Algebraic fractions

Junior Cycle · Higher Level

Understand the idea

How to start

The letters do not change the fraction rules. The whole numerator must stay above its fraction bar.

1. Check which values would make any original denominator zero.

2. For adding or subtracting, make the denominators match by multiplying top and bottom by the same factor.

3. Combine the numerators. For subtraction, bracket the whole second numerator.

Key terms & definitions

Restriction
A value excluded because it makes an original denominator zero.

Common denominator
A denominator shared by equivalent forms of the fractions.

Rational expression
A ratio of polynomials with a non-zero denominator.

Key ideas

• An algebraic fraction follows the same rules as a numerical fraction, but you must also state excluded values.

• A denominator cannot be zero. Find restrictions from every original denominator before simplifying.

• Cancel common factors of the whole numerator and denominator. Do not cancel terms across addition.

• For addition or subtraction, choose a common denominator and scale each numerator by the missing factor.

• For subtraction, put the whole second numerator in brackets before collecting terms.

Quick reminder

• State excluded values from the original denominators.

• Use equivalent fractions with a common denominator.

• Cancel factors, never separate terms in a sum.

Worked examples

Example 01 · Start here: the same denominator

Simplify $\frac{x}{5} + \frac{2x}{5}$.

Why this method?

Start with equal denominators. Adding x fifths and $2x$ fifths gives $3x$ fifths, just as adding ordinary fractions does.

$\frac{x+2x}{5}$	**Step 1** Both fractions are already in fifths, so the denominator can stay 5.
$\frac{3x}{5}$	**Step 2** The numerator contains like terms: one x plus two x makes three x.
$\frac{x}{5} + \frac{2x}{5} = \frac{3x}{5}$	**Step 3** We do not add the denominators. They describe the size of the parts we are counting.

Example 02 · Different denominators, one line at a time

Simplify $\frac{x+3}{5} - \frac{2x-1}{3}$.

Blue follows the first whole fraction and purple the second as their denominators are made equal. Each numerator stays above its own fraction bar.

Why this method?

The smallest common denominator is 15. Multiply top and bottom of the first fraction by 3 and of the second by 5. Each fraction keeps its value.

$\frac{x+3}{5} - \frac{2x-1}{3}$	**Step 1** Start with the original expression.

$\frac{3(x+3)}{15} - \frac{5(2x-1)}{15}$	**Step 2** The factors 3 and 5 multiply the whole numerators and stay above their bars.
$\frac{3x+9}{15} - \frac{10x-5}{15}$	**Step 3** Expand each numerator.
$\frac{3x+9-(10x-5)}{15}$	**Step 4** Combine the numerators. Bracket the whole second numerator.
$\frac{3x+9-10x+5}{15}$	**Step 5** Subtracting -5 gives $+5$.
$\frac{-7x+14}{15}$	**Step 6** Collect the x terms and the number terms.

Example 03 · Adding algebraic fractions with linear denominators

Write as a single fraction
$: \frac{4}{(x+4)} + \frac{2}{(x+2)}$
.

Blue follows the first whole fraction and purple the second as their denominators are made equal. Each numerator stays above its own fraction bar.

Why this method?

Find values that would make a denominator zero. Then build equivalent fractions with the same denominator before combining them.

$\frac{4}{x+4} + \frac{2}{x+2}$	**Step 1** The denominators differ. Use $(x+4)(x+2)$, keeping $x \neq -4$ and $x \neq -2$.
$\frac{4(x+2)}{(x+4)(x+2)} + \frac{2(x+4)}{(x+4)(x+2)}$	**Step 2** Multiply the whole first fraction by $\frac{(x+2)}{(x+2)}$, and the whole second fraction by $\frac{(x+4)}{(x+4)}$. Keep both multipliers in their numerators.
$\frac{4x+8}{(x+4)(x+2)} + \frac{2x+8}{(x+4)(x+2)}$	**Step 3** Expand each numerator while keeping the common denominator underneath.

$\frac{4x+8+2x+8}{(x+4)(x+2)}$	**Step 4** The denominators now match, so add the numerators.
$\frac{6x+16}{(x+4)(x+2)}$	**Step 5** Collect like terms on top. Keep the original restrictions $x \neq -4, -2$.

Think about it: Before moving on: why can the numerators not simply be added when the denominators are different?

Example 04 · Subtracting algebraic fractions with linear denominators

Write as a single fraction
$: \frac{4}{(x+2)} - \frac{2}{(x+4)}$
.

Blue follows the first whole fraction and purple the second as their denominators are made equal. Each numerator stays above its own fraction bar.

Why this method?

Find values that would make a denominator zero. Then build equivalent fractions with the same denominator before combining them.

$\frac{4}{x+2} - \frac{2}{x+4}$	**Step 1** Use $(x+2)(x+4)$ as the common denominator. Original restrictions $: x \neq -2, -4$.
$\frac{4(x+4)}{(x+2)(x+4)} - \frac{2(x+2)}{(x+2)(x+4)}$	**Step 2** Multiply top and bottom of each fraction by its missing factor.
$\frac{4x+16}{(x+2)(x+4)} - \frac{2x+4}{(x+2)(x+4)}$	**Step 3** Expand the numerators. Keep each denominator in place.
$\frac{4x+16-(2x+4)}{(x+2)(x+4)}$	**Step 4** Subtract the entire second numerator, so put it in brackets.
$\frac{4x+16-2x-4}{(x+2)(x+4)}$	**Step 5** The minus sign changes both signs in the second bracket.
$\frac{2x+12}{(x+2)(x+4)}$	**Step 6** Collect like terms. Keep $x \neq -2, -4$.

Think about it: Before moving on: why must the whole second numerator be subtracted, and not just its first term?

Example 05 · Algebraic fractions with coefficients on x

Write as a single fraction
$: \frac{1}{(2x+1)} + \frac{2}{(5x-1)}$
. Exclude values of x that make either original denominator zero.

Why this method?

Find values that would make a denominator zero. Then build equivalent fractions with the same denominator before combining them.

$\frac{1}{2x+1} + \frac{2}{5x-1}$	**Step 1** Use the product of the different denominators.
$\frac{1(5x-1)}{(2x+1)(5x-1)} + \frac{2(2x+1)}{(2x+1)(5x-1)}$	**Step 2** Multiply each numerator and denominator by the same missing factor.
$\frac{5x-1+4x+2}{(2x+1)(5x-1)}$	**Step 3** Expand on top, then combine over the common denominator.
$\frac{9x+1}{(2x+1)(5x-1)}$	**Step 4** Collect like terms. Keep $x \neq -\frac{1}{2}$ and $x \neq \frac{1}{5}$.

Think about it: Why must the coefficient of x be retained when multiplying by the missing denominator?

Example 06 · Algebraic fractions with coefficients on x — another example

Write as a single fraction
$: \frac{2}{(2x+5)} - \frac{5}{(4x+3)}$
. Exclude values of x that make either original denominator zero.

Why this method?

Find values that would make a denominator zero. Then build equivalent fractions with the same denominator before combining them.

$\frac{2}{2x+5} - \frac{5}{4x+3}$	**Step 1** Use $(2x+5)(4x+3)$ as the common denominator.
$\frac{2(4x+3)}{(2x+5)(4x+3)} - \frac{5(2x+5)}{(2x+5)(4x+3)}$	**Step 2** Keep each multiplier above the fraction bar.

$\frac{8x+6-(10x+25)}{(2x+5)(4x+3)}$	**Step 3** Expand and bracket the whole second numerator.
$\frac{8x+6-10x-25}{(2x+5)(4x+3)}$	**Step 4** Distribute the subtraction sign to both terms.
$\frac{-2x-19}{(2x+5)(4x+3)}$	**Step 5** Collect like terms. Keep $x \neq -\frac{5}{2}$ and $x \neq -\frac{3}{4}$.

Example 07 · Algebraic fractions with repeated or proportional denominators

Simplify $\frac{2}{(5x+(11))}+\frac{8}{(5x+(11))}$. State the excluded x-value from the original denominators.

Why this method?

Find values that would make a denominator zero. Then build equivalent fractions with the same denominator before combining them.

$\frac{2}{5x+11}+\frac{8}{5x+11}$	**Step 1** The denominators already match.
$\frac{2+8}{5x+11}$	**Step 2** Add only the numerators.
$\frac{10}{5x+11}$	**Step 3** Keep the original restriction $x \neq -\frac{11}{5}$.

Think about it: Why is multiplying two identical denominators unnecessary when adding their fractions?

Example 08 · Algebraic fractions with repeated or proportional denominators — another example

Simplify $\frac{8}{(5x+(9))} - \frac{4}{(10x+(18))}$. State the excluded x-value from the original denominators.

Why this method?

Find values that would make a denominator zero. Then build equivalent fractions with the same denominator before combining them.

$\frac{8}{5x+9} - \frac{4}{2(5x+9)}$	**Step 1** Factor the second denominator. It is twice the first.
$\frac{2(8)}{2(5x+9)} - \frac{4}{2(5x+9)}$	**Step 2** Multiply the first numerator and denominator by 2.
$\frac{16-4}{2(5x+9)}$	**Step 3** The denominators now match.
$\frac{12}{2(5x+9)} = \frac{6}{5x+9}$	**Step 4** Cancel the common factor 2. Keep $x \neq -\frac{9}{5}$.

Common mistake

Cancelling a term that is joined by addition. $\frac{x+3}{x}$ cannot become 3. Only common factors cancel: $\frac{x(x+3)}{x} = x+3$ when $x \neq 0$.

Exam technique

Keep the original restrictions even if a factor cancels.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Write as a single fraction $: \frac{3}{(x+5)} + \frac{2}{(x+1)}$.

A.
$$\frac{5}{((x+5)(x+1))}$$

B.
$$\frac{(5x+13)}{(x+6)}$$

C.
$$\frac{(5x+13)}{((x+5)(x+1))}$$

D.
$$\frac{(5x+17)}{((x+5)(x+1))}$$

Question 2

Write as a single fraction
$$:\frac{3}{(x+2)}-\frac{2}{(x+5)}$$
.

A.
$$\frac{((1)x+11)}{((x+2)(x+5))}$$

B.
$$\frac{(1)}{((x+2)(x+5))}$$

C.
$$\frac{((5)x+19)}{((x+2)(x+5))}$$

D.
$$\frac{((1)x-4)}{((x+2)(x+5))}$$

Question 3

Write as a single fraction
$$:\frac{1}{(2x+1)}+\frac{3}{(2x-2)}$$
. Exclude values of x that make either original denominator zero.

A.
$$\frac{(8x-5)}{((2x+1)(2x-2))}$$

B.
$$\frac{(4)}{((2x+1)(2x-2))}$$

C.
$$\frac{(8x+1)}{((2x+1)(2x-2))}$$

D.
$$\frac{(-4x-5)}{((2x+1)(2x-2))}$$

Question 4

Simplify
$$\frac{7}{(5x+(6))} - \frac{8}{(10x+(12))}$$
. State the excluded x-value from the original denominators.

A.
$$\frac{(3)}{(5x+(6))}$$
; $x \neq \frac{6}{5}$.

B.
$$\frac{(11)}{(5x+(6))}$$
; $x \neq -\frac{6}{5}$.

C.
$$\frac{(3)}{(5x+(6))}$$
; $x \neq -\frac{6}{5}$.

D.
$$\frac{\left(-\frac{1}{2}\right)}{(5x+(6))}$$
; $x \neq -\frac{6}{5}$.

Watch an explanation

Cognito — Simplifying Algebraic Fractions

Corbettmaths — Algebraic fractions: addition

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

$$\frac{(5x+13)}{((x+5)(x+1))}$$

Question 2

$$\frac{((1)x+11)}{((x+2)(x+5))}$$

Question 3

$$\frac{(8x+1)}{((2x+1)(2x-2))}$$

Question 4

$$\frac{(3)}{(5x+(6))}$$
; $x \neq -\frac{6}{5}$.

Open the full chapter online