Surds and exact answers

Leaving Certificate · Ordinary Level

Understand the idea

How to start

A square root asks which non-negative number squares to give the number inside. For example, $\sqrt{9} = 3$.

1. To simplify a root, look for a square factor such as $4, 9, 16$ or 25.
2. Take the square root of that factor, leaving the other factor inside.
3. Add surds only when their root parts match. Keep exact roots until a decimal is requested.

Key terms & definitions

Exact answer
An answer without rounding, such as $3\sqrt{2}$.

Like surds
Terms with the same simplified root part.

Conjugates
A pair such as $a + \sqrt{b}$ and $a - \sqrt{b}$.

Key ideas

• A surd is an irrational root written exactly. Keep it exact unless a decimal approximation is requested.

• Find a square factor to simplify: $\sqrt{72} = \sqrt{36}\sqrt{2} = 6\sqrt{2}$.

• Collect like surds after simplifying. Different surds cannot usually be added into one term.

• For non-negative real a, b, $\sqrt{a}\sqrt{b} = \sqrt{ab}$. Expand brackets carefully when multiplying surd expressions.

• The positive square root symbol means the non-negative root. Solving $x^2 = 9$ gives $x = \pm 3$.

Quick reminder

• Find a square factor to simplify a root.

• Collect only terms with the same simplified root part.

• Keep exact surds until a decimal answer is requested.

Worked examples

Example 01 · Simplifying a surd

Simplify $\sqrt{8}$.

Why this method?

Keep the answer exact. Look for square factors and matching root parts before combining any terms.

$8 = 4 \times 2$	**Step 1** Look for a square factor. Four is a square because $2 \times 2 = 4$.
$\sqrt{8} = \sqrt{4} \times \sqrt{2}$	**Step 2** The square root of a product of non-negative numbers can be split into the product of the roots.
$\sqrt{8} = 2\sqrt{2}$	**Step 3** The root of 4 is 2. Two has no square factor greater than 1, so its root stays inside.

Think about it: Why can 2 come out of the square root, while the other factor stays inside?

Example 02 · Simplifying a surd — another example

Simplify $\sqrt{12}$.

Why this method?

Keep the answer exact. Look for square factors and matching root parts before combining any terms.

$12 = 2^2 \times 3$	**Step 1** Find the largest square factor of 12

$\sqrt{12} = \sqrt{2^2} \times \sqrt{3} = 2\sqrt{3}$	**Step 2** Take the square root of the square factor out

Example 03 · Multiplying surds

Simplify $\sqrt{2} \times \sqrt{3}$.

Why this method?

Keep the answer exact. Look for square factors and matching root parts before combining any terms.

$\sqrt{2} \times \sqrt{3} = \sqrt{6}$	**Step 1** Multiply the numbers under one root
$\sqrt{6}$	**Step 2** There is no square factor greater than 1 to take outside, so this is already simplified.

Think about it: Before moving on: why can the two numbers be multiplied under one root, but not added?

Example 04 · Give an exact trigonometric length

A right triangle has hypotenuse 8 cm and an angle of 45°. Find the opposite side exactly.

$x = 8\sin 45^\circ$	**Step 1** Use sine: opposite over hypotenuse.
$x = 8\left(\frac{\sqrt{2}}{2}\right) = 4\sqrt{2}$ cm	**Final step** Use the exact value rather than a rounded decimal.

Common mistake

Splitting a square root across addition. $\sqrt{9+16} = \sqrt{25} = 5$, but $\sqrt{9} + \sqrt{16} = 3 + 4 = 7$. These are different calculations.

Exam technique

Simplify each root before deciding whether terms can be collected.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Simplify $\sqrt{98}$.

A. $\sqrt{14}$

B. $7\sqrt{14}$

C. $49\sqrt{2}$

D. $7\sqrt{2}$

Question 2

Simplify $\sqrt{3} \times \sqrt{12}$.

A. $\sqrt{15}$

B. 36

C. $3\sqrt{12}$

D. 6

Watch an explanation

Cognito — Surds - Introduction & how to Simplify

Cognito — Surds - Rationalising the Denominator

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

$7\sqrt{2}$

Correct : $98 = 7^2 \times 2$, so $\sqrt{98} = 7\sqrt{2}$.

Question 2

6

Correct : $\sqrt{3} \times \sqrt{12} = \sqrt{36} = 6$.

Open the full chapter online