Patterns and nth terms

Leaving Certificate · Ordinary Level

Understand the idea

How to start

The term number tells you the position. The term value tells you the number at that position.

1. Write the position numbers above the terms, starting with 1 unless told otherwise.
2. Find the differences between neighbouring terms.
3. Test a proposed rule using more than one position.

Key terms & definitions

Sequence
An ordered list of terms.

First difference
The change from one term to the next.

Nth term
A rule for the term at position n.

Key ideas

• The position number n starts at 1 unless the question states otherwise.

• A constant first difference suggests a linear sequence. For first term a and difference d, $T_n = a + (n-1)d$.

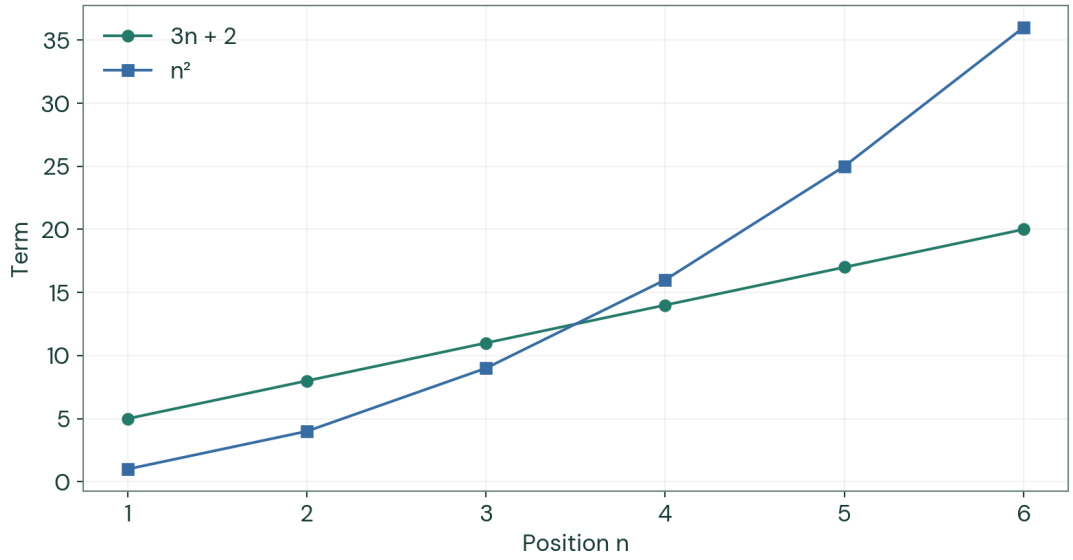
• A constant non-zero second difference suggests a quadratic sequence $T_n = an^2 + bn + c$; its second difference is $2a$.

• A constant ratio suggests a geometric pattern, such as repeated doubling or tripling.

• Check a proposed rule against several terms and the context. A short list alone does not prove a unique general rule.

• Arithmetic term: $T_n = a + (n-1)d$. Arithmetic sum: $S_n = \frac{n}{2}(2a + (n-1)d)$. Here a is the first term and d is the common difference.

• An arithmetic sequence adds the same amount each time. A geometric sequence multiplies by the same amount each time.



Linear pattern $3n + 2$ has constant first difference; quadratic pattern n^2 has constant second difference.

Quick reminder

• Write the term positions, starting at the stated n-value.

• Constant first difference → linear; constant non-zero second difference → quadratic.

• Check your rule against the original terms.

Worked examples

Example 01 · Continuing a numerical pattern

The pattern starts $3, 8, 13, 18, \ldots$ What is the next term?

Why this method?

Write the position numbers beside the terms. Check differences or ratios before choosing the form of the rule.

$8 - 3 = 5$	**Step 1** Find the change from the first term to the second.
$13 - 8 = 5, \quad 18 - 13 = 5$	**Step 2** Check the same change happens in the next two steps.

$18 + 5 = 23$	**Step 3** Continue the pattern by adding 5 to the last term shown.

Think about it: Before moving on: how can you check the common difference is right for every pair of neighbouring terms?

Example 02 · The term at any position

A pattern starts with 2 and goes up by 4 each time. What is term 10?

Position	1	2	3	4	5
Term	2	6	10	14	18

Why this method?

Write the position numbers beside the terms. Check differences or ratios before choosing the form of the rule.

$T_1 = 2$	**Step 1** The first term is 2. No increases have happened yet.
$10 - 1 = 9$	**Step 2** From position 1 to position 10 there are nine steps.
$9 \times 4 = 36$	**Step 3** Each step adds 4, so the total increase is 36.
$T_{10} = 2 + 36 = 38$	**Step 4** Add the increase to the first term.

Think about it: Before moving on: how many differences are added to reach term 10?

Example 03 · Finding a term of a linear pattern

The pattern starts $3, 9, 15, 21, \ldots$ What is term number 12?

Why this method?

Write the position numbers beside the terms. Check differences or ratios before choosing the form of the rule.

$9 - 3 = 6$	**Step 1** The pattern adds 6 each time.
$12 - 1 = 11$	**Step 2** Term 12 is eleven steps after term 1.

$11 \times 6 = 66$	**Step 3** Find the total increase across eleven steps.
$T_{12} = 3 + 66 = 69$	**Step 4** Start at the first term , 3, then add the increase.

Think about it: Before moving on: why is there one fewer difference than the term number?

Example 04 · Sum of an arithmetic series

Find the sum of the first 5 terms of $2, 5, 8, \ldots$

Why this method?

Look for a constant difference or ratio before choosing the matching rule.

$a = 2, \quad d = 3, \quad n = 5$	**Step 1** Identify the first term, difference and number of terms.
$S_5 = \frac{5}{2}(2(2) + (5 - 1)3)$	**Step 2** Substitute into the sum formula.
$S_5 = \frac{5}{2}(16) = 40$	**Step 3** Calculate. Check by adding $2 + 5 + 8 + 11 + 14$.

Example 05 · Find which term has a given value

The sequence is $5, 9, 13, 17, \ldots$. Which term is 81?

Why this method?

The term value and its position are different. First write a rule for the value, then solve for the position n.

$a = 5, \quad d = 4$	**Step 1** The first term is 5 and the sequence adds 4 each time.
$T_n = 5 + 4(n - 1) = 4n + 1$	**Step 2** There are $n - 1$ jumps from the first term to term n.
$4n + 1 = 81$	**Step 3** Set the term value equal to 81.

$4n = 80$	**Step 4** Subtract 1 from both sides.
$n = 20$	**Step 5** Divide by 4. Therefore 81 is the 20th term.

Example 06 · Find the sum, not the last term

Find the total of the first 12 terms of $8, 13, 18, \ldots$.

$T_{12} = 8 + (12 - 1)5 = 63$	**Step 1** Find the last term needed.
$S_{12} = \frac{12}{2}(8 + 63) = 426$	**Final step** Multiply the average of first and last terms by the number of terms.

Example 07 · Recognise a geometric sequence

Classify $81, 27, 9, 3, \ldots$ and find the next term.

$\frac{27}{81} = \frac{9}{27} = \frac{3}{9} = \frac{1}{3}$	**Step 1** Successive terms share a constant ratio.
$3 \times \frac{1}{3} = 1$	**Final step** It is geometric; multiply by one third again.

Common mistake

Starting at $n = 0$ when the question numbers the first term as 1. For $T_n = 3n + 2$, the first term is $3(1) + 2 = 5$, not 2.

Exam technique

Write a position row and a value row before finding a rule.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

The pattern starts $6, 11, 16, 21, \ldots$ What is the next term?

Question 2

A pattern starts with 8 and goes up by 3 each time. What is term 15?

Position	1	2	3	4
Term	8	11	14	17

Question 3 · Build confidence

For $7, 10, 13, \ldots$, write a term rule.

Question 4 · Apply or explain

Which term of $7, 10, 13, \ldots$ is 79?

Before you move on

Try these without looking at a worked example. Revisit any part that is still difficult.

- Identify the first term and common difference.
- Use n for the position of a term.
- Check a term rule against the first few terms.

Watch an explanation

Cognito — Finding the nth Term (Arithmetic Sequences)

Cognito — Finding the nth Term (Quadratic Sequences)

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

26

Question 2

50

Question 3

$T_n = 7 + 3(n-1) = 3n + 4.$

The first position is $n = 1$.

Question 4

$3n + 4 = 79$, so $n = 25$.

Solve for the position, rather than giving the term value again.

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