Volume, surface area and nets

Leaving Certificate · Ordinary Level

Understand the idea

How to start

Volume is the space inside a solid. Surface area is the area of its outside faces.

1. Read whether the question asks for volume or surface area.
2. For a prism, find the cross-section area, then multiply it by the prism length.
3. Use cubic units for volume and square units for surface area.

Key terms & definitions

Prism
A solid with a constant polygonal cross-section along its length.

Net
A flat arrangement of the surfaces that fold into a solid.

Exposed surface
A face or curved surface visible on the outside.

Key ideas

• Volume measures space inside a solid. For a prism, $V = A_{\text{cross}-\text{section}} l$.

• Cuboid: $V = lwh$. Cylinder: $V = \pi r^2 h$. Sphere: $V = \frac{4}{3}\pi r^3$.

• Surface area is the total exposed area. A net helps identify every face.

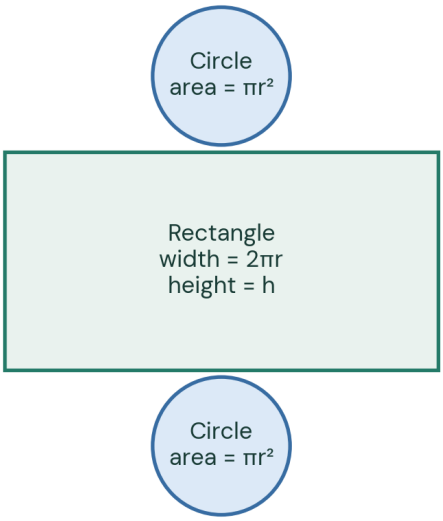
• Cylinder curved surface area is $2\pi rh$; add $2\pi r^2$ for two closed circular ends. Sphere surface area is $4\pi r^2$.

• In a composite solid, add or subtract volumes as required. For exposed surface area, exclude joined internal faces.

• A cylinder net has a rectangle of length $2\pi r$ and height h, plus two circles when both ends are closed.

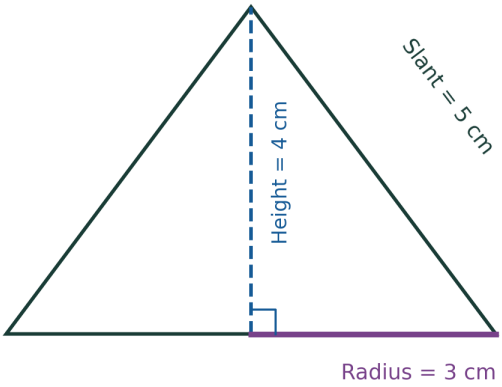
• Cone volume: $V = \frac{1}{3}\pi r^2 h$. Here h is the perpendicular height.

• Hemisphere volume: $V = \frac{2}{3}\pi r^3$. Total surface area of a closed hemisphere is $3\pi r^2$, including its flat circular base.



Closed cylinder net: one rectangle and two circular ends. This schematic shows the surfaces separately.

Cone: use the perpendicular height



A cross-section through the centre of the cone. For volume, use the perpendicular height of 4 cm, not the slant height of 5 cm. The radius is half the base width.

Quick reminder

• Volume uses cubic units; surface area uses square units.

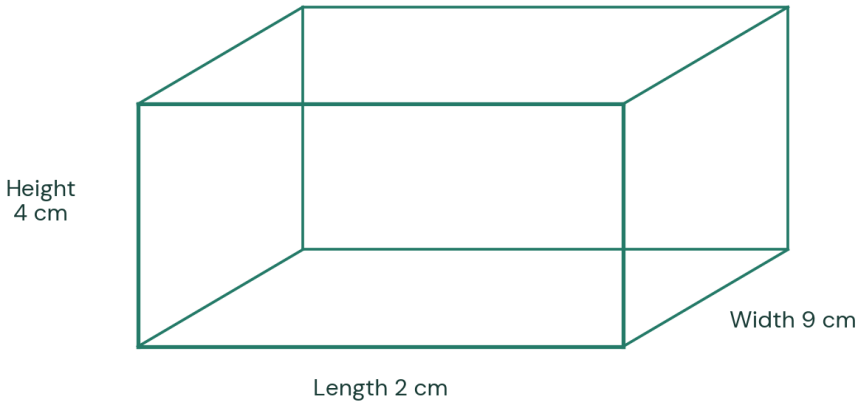
• Prism volume $=$ cross-section area $\times$ length.

• Count exposed surfaces only; do not count joined internal faces.

Worked examples

Example 01 · Volume of a cuboid

A cuboid has length 2 cm, width 9 cm and height 4 cm. What is its volume?



Cuboid with length 2 cm, width 9 cm and height 4 cm. Schematic; use stated dimensions.

Why this method?

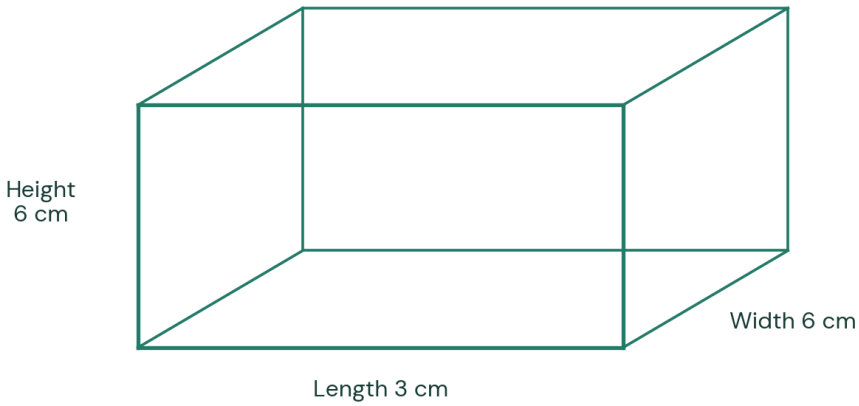
A cuboid has rectangular layers. Its volume is the area of one layer multiplied by the height.

$V = lwh$	**Step 1** V means volume. Here $l = 2$ cm, $w = 9$ cm and $h = 4$ cm.
$V = 2 \times 9 \times 4$	**Step 2** Put each length into the matching place in the formula.
$V = 18 \times 4 = 72$	**Step 3** Multiply the lengths.
$V = 72\text{cm}^3$	**Step 4** Three lengths were multiplied, so the result is in cubic centimetres.

Think about it: Before moving on: why does a volume need all three dimensions?

Example 02 · Total surface area of a cuboid

A cuboid has length 3 cm, width 6 cm and height 6 cm. What is its total surface area?



Cuboid with length 3 cm, width 6 cm and height 6 cm. Schematic; use stated dimensions.

Why this method?

Decide whether the question asks for space inside or area outside. List the required volumes or exposed surfaces before calculating.

$3 \times 6, 3 \times 6, 6 \times 6$	**Step 1** A cuboid has three pairs of equal faces
$2 \times (18 + 18 + 36) = 144 \text{ cm}^2$	**Step 2** Add the three face areas and double

Think about it: Before moving on: why are the faces counted in pairs?

Example 03 · Volume of a cylinder

A cylinder has radius 2 cm and height 4 cm. Find its volume, giving your answer in terms of π.

Why this method?

Decide whether the question asks for space inside or area outside. List the required volumes or exposed surfaces before calculating.

$V = \pi \times 2^2 \times 4$	**Step 1** Volume of a cylinder $= \pi \times \text{radius}^2 \times$ height
$V = 16\pi \text{ cm}^3$	**Step 2** Work out the square and the product

Think about it: Before moving on: why do we square the radius, and why is the answer given in terms of π?

Example 04 · A cone: identify perpendicular height

Find the volume of a cone with radius 3 cm and perpendicular height 4 cm.

Why this method?

A cone occupies one third of the volume of a cylinder with the same base and perpendicular height.

$V = \frac{1}{3}\pi r^2 h$	**Step 1** Choose the cone formula, not the cylinder formula.
$V = \frac{1}{3}\pi(3)^2(4)$	**Step 2** The height here is perpendicular to the base; it is not the sloping length.
$V = \frac{1}{3}\pi(9)(4) = 12\pi \text{cm}^3$	**Step 3** Square the radius and multiply. This is an exact answer.
$V \approx 37.7 \text{cm}^3$	**Step 4** Round only if a decimal answer is required.

Example 05 · A hemisphere is half a sphere

Find the volume of a hemisphere with radius 3 cm.

Why this method?

A hemisphere is half a sphere. Apply the one-half factor to the entire sphere-volume formula.

$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$	**Step 1** Begin with one half of the sphere formula.
$V = \frac{2}{3}\pi(3)^3$	**Step 2** Simplify the numerical factor and put in the radius.
$V = \frac{2}{3}\pi(27) = 18\pi \text{cm}^3$	**Step 3** Cube 3 to get 27, then multiply.

Example 06 · Recasting: conserve the volume

A metal cylinder of radius 3 cm and height 8 cm is melted to make small spheres of radius 1 cm. No metal is lost. How many complete spheres can be made?

Why this method?

Melting changes the shape but preserves the volume when no metal is lost. Divide the starting volume by the volume of one sphere.

$V_{\text{cylinder}} = \pi(3)^2(8) = 72\pi \text{ cm}^3$	**Step 1** Calculate the total volume of metal.
$V_{\text{sphere}} = \frac{4}{3}\pi(1)^3 = \frac{4\pi}{3} \text{ cm}^3$	**Step 2** Calculate the volume needed for just one small sphere.
$\text{Number} = \frac{72\pi}{4\pi/3} = 72\pi \times \frac{3}{4\pi} = 54$	**Step 3** Divide by a fraction by multiplying by its reciprocal. The π factors cancel.
$\text{Number of complete spheres} = 54$	**Step 4** A number of objects must be a whole number. Here there is exactly enough metal for 54.

Example 07 · Surface area of a hemisphere

A solid hemisphere has radius 3 cm. Find its total surface area.

$\frac{1}{2}(4\pi r^2) = 18\pi$	**Step 1** The curved surface is half that of a sphere.
$18\pi + \pi(3^2) = 27\pi \text{ cm}^2$	**Final step** Add the flat circular base.

Common mistake

Giving surface area in cubic units or volume in square units. Surface area measures covering and uses units such as cm^2; volume measures space and uses cm^3.

Exam technique

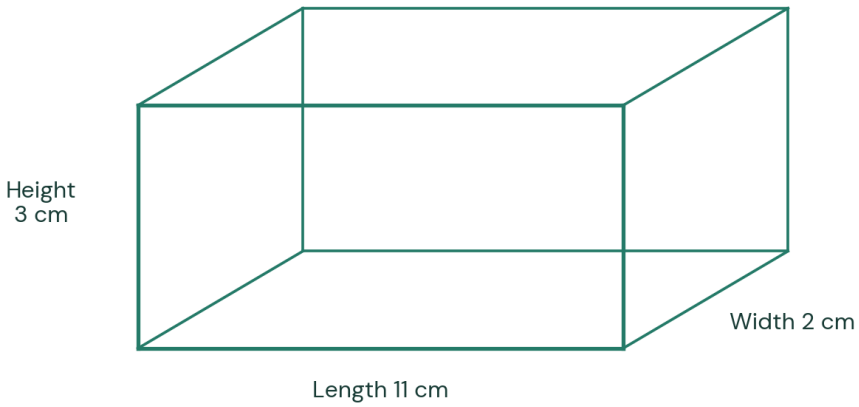
Make a face list and cross off hidden joined surfaces before adding areas.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

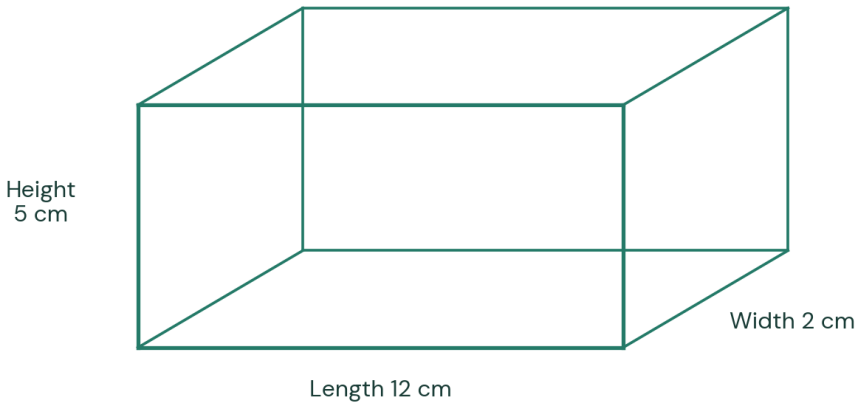
A cuboid has length 11 cm, width 2 cm and height 3 cm. What is its volume?



Cuboid with length 11 cm, width 2 cm and height 3 cm. Schematic; use stated dimensions.

Question 2

A cuboid has length 12 cm, width 2 cm and height 5 cm. What is its total surface area?



Cuboid with length 12 cm, width 2 cm and height 5 cm. Schematic; use stated dimensions.

Question 3

Find the volume of a cone with radius 2 cm and perpendicular height 3 cm.

Question 4 · Build confidence

A cylinder of radius 2 cm and height 6 cm is recast into spheres of radius 1 cm. No metal is lost. How many spheres?

Before you move on

Try these without looking at a worked example. Revisit any part that is still difficult.

- Choose the correct volume formula.
- Use perpendicular height for a cone.
- Use equal volumes when material is recast without loss.

Watch an explanation

Cognito — Volumes of Cylinders & Prisms

Corbettmaths — Surface area: cylinders

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

66

Question 2

188

Question 3

$4\pi \text{cm}^3$.

Use one third of π times radius squared times height.

Question 4

$\frac{24\pi}{4\pi/3} = 18$ spheres.

Volume is conserved; divide by the volume of one sphere.

Open the full chapter online