Standard deviation, normal data and margin of error

Leaving Certificate · Ordinary Level

Understand the idea

How to start

Statistics use a sample to learn about a larger population. A sample estimate has uncertainty.

1. Identify the sample size and the sample proportion or percentage.
2. Calculate the margin of error, keeping proportions separate from percentage points.
3. Build the interval, then interpret it in the population context.

Key terms & definitions

Standard deviation
A measure of how spread out data are around their mean.

Margin of error
An allowance for uncertainty in a sample estimate.

Null hypothesis
The population claim being tested.

Key ideas

• Standard deviation describes spread around the mean. Use the population standard-deviation setting when that is the quantity requested.

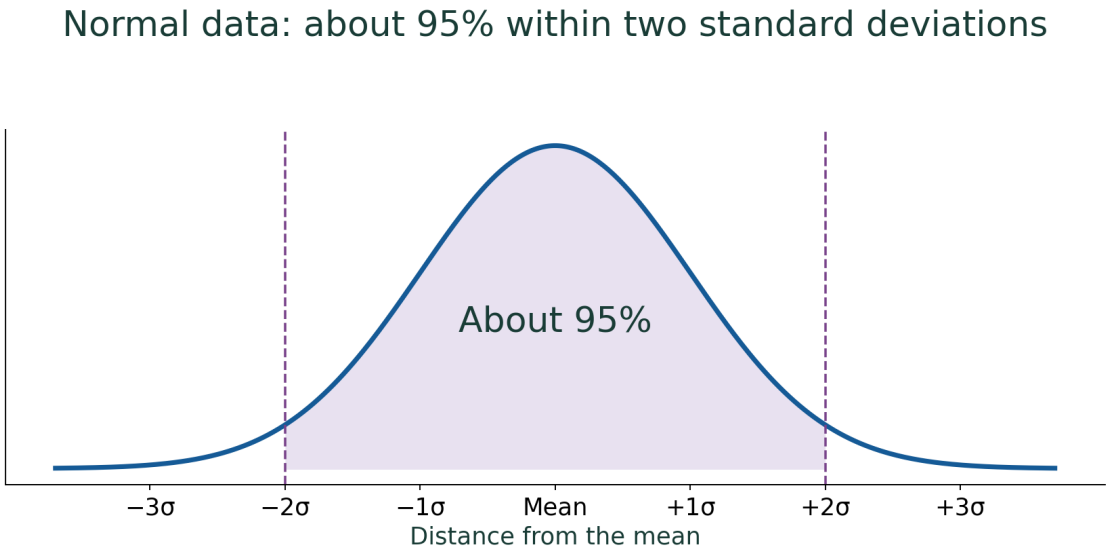
• For an approximately normal distribution, about 68% is within one standard deviation of the mean, 95% within two, and 99.7% within three.

• The approximate 95% margin of error for a population proportion is $\frac{1}{\sqrt{n}}$. This is a proportion; multiply by 100 for percentage points.

• An approximate interval is $\hat{p} \pm \frac{1}{\sqrt{n}}$. State the population and sampling assumptions.

• For the syllabus margin-of-error test, a hypothesised proportion outside the sample interval gives evidence against it at the approximate 5% level. Inside means insufficient evidence to reject it.

• Failure to reject a claim does not prove it is true. Random sampling and representativeness matter.



For approximately normal data, about 95% lie between mean minus two standard deviations and mean plus two standard deviations. This is an approximation, not a guarantee for every sample.

Quick reminder

• Standard deviation describes spread around the mean. Use the population standard-deviation setting when that is the quantity requested.

• For an approximately normal distribution, about 68% is within one standard deviation of the mean, 95% within two, and 99.7% within three.

Worked examples

Example 01 · Start here: a bigger sample gives a smaller margin

Estimate the 95% margin of error for a random sample of $n = 100$.

Why this method?

Use the syllabus approximation 1 divided by the square root of the sample size. Convert the proportion to percentage points at the end.

$\sqrt{n} = \sqrt{100} = 10$	**Step 1** Take the square root before dividing.
$\text{Marginoferror} \approx \frac{1}{10} = 0.10$	**Step 2** This is a proportion.

$0.10 \times 100 = 10 \text{percentagepoints}$	**Step 3** For example, an estimate of 60% would give about 50% to 70%.

Example 02 · A simple standard deviation

Find the population standard deviation of $2, 4, 6$.

Why this method?

Identify the sample information first, calculate the uncertainty, then interpret the result in context.

$\bar{x} = \frac{2+4+6}{3} = 4$	**Step 1** Find the mean first.
$(2-4)^2 + (4-4)^2 + (6-4)^2 = 8$	**Step 2** Find each distance from the mean, square it and add.
$\sigma = \sqrt{\frac{8}{3}} \approx 1.63$	**Step 3** Divide by the number of observations, then take the square root.

Example 03 · Use the empirical rule

Approximately normal scores have mean 50 and standard deviation 5. Find the interval containing about 95%.

Why this method?

Identify the sample information first, calculate the uncertainty, then interpret the result in context.

$50 - 2(5) = 40$	**Step 1** Two standard deviations below the mean.
$50 + 2(5) = 60$	**Step 2** Two standard deviations above the mean.
$40 \leq x \leq 60$	**Step 3** About 95% lie in this interval.

Example 04 · A margin-of-error interval

A random sample of 400 gives a proportion 0.60. Estimate $a95\%$ interval.

Why this method?

Identify the sample information first, calculate the uncertainty, then interpret the result in context.

$\text{MOE} = \frac{1}{\sqrt{400}} = 0.05$	**Step 1** Use the number of people sampled , 400.
$0.60 - 0.05 = 0.55, \quad 0.60 + 0.05 = 0.65$	**Step 2** Subtract and add the margin to the sample proportion.
$55\%\text{to}65\%$	**Step 3** Convert the proportion endpoints to percentages.

Example 05 · Test a population claim

For the previous sample, test the claim that the population proportion is 0.50.

Why this method?

Identify the sample information first, calculate the uncertainty, then interpret the result in context.

$0.50 < 0.55$	**Step 1** The claimed value lies below the sample interval 0.55 to 0.65.
$\text{Rejecttheclaimattheapproximate}5\%\text{level.}$	**Step 2** Under this margin-of-error method, the sample gives evidence against the claim.

Example 06 · A claim inside the sample interval

A sample estimate is 55%, with an approximate 95% margin of error of 5 percentage points. Test a population claim of 52%.

Why this method?

Build the interval first. Then check whether the claimed population percentage falls inside it.

$55\% - 5\% = 50\%, \quad 55\% + 5\% = 60\%$	**Step 1** The interval runs from 50% to 60%.
$50\% \leq 52\% \leq 60\%$	**Step 2** The claimed 52% is inside the interval.

Insufficientevidencetorejecttheclaim.	**Step 3** This does not prove that 52% is the true population percentage.

Example 07 · Test a population proportion using the OL margin of error

In a random sample of 400, 54% agree. Does this challenge a claim that 60% agree?

$\text{MOE} = \frac{1}{\sqrt{400}} = 0.05$	**Step 1** Use the Ordinary Level rule for a population proportion.
From 49 per cent to 59 per cent	**Step 2** Add and subtract five percentage points from 54%.
60% is outside the interval.	**Final step** The result gives evidence against the claim by this margin-of-error method.

Common mistake

Writing a margin of error of 0.05 as 0.05%. A proportion of 0.05 is 5 percentage points. A sample estimate of 60% with this margin gives approximately 55% to 65%.

Exam technique

Explain the conclusion in the context of the population claim.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Find the margin of error for $n = 100$.

Question 2

Estimate the margin of error for a random sample of $n = 400$.

Question 3

A sample estimate is 70% with margin of error 4 percentage points. State the interval.

Self-check answers

Question 1

0.10, or 10 percentage points.

Question 2

$\frac{1}{\sqrt{400}} = 0.05$, or 5 percentage points.

A proportion of 0.05 is five percentage points.

Question 3

Approximately 66% to 74%.

Subtract and add four percentage points.

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