Congruence, similarity and triangle results

Leaving Certificate · Ordinary Level

Understand the idea

How to start

Congruent triangles match exactly in size and shape. Similar triangles have the same shape but may differ in size.

1. Match the corresponding vertices and sides before calculating.
2. Check the stated conditions, such as parallel lines or equal angles.
3. Name the geometrical fact you use and say what it tells you here.

Key terms & definitions

Congruent
The same shape and size.

Similar
The same shape, with a common scale factor between corresponding lengths.

Corresponding
Matching positions in two figures.

Key ideas

• Congruent triangles have corresponding sides and angles equal. Useful criteria are SSS, SAS and ASA, and RHS for right triangles.

• Similar triangles have equal corresponding angles and proportional corresponding sides. Match the vertices in the same order.

• The midpoint theorem links midpoints of two triangle sides: the joining segment is parallel to the third side and half its length.

• Parallel cuts create equal intercepts or proportional segments under the relevant conditions.

• In a parallelogram, opposite sides are equal and diagonals bisect each other. A longer triangle side is opposite a larger angle.

• Explain why a result applies, including its conditions; memorising theorem numbers alone is not enough.

• A diagonal divides a parallelogram into congruent triangles. The converse is not generally valid: a kite can have this diagonal property without being a parallelogram.

• Triangle inequality: any two side lengths must add to more than the third. This is useful when checking whether an SSS construction is possible.

Quick reminder

• Congruent: same size and shape. Similar: same shape.

• Match corresponding vertices before using ratios.

• Check the conditions of the theorem or congruence test.

Worked examples

Example 01 · Match sides before naming congruence

Two triangles each have side lengths 3 cm, 4 cm and 5 cm. Are they congruent?

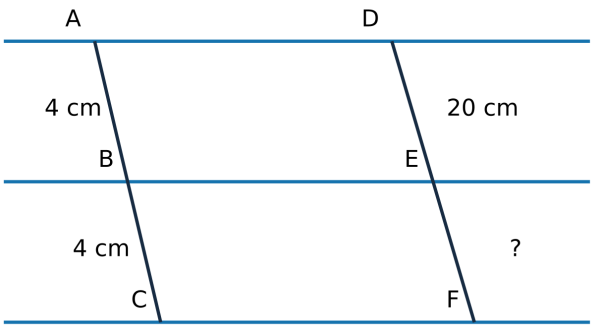
Why this method?

Match the corresponding sides and angles before applying a congruence or similarity rule.

$3 = 3, \quad 4 = 4, \quad 5 = 5$	**Step 1** Match each side with the side of the same length.
Congruent by SSS.	**Step 2** All three corresponding sides match. The triangles have the same size and shape.

Example 02 · Equal gaps made by parallel lines

Three parallel lines meet one transversal at A, B, C in that order and another at D, E, F in that order. AB $=$ BC $=4$ cm and DE $=20$ cm. Find EF.



Three blue lines are parallel.

Equal intercepts on two transversals. Diagram not to scale.

Why this method?

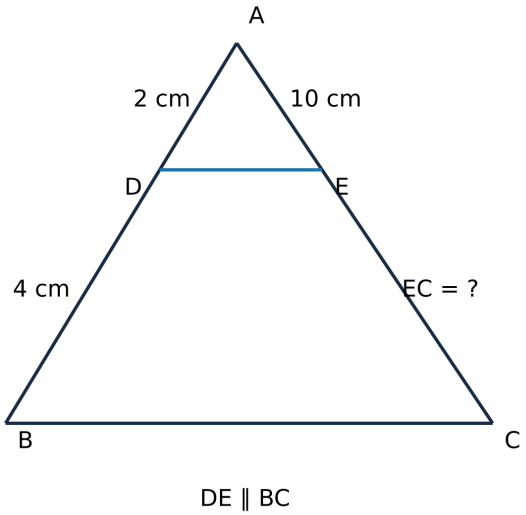
Mark the matching sides or angles. Check the conditions of the geometry result before using it.

The three lines are parallel. AB $=$ BC $=4$ cm.	**Step 1** A transversal is a line crossing the parallel lines. The first transversal has two equal gaps.
$DE = EF$	**Step 2** Three parallel lines that cut equal gaps on one transversal also cut equal gaps on another transversal.
$20\text{cm} = EF$	**Step 3** DE is given as 20 cm. Replace DE by that known length.
$EF = 20\text{cm}$	**Step 4** State the length asked for. The gaps on the two different transversals need not have the same size.

Think about it: Why do DE and EF have to match each other, but do not have to equal 4 cm?

Example 03 · A parallel line divides two sides in the same ratio

In triangle ABC, D is between A and B, E is between A and C, and DE is parallel to BC. AD $= 2$ cm, DB $= 4$ cm and AE $= 10$ cm. Find EC.



Triangle with a parallel cross-section. Diagram not to scale.

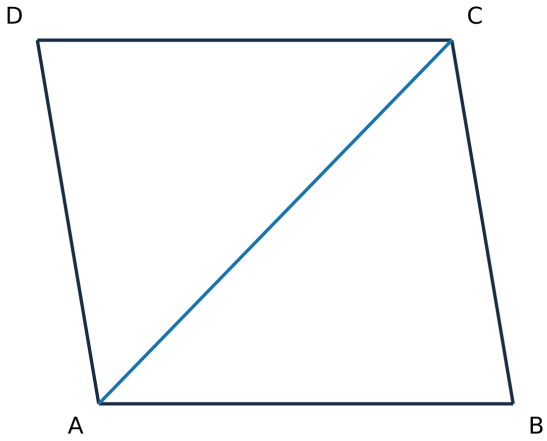
Why this method?

Mark the matching sides or angles. Check the conditions of the geometry result before using it.

$\frac{AD}{DB} = \frac{AE}{EC}$	**Step 1** DE is parallel to BC, so it divides the two sides in the same ratio. Keep upper segment over lower segment on both sides.
$\frac{2}{4} = \frac{10}{EC}$	**Step 2** Replace AD, DB and AE by their given lengths.
$2 \times EC = 4 \times 10$	**Step 3** Multiply both sides by $4 \times$ EC to clear both denominators.
$2EC = 40$	**Step 4** Calculate the right-hand side.
$EC = 20\text{cm}$	**Step 5** Divide both sides by 2. Check : $\frac{2}{4}$ and $\frac{10}{20}$ are both one half.

Example 04 · Corollaries 1, 2 and 5

ABCD is a parallelogram with diagonal AC. AB $= 9$ cm, BC $= 8$ cm and AC $= 11$ cm. Find the perimeter of triangle CDA.



Parallelogram divided by a diagonal. Diagram not to scale.

Why this method?

Mark the matching sides or angles. Check the conditions of the geometry result before using it.

Triangles ABC and CDA are congruent.	**Step 1** Use the diagonal congruence corollary
CD $=$ AB $= 9$ cm; DA $=$ BC $= 8$ cm; CA $= 11$ cm.	**Step 2** Match the corresponding sides
$9 + 8 + 11 = 28$ cm.	**Step 3** Add the three sides of triangle CDA

Think about it: Which sides in the two triangles correspond, and why does the shared diagonal have the same length in both?

Example 05 · A parallel cut gives proportional sides

DE is parallel to BC in triangle ABC. AD $= 4$, DB $= 6$, AE $= 3$. Find EC.

$\frac{AD}{DB} = \frac{AE}{EC}$	**Step 1** The parallel cut divides the two sides in the same ratio.
$\frac{4}{6} = \frac{3}{EC}$	**Step 2** Match top segments and lower segments.

$EC = 4.5$	**Final step** Cross-multiply and divide by 4.

Common mistake

Using equal angles to claim two triangles are congruent. Equal corresponding angles show similarity, so the shape is the same; the triangles may still be different sizes.

Exam technique

Mark corresponding sides before writing a ratio.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Three parallel lines meet one transversal at A, B, C in that order and another at D, E, F in that order. AB $=$ BC $=4$ cm and DE $=6$ cm. Find EF.

A. 4 cm

B. 12 cm

C. 6 cm

D. 3 cm

Question 2

ABCD is a parallelogram with diagonal AC. AB $=7$ cm, BC $=10$ cm and AC $=9$ cm. Find the perimeter of triangle CDA.

A. 17 cm

B. 26 cm

C. 35 cm

D. 34 cm

Watch an explanation

Cognito — Proving Congruence in Triangles (SSS, SAS, AAS & RHS Rules)

Cognito — Similar Shapes

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

6 cm

Correct.

Question 2

26 cm

Correct.

Open the full chapter online