Probability and relative frequency

Leaving Certificate · Ordinary Level

Understand the idea

How to start

Probability describes how likely an event is. It runs from 0, impossible, to 1, certain.

1. Describe the event you want.
2. Count all possible outcomes and the successful ones. Check that the outcomes are equally likely.
3. Write successful outcomes over total outcomes, then simplify if possible.

Key terms & definitions

Sample space
The set of all possible outcomes.

Event
A set of outcomes of interest.

Equally likely
Having the same chance of occurring.

Relative frequency
The proportion of trials in which an event occurs.

Key ideas

• Probability lies between 0 and 1. Zero means impossible and 1 means certain.

• For equally likely outcomes, $P(E) = \frac{\text{favourable outcomes}}{\text{all outcomes}}$.

• List a sample space systematically so outcomes are not omitted or counted twice.

• The complement rule is $P(\text{not } E) = 1 - P(E)$.

• Relative frequency is $\frac{\text{observed successes}}{\text{trials}}$. It estimates probability; more trials often improve the estimate but do not guarantee a particular result.

• $n!$ counts arrangements of n distinct objects. For r positions chosen without replacement, multiply the number of remaining choices.

• Expected value is the sum of each outcome multiplied by its probability. Subtract any cost when asked for expected net gain.

Quick reminder

• Use equally likely outcomes when counting probabilities.

• Probability $=$ favourable outcomes $\div$ all outcomes.

• Check that the result lies between 0 and 1.

Worked examples

Example 01 · Probability on the $0-1$ scale

A bag has 3 red and 7 blue balls. Each ball is equally likely to be drawn. What is the probability of drawing a red ball? Give it as a decimal.

Why this method?

Identify equally likely outcomes, then count the whole sample space and the outcomes that match the event.

$3+7=10$	**Step 1** Count all the balls that could be selected.
$P(\text{red})=\frac{3}{10}$	**Step 2** Three balls are red out of ten equally likely balls.
$P(\text{red})=0.3$	**Step 3** Divide 3 by 10 because the question asks for a decimal.

Think about it: Before moving on: where would a probability of 0.5 sit on the scale, and why?

Example 02 · Relative frequency as an estimate of probability

In an experiment, a coin lands on heads in 12 of 50 trials. Use the relative frequency to estimate the probability that it lands on heads in one trial. Give your answer as a decimal.

Why this method?

Identify equally likely outcomes, then count the whole sample space and the outcomes that match the event.

$12 \div 50 = 0.24$	**Step 1** Relative frequency $=$ number of times it happened $\div$ number of trials
Estimate : 0.24	**Step 2** With more trials, the estimate is usually more reliable

Think about it: Before moving on: why would more trials usually give a better estimate?

Example 03 · Sample spaces and equally likely outcomes

When two fair dice are rolled, how many of the 36 equally likely outcomes give a total of 4?

Why this method?

Identify equally likely outcomes, then count the whole sample space and the outcomes that match the event.

$(1, 3), (2, 2), (3, 1)$	**Step 1** Write out the outcomes with that total
3 of 36	**Step 2** Count them, out of the 36 equally likely outcomes

Think about it: Before moving on: why are there 36 outcomes for two dice, not 11?

Example 04 · The fundamental counting principle

A cafe offers 3 sandwiches and 4 drinks. A meal is one sandwich and one drink. How many different meals are possible?

Why this method?

Identify equally likely outcomes, then count the whole sample space and the outcomes that match the event.

3 sandwiches $\times 4$ drinks	**Step 1** Count the choices for each part

12 meals	**Step 2** Multiply, because each sandwich can go with every drink

Think about it: Before moving on: why multiply, not add, for two separate choices?

Example 05 · Order three different objects

How many orders can A, B and C have?

Why this method?

List the possible outcomes or choices systematically so none is missed or counted twice.

$3 \times 2 \times 1 = 6$	**Step 1** There are 3 choices for first, 2 remaining for second and 1 for last. This is $3!$.

Example 06 · Expected return

A fair coin gives a €4 prize for heads and €0 for tails. Find the expected prize.

Why this method?

List the possible outcomes or choices systematically so none is missed or counted twice.

$E = 4(0.5) + 0(0.5)$	**Step 1** Multiply each prize by its probability.
$E = 2$	**Step 2** The expected prize is €2 per play over many plays. A fee of €2 would make the expected net gain zero.

Example 07 · Check whether a game is fair

A game pays €6 on a six and nothing otherwise. It costs €1.20 to play. Find the expected net result.

$6\left(\frac{1}{6}\right) + 0\left(\frac{5}{6}\right) = 1$ euro	**Step 1** Weight the payouts by their probabilities.
€$1 -$ €$1.20 = -$€0.20	**Final step** Subtract the stake. The game is not fair.

Common mistake

Assuming all possible totals of two fair dice are equally likely. Total 2 has one outcome, $(1, 1)$; total 7 has six outcomes. Count the equally likely ordered dice outcomes, not just the names of the totals.

Exam technique

Check that a final probability is between zero and one.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

A bag has 4 red and 21 blue balls. Each ball is equally likely to be drawn. What is the probability of drawing a red ball? Give it as a decimal.



Probability scale from 0 to 1 in tenths, with 0.16 marked. The independent question is 4 red out of 25 balls.

Question 2

A coin is flipped 80 times and lands on heads 36 times. Use the relative frequency to estimate the probability of heads. Give your answer as a decimal.

Watch an explanation

Cognito — Probability Basics

Cognito — Probability Experiments

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

0.16

Question 2

0.45

Open the full chapter online