Find the expected net result.

$6\left(\frac{1}{6}\right) + 0\left(\frac{5}{6}\right) = 1$ euro	**Step 1** Weight the payouts by their probabilities.
€$1 -$ €$1.20 = -$€0.20	**Final step** Subtract the stake. The game is not fair.

Common mistake

Assuming all possible totals of two fair dice are equally likely. Total 2 has one outcome, $(1, 1)$; total 7 has six outcomes. Count the equally likely ordered dice outcomes, not just the names of the totals.

Exam technique

Check that a final probability is between zero and one.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

A bag has 4 red and 21 blue balls. Each ball is equally likely to be drawn. What is the probability of drawing a red ball? Give it as a decimal.



Probability scale from 0 to 1 in tenths, with 0.16 marked. The independent question is 4 red out of 25 balls.

Question 2

A coin is flipped 80 times and lands on heads 36 times. Use the relative frequency to estimate the probability of heads. Give your answer as a decimal.

Watch an explanation

Cognito — Probability Basics

Cognito — Probability Experiments

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

0.16

Question 2

0.45

Open the full chapter online

Combined events, trees and tables

Leaving Certificate · Ordinary Level

Understand the idea

How to start

For two events, keep track of what happens first and what happens second.

1. List the complete paths that give the event you want.
2. For independent events, multiply the probabilities along each path.
3. Add different successful paths when they cannot happen together.

Key terms & definitions

Independent events
Knowing one outcome does not change the probability of the other.

Mutually exclusive
Events that cannot happen together.

Tree diagram
A branching display of successive outcomes and their probabilities.

Key ideas

• The counting principle multiplies the number of choices at successive stages when each stage has the stated choices.

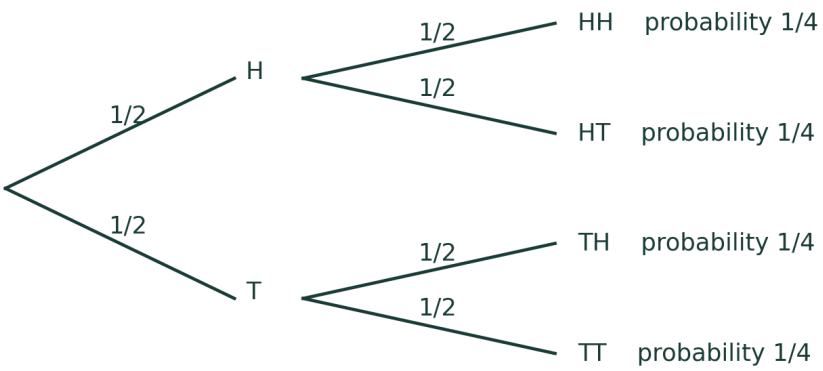
• For independent events, multiply probabilities along a tree branch.

• For mutually exclusive complete paths to the required event, add the path probabilities.

• A two-way table shows overlaps. Use the correct total for the group named in the question.

• Ordinary Level focuses on independent events, up to three Bernoulli trials and the first success on a specified trial. The dependent-event examples are optional extension.

Two fair coin tosses: four equally likely paths



Two independent fair coin tosses give HH, HT, TH and TT. Each complete path has probability one half times one half, or one quarter.

Quick reminder

- Multiply along an independent sequence of events.
- Add mutually exclusive successful paths.
- Use the total of the group actually selected.

Worked examples

Example 01 · Start here: count choices in stages

A café offers 3 starters and 4 main courses. How many different starter-and-main meals can be chosen?

Why this method?

Each starter can be paired with any of the 4 mains. Count 4 meals for each of the 3 starters.

$4 + 4 + 4 = 12$	**Step 1** Three groups, each containing four possible meals.
$3 \times 4 = 12$	**Step 2** Multiplication counts the same choices more quickly. Order the choices consistently: starter, then main.

Example 02 · Two independent spins

A spinner has 10 equal-sized sections, with 3 red sections. The probability of red is 0.3. It is spun twice independently. Find the probability of exactly one red.

Blue follows red then not red. Purple follows not red then red. Add the two complete paths for exactly one red.

Why this method?

List the possible paths or groups first. Decide when we are multiplying along a path and when we are adding different successful paths.

$P(\text{notred}) = 1 - 0.3 = 0.7$	**Step 1** Red and not red cover all possible outcomes of one spin.
Successful paths: red then not red, or not red then red.	**Step 2** Exactly one red means one red spin and one non-red spin. Both orders count.
$P(\text{redthennotred}) = 0.3 \times 0.7 = 0.21$	**Step 3** The spins are independent, so multiply along this path.
$P(\text{notredthenred}) = 0.7 \times 0.3 = 0.21$	**Step 4** Calculate the other successful order.
$P(\text{exactlyonered}) = 0.21 + 0.21 = 0.42$	**Step 5** Add these paths because the same pair of spins cannot follow both orders.

Example 03 · Two dice

Two fair six-sided dice are rolled. Find the probability of a total of 8.

Why this method?

List the possible paths or groups first. Decide when we are multiplying along a path and when we are adding different successful paths.

$6 \times 6 = 36$	**Step 1** Count equally likely ordered outcomes
$(2,6), (3,5), (4,4), (5,3), (6,2).$	**Step 2** List favourable pairs
$P(8) = \frac{5}{36}$	**Step 3** Form the probability

Example 04 · A two-way table

Of 24 students , 14 cycle. Of those 14, 6 wear glasses. If a cyclist is selected, what is the probability they wear glasses?

Why this method?

List the possible paths or groups first. Decide when we are multiplying along a path and when we are adding different successful paths.

Choose from the 14 cyclists.	**Step 1** Only cyclists can be selected here, so use 14 as the total, not 24.
$P(\text{glassesamongcyclists}) = \frac{6}{14}$	**Step 2** Six of the fourteen cyclists wear glasses.
$\frac{6}{14} = \frac{3}{7}$	**Step 3** Divide numerator and denominator by 2. The denominator still represents the cyclist group.

Example 05 · First success on trial 3

A trial succeeds with probability 0.4, independently each time. Find the probability of first success on trial 3.

Why this method?

Identify whether each stage changes the available outcomes before multiplying probabilities.

$P(\text{failure}) = 1 - 0.4 = 0.6$	**Step 1** The first two trials must fail.
$P = 0.6(0.6)(0.4)$	**Step 2** The required path is failure, failure, success.
$P = 0.144$	**Step 3** Multiply along this independent path.

Example 06 · Exactly two successes in three trials

Each trial succeeds independently with probability 0.4. Find the chance of exactly two successes.

SSF, SFS, FSS	**Step 1** List the three different orders.
$0.4^2(0.6) = 0.096$	**Step 2** Multiply along one order.

$3(0.096) = 0.288$	**Final step** Add the three equally probable orders.

Example 07 · Arrange, rather than choose

How many ways can 8 runners fill gold, silver and bronze?

$8 \times 7 \times 6 = 336$	**Final step** Order matters, and a runner cannot occupy a second place.

Optional extension · not required for Ordinary Level

Example 08 · Without replacement: the second probability changes

A bag contains 3 red counters and 2 blue counters. Two counters are taken without replacement. Find the probability that both are red.

Why this method?

Follow the red-then-red path. The first red counter is kept out, so the second draw has fewer counters and fewer reds.

$P(\text{first red}) = \frac{3}{5}$	**Step 1** At the start, 3 of the 5 counters are red.
$P(\text{second red} \mid \text{first red}) = \frac{2}{4}$	**Step 2** After taking one red, 2 red counters remain among 4 counters.
$P(\text{both red}) = \frac{3}{5} \times \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$	**Step 3** Multiply along this complete path. Do not reuse $\frac{3}{5}$ for the second draw.

Common mistake

Confusing independent with mutually exclusive. Two coin tosses are independent. Getting heads and tails on the same single toss is mutually exclusive: both cannot happen on that toss.

Exam technique

Check that the probabilities leaving each tree point add to one.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

A spinner has probability 0.2 of red. It is spun twice independently. Find the probability of exactly one red.

Question 2

Two fair six-sided dice are rolled. Find the probability of a total of 9.

Question 3 · Build confidence

Choose one of 2 soups and one of 5 sandwiches. How many meals?

Question 4 · Optional extension

A bag contains 4 red and 2 blue counters. Two are taken without replacement. Find the probability of two reds.

Before you move on

Try these without looking at a worked example. Revisit any part that is still difficult.

- Count successive choices systematically.
- Change the second probability when an item is not replaced.
- Multiply probabilities along one complete path.

Watch an explanation

Cognito — AND / OR Rules

Cognito — Tree Diagrams

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

$0.2(0.8) + 0.8(0.2) = 0.32$.

There are two disjoint paths: red then not red, or not red then red. Multiply along each path and add the results.

Question 2

$\frac{4}{36} = \frac{1}{9}$.

The ordered pairs are $(3, 6), (4, 5), (5, 4)$ and $(6, 3)$. Count outcomes out of 36, not out of the possible totals.

Question 3

$2 \times 5 = 10$.

Each of the two soups can be paired with all five sandwiches.

Question 4

$\frac{4}{6} \times \frac{3}{5} = \frac{2}{5}$.

After the first red, there are 3 reds among 5 remaining counters.

Open the full chapter online

Standard deviation, normal data and margin of error

Leaving Certificate · Ordinary Level

Understand the idea

How to start

Statistics use a sample to learn about a larger population. A sample estimate has uncertainty.

1. Identify the sample size and the sample proportion or percentage.
2. Calculate the margin of error, keeping proportions separate from percentage points.
3. Build the interval, then interpret it in the population context.

Key terms & definitions

Standard deviation
A measure of how spread out data are around their mean.

Margin of error
An allowance for uncertainty in a sample estimate.

Null hypothesis
The population claim being tested.

Key ideas

• Standard deviation describes spread around the mean. Use the population standard-deviation setting when that is the quantity requested.

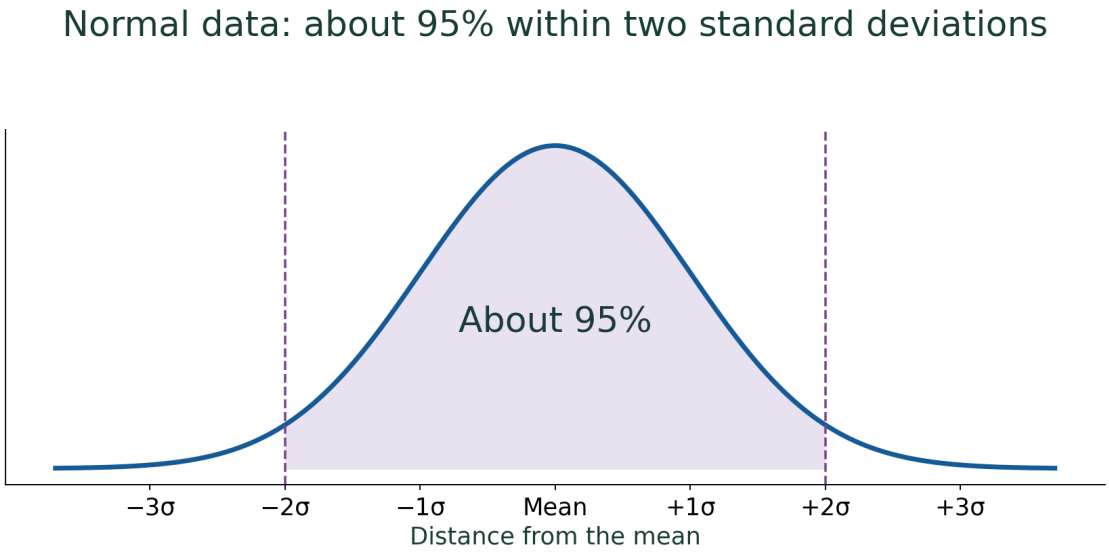
• For an approximately normal distribution, about 68% is within one standard deviation of the mean, 95% within two, and 99.7% within three.

• The approximate 95% margin of error for a population proportion is $\frac{1}{\sqrt{n}}$. This is a proportion; multiply by 100 for percentage points.

• An approximate interval is $\hat{p} \pm \frac{1}{\sqrt{n}}$. State the population and sampling assumptions.

• For the syllabus margin-of-error test, a hypothesised proportion outside the sample interval gives evidence against it at the approximate 5% level. Inside means insufficient evidence to reject it.

• Failure to reject a claim does not prove it is true. Random sampling and representativeness matter.



For approximately normal data, about 95% lie between mean minus two standard deviations and mean plus two standard deviations. This is an approximation, not a guarantee for every sample.

Quick reminder

• Standard deviation describes spread around the mean. Use the population standard-deviation setting when that is the quantity requested.

• For an approximately normal distribution, about 68% is within one standard deviation of the mean, 95% within two, and 99.7% within three.

Worked examples

Example 01 · Start here: a bigger sample gives a smaller margin

Estimate the 95% margin of error for a random sample of $n = 100$.

Why this method?

Use the syllabus approximation 1 divided by the square root of the sample size. Convert the proportion to percentage points at the end.

$\sqrt{n} = \sqrt{100} = 10$	**Step 1** Take the square root before dividing.
$\text{Marginoferror} \approx \frac{1}{10} = 0.10$	**Step 2** This is a proportion.

$0.10 \times 100 = 10 \text{percentagepoints}$	**Step 3** For example, an estimate of 60% would give about 50% to 70%.

Example 02 · A simple standard deviation

Find the population standard deviation of $2, 4, 6$.

Why this method?

Identify the sample information first, calculate the uncertainty, then interpret the result in context.

$\bar{x} = \frac{2+4+6}{3} = 4$	**Step 1** Find the mean first.
$(2-4)^2 + (4-4)^2 + (6-4)^2 = 8$	**Step 2** Find each distance from the mean, square it and add.
$\sigma = \sqrt{\frac{8}{3}} \approx 1.63$	**Step 3** Divide by the number of observations, then take the square root.

Example 03 · Use the empirical rule

Approximately normal scores have mean 50 and standard deviation 5. Find the interval containing about 95%.

Why this method?

Identify the sample information first, calculate the uncertainty, then interpret the result in context.

$50 - 2(5) = 40$	**Step 1** Two standard deviations below the mean.
$50 + 2(5) = 60$	**Step 2** Two standard deviations above the mean.
$40 \leq x \leq 60$	**Step 3** About 95% lie in this interval.

Example 04 · A margin-of-error interval

A random sample of 400 gives a proportion 0.60. Estimate $a95\%$ interval.

Why this method?

Identify the sample information first, calculate the uncertainty, then interpret the result in context.

$\text{MOE} = \frac{1}{\sqrt{400}} = 0.05$	**Step 1** Use the number of people sampled , 400.
$0.60 - 0.05 = 0.55, \quad 0.60 + 0.05 = 0.65$	**Step 2** Subtract and add the margin to the sample proportion.
$55\%\text{to}65\%$	**Step 3** Convert the proportion endpoints to percentages.

Example 05 · Test a population claim

For the previous sample, test the claim that the population proportion is 0.50.

Why this method?

Identify the sample information first, calculate the uncertainty, then interpret the result in context.

$0.50 < 0.55$	**Step 1** The claimed value lies below the sample interval 0.55 to 0.65.
$\text{Rejecttheclaimattheapproximate}5\%\text{level.}$	**Step 2** Under this margin-of-error method, the sample gives evidence against the claim.

Example 06 · A claim inside the sample interval

A sample estimate is 55%, with an approximate 95% margin of error of 5 percentage points. Test a population claim of 52%.

Why this method?

Build the interval first. Then check whether the claimed population percentage falls inside it.

$55\% - 5\% = 50\%, \quad 55\% + 5\% = 60\%$	**Step 1** The interval runs from 50% to 60%.
$50\% \leq 52\% \leq 60\%$	**Step 2** The claimed 52% is inside the interval.

Insufficientevidencetorejecttheclaim.	**Step 3** This does not prove that 52% is the true population percentage.

Example 07 · Test a population proportion using the OL margin of error

In a random sample of 400, 54% agree. Does this challenge a claim that 60% agree?

$\text{MOE} = \frac{1}{\sqrt{400}} = 0.05$	**Step 1** Use the Ordinary Level rule for a population proportion.
From 49 per cent to 59 per cent	**Step 2** Add and subtract five percentage points from 54%.
60% is outside the interval.	**Final step** The result gives evidence against the claim by this margin-of-error method.

Common mistake

Writing a margin of error of 0.05 as 0.05%. A proportion of 0.05 is 5 percentage points. A sample estimate of 60% with this margin gives approximately 55% to 65%.

Exam technique

Explain the conclusion in the context of the population claim.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Find the margin of error for $n = 100$.

Question 2

Estimate the margin of error for a random sample of $n = 400$.

Question 3

A sample estimate is 70% with margin of error 4 percentage points. State the interval.

Self-check answers

Question 1

0.10, or 10 percentage points.

Question 2

$\frac{1}{\sqrt{400}} = 0.05$, or 5 percentage points.

A proportion of 0.05 is five percentage points.

Question 3

Approximately 66% to 74%.

Subtract and add four percentage points.

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