Sets and Venn diagrams

Leaving Certificate · Ordinary Level

Understand the idea

How to start

A set is a collection of items. An overlap contains items that belong to both sets.

1. Name the whole group, called the universal set.
2. Put the shared items in the overlap first.
3. Fill the remaining regions and check that every item is counted once.

Key terms & definitions

Universal set
All elements under consideration, usually labelled U.

Complement
Elements in U that are not in the named set.

Cardinality
The number of elements in a finite set.

Empty set
A set with no elements.

Key ideas

• A set is a collection of distinct elements. State the universal set before finding a complement.

• $A \cup B$ means in A or B or both. $A \cap B$ means in both. $A \setminus B$ means in A but not B.

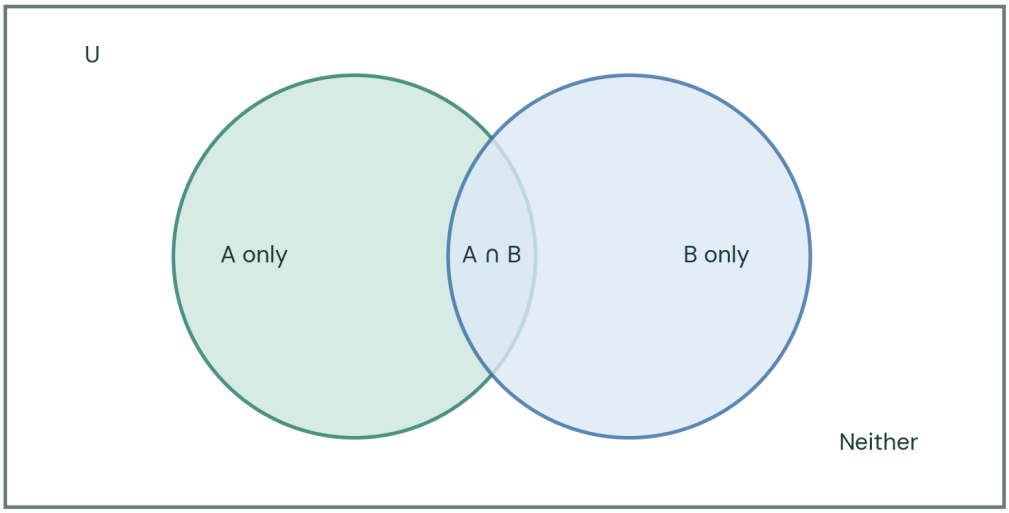
• Fill a Venn diagram from the most specific overlap outwards: the triple intersection first, then pair-only regions, then one-set-only regions.

• For two sets, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. Subtract the union from the universal total to find neither.

• Union and intersection are commutative and associative. These laws regroup sets; they do not change which elements belong.

• A subset contains only elements of another set: $A \subseteq B$. Distinguish an element belonging to a set, $x \in A$, from a set being a subset.

• For three sets,
$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$.



Four separate regions: A only, both A and B, B only, and neither. Together they make the universal set.

Quick reminder

• Union: in either set or both. Intersection: in both.

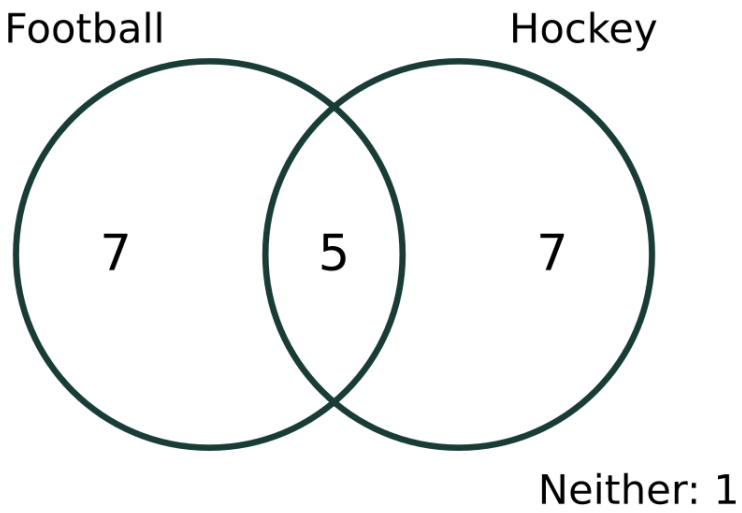
• In Venn diagrams, fill the deepest overlap first.

• Neither = universal total minus the union.

Worked examples

Example 01 · Two-set Venn diagrams

In a group of 20 students, 12 play football, 12 play hockey, and 5 play both. How many play football only?



Two-set Venn diagram for 20 students. Football only : 7. Both : 5. Hockey only : 7. Neither : 1.

Why this method?

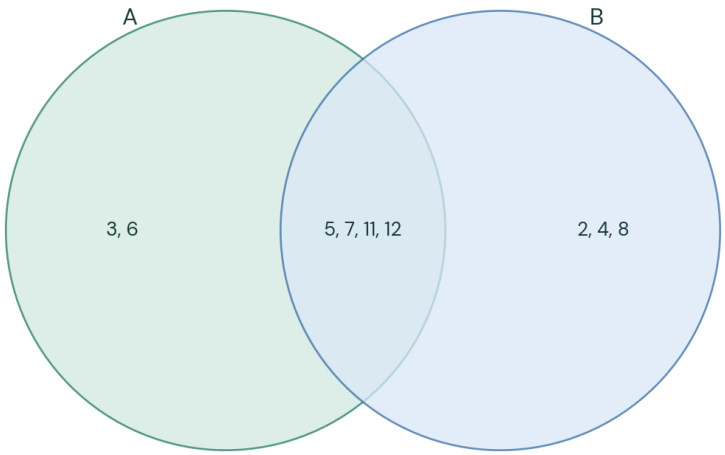
Identify which region or operation the question asks for. Count each element or person only in the regions where they belong.

$\text{Football total} = \text{football only} + \text{both}.$	**Step 1** The 12 football players already include the 5 who play both sports.
$12 = \text{footballonly} + 5$	**Step 2** Put the given numbers into that relationship.
$\text{footballonly} = 12 - 5 = 7$	**Step 3** Subtract the overlap once to leave only the football-only region.

Think about it: Before moving on: why must the overlap be taken away from each circle total?

Example 02 · Set notation: union, intersection and difference

$A = \{3, 5, 6, 7, 11, 12\}$ and $B = \{2, 4, 5, 7, 8, 11, 12\}$. How many elements are in $A \cup B$?



Venn diagram matching the sets in this question. A only : [3, 6]. Both : [5, 7, 11, 12]. B only : [2, 4, 8].

Why this method?

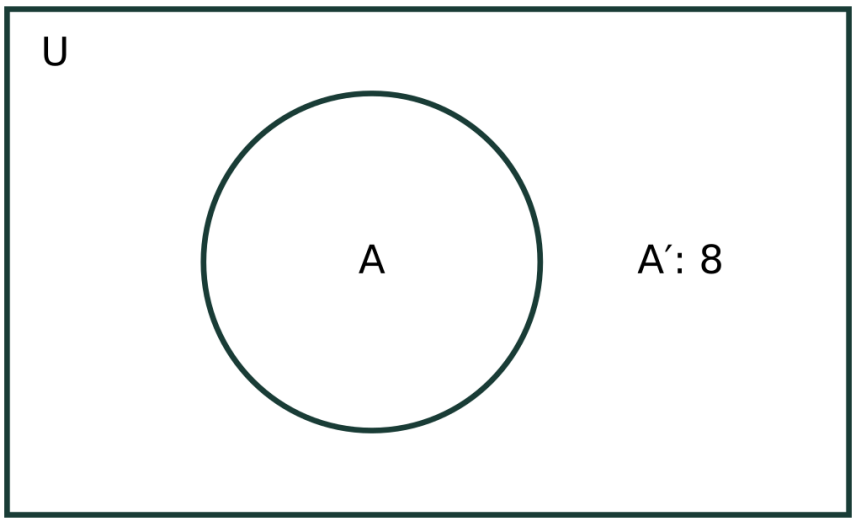
Identify which region or operation the question asks for. Count each element or person only in the regions where they belong.

$A \cup B = \{2, 3, 4, 5, 6, 7, 8, 11, 12\}$	**Step 1** $A \cup B$ contains every element in A or in B, each listed once
9 elements	**Step 2** Count the elements

Think about it: Before moving on: why is an element that is in both sets listed only once in $A \cup B$?

Example 03 · Complement and universal set

The universal set is $U = \{1, 2\} \ldots, 12\}$. $A = \{2, 6, 8, 12\}$. How many elements are in A□, the complement of A?



Universal set $U = \{1\} \ldots, 12\}$ drawn as a rectangle, with $A = \{2, 6, 8, 12\}$ as a circle inside it. The complement A□ is the part of U outside the circle, with 8 elements.

Why this method?

Identify which region or operation the question asks for. Count each element or person only in the regions where they belong.

A□ $= \{1, 3, 4, 5, 7, 9, 10, 11\}$	**Step 1** The complement is the elements of U that are not in A
$12 - 4 = 8$	**Step 2** Count the elements of A□

Think about it: Before moving on: why is A□ found by taking A away from U rather than by adding A to U?

Example 04 · Cardinal number and the null set

Hannah writes the set $A = \{2, 2, 9, 12\}$. What is n(A), the number of elements in A?

Why this method?

Identify which region or operation the question asks for. Count each element or person only in the regions where they belong.

Different elements : $\{2, 9, 12\}$	**Step 1** Each element of a set is counted once, even if it is written more than once

$n(A) = 3$	**Step 2** Count the different elements

Think about it: Before moving on: why is the null set not the same as the set $\{0\}$?

Example 05 · **Count an overlap**

In a group of 32, 18 take French, 15 take Music and 7 take both. How many take neither?

$18 + 15 - 7 = 26$	**Step 1** Subtract the overlap once: it was counted twice.
$32 - 26 = 6$	**Final step** Subtract the union from the total.

Common mistake

Counting the overlap twice when finding how many are in A or B. If $n(A) = 12$, $n(B) = 10$ and $n(A \cap B) = 4$, the total is $12 + 10 - 4 = 18$, not 22.

Exam technique

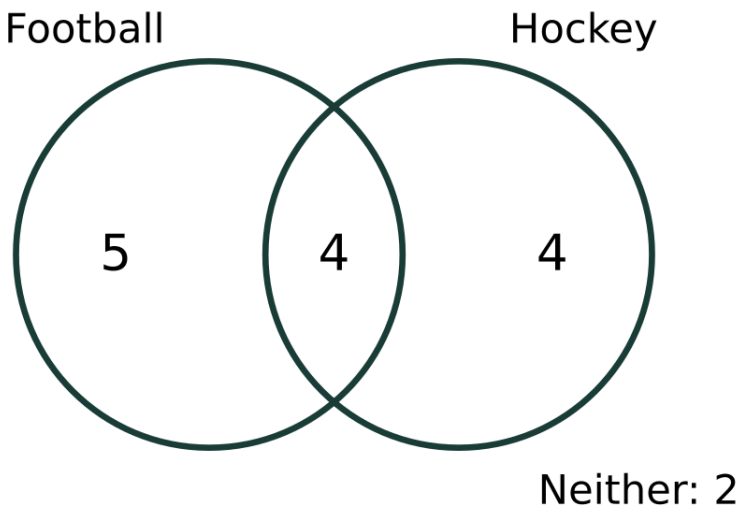
Check that all disjoint regions add to the universal total.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

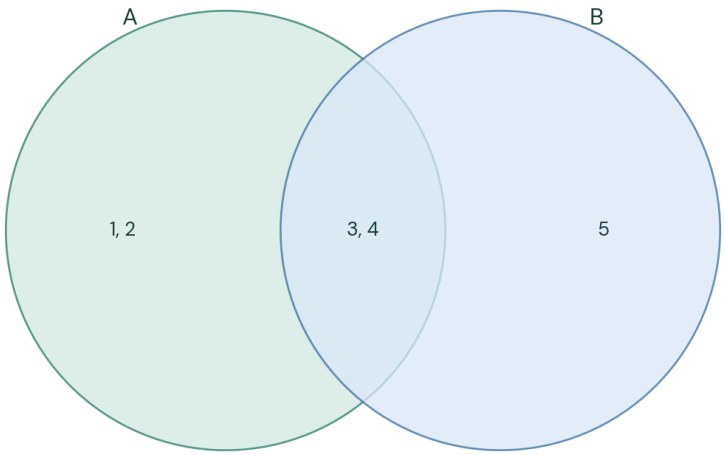
In a group of 15 students, 9 play football, 8 play hockey, and 4 play both. How many play neither sport?



Two-set Venn diagram for 15 students. Football only : 5. Both : 4. Hockey only : 4. Neither : 2.

Question 2

$A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5\}$. How many elements are in $A \cup B$?



Venn diagram matching the sets in this question. A only : [1, 2]. Both : [3, 4]. B only : [5].

Watch an explanation

Cognito — Set Notation & Venn Diagrams

Cognito — Venn Diagrams

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

2

Question 2

5

Open the full chapter online