Sine rule, cosine rule and triangle area

Leaving Certificate · Ordinary Level

Understand the idea

How to start

For a triangle without a right angle, choose a rule from the information given. Side a is opposite angle A.

1. Match every side with the angle opposite it.
2. Choose sine rule for opposite pairs, cosine rule for three sides or two sides with the included angle.
3. Write the formula, substitute the values and use degree mode.

Key terms & definitions

Included angle
The angle between two named sides.

Opposite pair
A side and the angle facing it.

Key ideas

• Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B}$. Match each side with its opposite angle.

• Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$. Use two sides and the angle between them, or all three sides.

• Triangle area: $A = \frac{1}{2}bc\sin A$ when A is the angle between the known sides.

• Use degree mode and keep calculator digits until the final rounding.

• The sine rule can sometimes give two possible triangles. Check the given diagram and angle conditions.

Quick reminder

• Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B}$. Match each side with its opposite angle.

• Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$. Use two sides and the angle between them, or all three sides.

Worked examples

Example 01 · Sine rule with clear opposite pairs

$A = 30^\circ$, $a = 5$ cm and $B = 60^\circ$. Find b.

Why this method?

Identify the known sides and angles first, then match them to the rule used below.

$\frac{b}{\sin 60^\circ} = \frac{5}{\sin 30^\circ}$	**Step 1** Pair b with 60° and 5 with 30°.
$b = \frac{5\sin 60^\circ}{\sin 30^\circ}$	**Step 2** Multiply both sides by $\sin 60^\circ$.
$b \approx 8.66\text{cm}$	**Step 3** Calculate in degree mode, then round.

Example 02 · Cosine rule to find a side

Two sides are 3 cm and 4 cm, with included angle 60°. Find the opposite side a.

Why this method?

Identify the known sides and angles first, then match them to the rule used below.

$a^2 = 3^2 + 4^2 - 2(3)(4)\cos 60^\circ$	**Step 1** Use the two sides and the angle between them.
$a^2 = 25 - 12 = 13$	**Step 2** Cos 60° is one half.
$a = \sqrt{13} \approx 3.61\text{cm}$	**Step 3** Take the positive square root.

Example 03 · Area using an included angle

Find the area with sides 4 cm, 6 cm and included angle 30°.

Why this method?

Identify the known sides and angles first, then match them to the rule used below.

$A = \frac{1}{2}(4)(6)\sin 30^\circ$	**Step 1** The given angle is between the known sides.
$A = 12(0.5) = 6\text{cm}^2$	**Step 2** Use $\sin 30^\circ = 0.5$ and include square units.

Example 04 · Find an angle with the cosine rule

A triangle has sides 3 cm, 4 cm and 5 cm. Find the angle A opposite the 5 cm side.

Why this method?

All three sides are known. Use the cosine rule and rearrange it before pressing inverse cosine.

$a = 5, \quad b = 3, \quad c = 4$	**Step 1** a must be the side opposite the angle A that we want.
$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$	**Step 2** Write the angle version of the cosine rule.
$\cos A = \frac{3^2 + 4^2 - 5^2}{2(3)(4)}$	**Step 3** Keep the entire numerator above the fraction bar.
$\cos A = \frac{9 + 16 - 25}{24} = 0$	**Step 4** Square the side lengths, then simplify.
$A = \cos^{-1}(0) = 90^\circ$	**Step 5** Use inverse cosine in degree mode.

Example 05 · Cosine rule with two sides and the included angle

Two sides are 5 cm and 7 cm, with included angle 60°. Find the opposite side.

$a^2 = 5^2 + 7^2 - 2(5)(7)\cos 60^\circ$	**Step 1** Use the included angle in the cosine rule.
$a^2 = 25 + 49 - 35 = 39$	**Step 2** Evaluate in degree mode.

$a = \sqrt{39} \approx 6.24$ cm	**Final step** Take the positive square root.

Common mistake

Pairing a side with an angle beside it in the sine rule. In $\frac{a}{\sin A} = \frac{b}{\sin B}$, side a must be opposite angle A, and side b opposite angle B.

Exam technique

Label a, b, c and their opposite angles before substituting.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Find the area with sides 5 cm and 8 cm and included angle 90°.

Question 2

Find the area of a triangle with sides 6 cm and 8 cm enclosing 30°.

Question 3

In a triangle, $A = 30^\circ$, $a = 4$ cm and $B = 90^\circ$. Find b.

Self-check answers

Question 1

20cm^2

Question 2

$A = \frac{1}{2}(6)(8)\sin 30^\circ = 12\text{cm}^2.$

The angle must be between the two known sides.

Question 3

$b = \frac{4\sin 90^\circ}{\sin 30^\circ} = 8\text{cm}.$

Match a with its opposite angle A and b with B.

Open the full chapter online