Simultaneous linear equations

Leaving Certificate · Ordinary Level

Understand the idea

How to start

We need one x-value and one y-value that work in both equations at the same time.

1. Label equations A and B, and line up the x terms, y terms and right-hand sides.
2. Choose the variable to remove. For addition, make its terms equal in size and opposite in sign.
3. Solve for the remaining variable, substitute back and check both original equations.

Key terms & definitions

Elimination
Combining equations to remove a variable.

Substitution method
Replacing a variable using an equivalent expression from another equation.

Key ideas

• A simultaneous solution is a pair of values that makes both original equations true. We must find both values, then check both equations.

• Label the equations A and B. Write x terms under x terms, y terms under y terms and the constants under the constants on the right of the equals signs.

• To eliminate a variable by addition, make its two terms the same size but opposite in sign. Their sum is zero: for example, $+2x + (-2x) = 0$.

• If a multiplier is needed, multiply every term in that equation, including the right-hand side. Choose a multiplier that creates the opposite term you need.

• Add the aligned equations. One variable disappears, leaving a one-variable equation you can solve.

• Substitute that value into one original equation to find the second variable. State the pair and check it in both original equations.

• You can also subtract equations with equal coefficients. We use addition with opposite terms in the early examples so the cancellation is easy to see.

• On a graph, the solution is the intersection of two lines: the same x-value gives the same y-value on both. Only after this is clear do we check a third line.

Quick reminder

• Choose which variable to remove.

• For addition, make equal-sized opposite terms and add.

• Substitute back to find the second value; check both equations.

Worked examples

Example 01 · Opposite terms are already ready to cancel

Solve the simultaneous equations $x + y = 7$ and $x - y = 1$.

Blue follows x terms and purple follows y terms in the aligned equations. Crossed-out terms add to zero.

Why this method?

We will remove y. One equation has $+y$ and the other has $-y$, so adding them removes y without any multiplication.

$x \quad +y \quad = \quad 7$ A $x \quad -y \quad = \quad 1$ B	**Step 1** Label and align the original equations. The y terms already have opposite signs.
$x \quad \cancel{+y} \quad = \quad 7$ A $x \quad \cancel{-y} \quad = \quad 1$ B $2x \quad = \quad 8$ $A + B$	**Step 2** Add A and B. The crossed terms sum to zero $: +y - y = 0$. Add the x terms and the right-hand sides too.
$x = 4$	**Step 3** Divide both sides by 2 because $2x$ means two lots of x.
$4 + y = 7$	**Step 4** Substitute $x = 4$ into original equation A. Now the only unknown is y.
$y = 3$	**Step 5** Subtract 4 from both sides to find y.
$4 + 3 = 7$ and $4 - 3 = 1$. Therefore $(x, y) = (4, 3)$.	**Step 6** Check the pair in both original equations. Both equalities must be true.

Example 02 · Multiply one equation to remove x

Solve $2x + y = 8$ and $x + 2y = 7$.

Blue follows x terms and purple follows y terms in the aligned equations. Crossed-out terms add to zero.

Why this method?

We will remove x. A contains $+2x$. B contains $+x$, so multiplying B by -2 creates $-2x$. Then adding the equations makes the x terms cancel.

$$2x + y = 8 \quad \text{A}$$
$$x + 2y = 7 \quad \text{B}$$

Step 1
Label the equations. Compare $+2x$ in A with $+x$ in B: their sizes are not yet equal.

$$-2(x + 2y) = -2(7),$$
so $-2x - 4y = -14$.

Step 2
Keep A unchanged. Multiply every term of B by -2, including the 7 on the right, to make $-2x$.

$$\begin{array}{rrcll} \cancel{2x} & +y & = & 8 & \text{A unchanged} \\ \cancel{-2x} & -4y & = & -14 & B \times (-2) \\ \hline & -3y & = & -6 & \text{Add} \end{array}$$

Step 3
Add A and $B \times (-2)$. The x terms cancel because $+2x - 2x = 0$. For y, $+y - 4y = -3y$.

$$y = 2$$

Step 4
Divide both sides by -3. A negative divided by a negative is positive.

$$x + 2(2) = 7$$

Step 5
Substitute $y = 2$ into original equation B, which has just one x.

$$x + 4 = 7$$

Step 6
Simplify, then subtract 4 from both sides.

$$x = 3,$$
so $(x, y) = (3, 2)$.

Step 7
Find x and state the pair in x, y order.

$$2(3) + 2 = 8$$
and $3 + 2(2) = 7$.

Step 8
Check A and B using both values.

Example 03 · When substitution is simpler

Solve $x = y + 1$ and $2x + y = 11$.

Why this method?

A already tells us what x equals. Replace x in B with the whole expression $y + 1$. This gives one equation containing only y.

$x = y + 1$ A $2x + y = 11$ B	**Step 1** Label the equations so it is clear which expression we will use.
$2(y + 1) + y = 11$	**Step 2** Replace the x in B with a bracket containing $y + 1$. The 2 multiplies that whole bracket.
$2y + 2 + y = 11$	**Step 3** Expand the bracket and collect the y terms.
$3y = 9$	**Step 4** Subtract 2 from both sides.
$y = 3$	**Step 5** Divide both sides by 3 to find y.
$x = 3 + 1 = 4$	**Step 6** Use A to find x from the y-value we have just found.
$4 = 3 + 1$ and $2(4) + 3 = 11$. Therefore $(x, y) = (4, 3)$.	**Step 7** Check the original equations.

Example 04 · Multiply both equations to remove y

Solve $5x + 3y = 67$ and $2x + 4y = 52$.

Why this method?

We will remove y. The coefficients are 3 and 4; their lowest common multiple is 12. Multiply A by 4 to get $+12y$ and B by -3 to get $-12y$. Then add.

$5x + 3y = 67$ A $2x + 4y = 52$ B	**Step 1** Label the original equations and line up the like terms.
$20x + 12y = 268$	**Step 2** Multiply every term of A by 4. We choose 4 because $3y \times 4$ becomes $+12y$.
$-6x - 12y = -156$	**Step 3** Multiply every term of B by -3. We choose -3 because $4y \times (-3)$ becomes $-12y$, the opposite of $+12y$.
$20x + 12y = 268$ $A \times 4$ $-6x - 12y = -156$ $B \times (-3)$ $14x = 112$ Add	**Step 4** Add the two new equations. $+12y - 12y = 0$, so only x remains. On the right, $268 - 156 = 112$.
$x = 8$	**Step 5** Divide both sides by 14 to find one x.
$2(8) + 4y = 52$	**Step 6** Substitute $x = 8$ into original equation B. Using an original equation helps avoid carrying a multiplication error forward.
$16 + 4y = 52$	**Step 7** Calculate 2×8. The equation is now written with a numerical constant and one unknown.
$4y = 36$	**Step 8** Subtract 16 from both sides to remove the constant 16.
$y = 9$, so $(x, y) = (8, 9)$.	**Step 9** Divide both sides by 4 to find y. State both values; finding y completes the solution.

$5(8) + 3(9) = 67$
and $2(8) + 4(9) = 52$.

Step 10

Check the pair in both A and B.

Example 05 · A line and a quadratic together

Solve $y = x + 2$ and $y = x^2$.

$x^2 = x + 2$	**Step 1** Both expressions equal y.
$x^2 - x - 2 = (x - 2)(x + 1) = 0$	**Step 2** Put zero on one side and factor.
$x = 2$ or $x = -1$	**Step 3** Solve each factor.
$(2, 4)$ and $(-1, 1)$	**Final step** Substitute both x values into $y = x + 2$.

Common mistake

Stopping after finding x when the question asks for x and y. Substitute x into an original equation to find y, then check the pair in both original equations.

Exam technique

Check the pair in both original equations.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Solve $x + y = 9$ and $x - y = 3$.

Question 2

Solve $2x + y = 7$ and $x + y = 5$.

Watch an explanation

tecmath — Simultaneous equations: another explanation

Cognito — Solving Simultaneous Equations 2 - Elimination Method

Cognito — Solving Simultaneous Equations 3 - Substitution Method

Cognito — Solving Simultaneous Equations 1 - Using Graphs

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

$x = 6,\ y = 3$. Add the equations to get $2x = 12$, then substitute into $x + y = 9$.

Check : $6 + 3 = 9$ and $6 - 3 = 3$.

Question 2

$x = 2,\ y = 3$. Subtract the second equation from the first, then substitute.

Check both original equations using the two values.

Open the full chapter online