Angles and geometrical reasoning

Junior Cycle · Higher Level

Understand the idea

How to start

An angle measures a turn. A straight angle is 180° and a full turn is 360°.

1. Mark the angle you need and the angles you know.
2. Name the angle fact that links them.
3. Write an equation or subtraction using that fact, then calculate.

Key terms & definitions

Axiom
An accepted starting statement in a mathematical system.

Theorem
A statement established by reasoning from accepted facts.

Converse
A statement made by reversing the condition and conclusion.

Corollary
A result that follows from a theorem.

Key ideas

• Angles on a straight line total 180°; angles around a point total 360°. Vertically opposite angles are equal.

• The interior angles of a triangle total 180°. An exterior angle equals the sum of the two opposite interior angles.

• With parallel lines, corresponding angles are equal, alternate angles are equal, and interior angles on the same side total 180°.

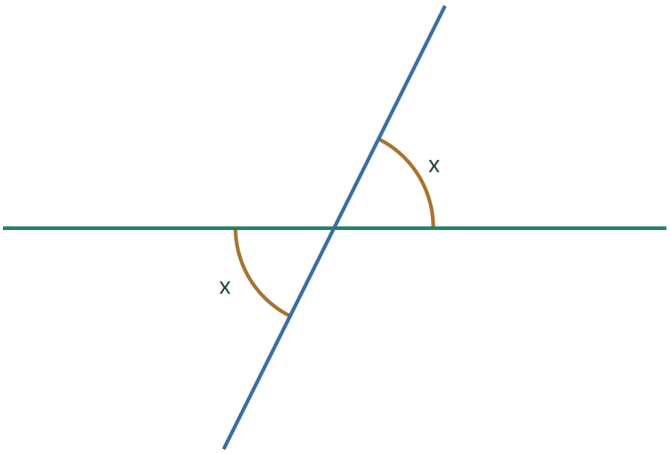
• An isosceles triangle has equal base angles; the converse also holds. An equilateral triangle has three equal sides and three 60° angles.

• An axiom is accepted as a starting point. A theorem is justified from earlier results. A converse reverses a statement and needs its own justification.

• Give a reason for each angle step. A diagram looking equal or parallel is not enough evidence.

• The five starting axioms concern the unique line through two distinct points, measurement of length, measurement of angles, triangle congruence and the unique parallel through a point off a line. Use them as agreed starting facts.

• Understand and explain reasoning, including what each step relies on. Formal theorem proofs are not examined as memorised reproductions.



Vertically opposite angles are equal. Each is supplementary to the same adjacent angle.

How the reasoning works

Vertically opposite angles

1. Name one angle x and the adjacent angle y. Together they make a straight angle, so $x + y = 180^\circ$.

2. The opposite angle z also forms a straight angle with y, so $z + y = 180^\circ$.

3. Both x and z equal $180^\circ - y$, so $x = z$.

Triangle angle sum

1. Draw a line through one vertex parallel to the opposite side.

2. Alternate angles transfer the two base angles to that new line.

3. Those two angles and the vertex angle make a straight angle, giving 180° in total.

Parallelogram properties

1. Draw a diagonal of the parallelogram. The two pairs of parallel sides give equal alternate angles.

2. The diagonal is common to both triangles. ASA establishes congruence.

3. Corresponding sides and angles are equal. A second congruence argument at the diagonal intersection shows that the diagonals bisect each other.

Pythagoras: an area argument

1. Arrange four congruent right triangles of legs a and b inside a square of side $a + b$, leaving a central square of side c.

2. The large square area is $(a + b)^2$. The four triangles total $2ab$, leaving c^2.

3. Thus $a^2 + 2ab + b^2 = 2ab + c^2$. Subtract $2ab$ to obtain $a^2 + b^2 = c^2$. The angle relationships ensure the central shape is a square.

Circle angle theorem

1. Join the centre to the circumference points. Radii make isosceles triangles, so the base angles in each are equal.

2. Use the exterior-angle rule in each triangle to express the central angle as twice the relevant base angle.

3. Add or subtract the parts to get the whole central angle as twice the circumference angle standing on the same arc. The cases depend on the position of the centre.

Similar triangle sides

1. For two triangles with equal corresponding angles, copy two sides of the smaller triangle onto the corresponding rays of the larger. SAS gives a congruent copy.

2. Equal corresponding angles make the joining segment parallel to the third side.

3. The parallel-line ratio result gives equal ratios of corresponding sides. This explains the scale factor rather than assuming it.

Isosceles triangle: why the base angles agree

1. Bisect the vertex angle. The two equal sides, the common bisector and the equal included half-angles give SAS congruence of the two smaller triangles.

2. Corresponding base angles are equal. Conversely, equal base angles allow an ASA argument that shows the opposite sides are equal.

Exterior angle: why we add the two opposite angles

1. The interior angle next to the exterior angle and the exterior angle total 180°.

2. The three interior triangle angles also total 180°. Subtract the shared interior angle from both totals to obtain the exterior-angle rule.

When a converse is valid

1. Equal alternate angles force parallel lines; parallel lines give equal alternate angles. The corresponding-angle statement also works in both directions.

2. A right triangle satisfies Pythagoras. Conversely, if its longest side satisfies the Pythagorean equation, compare it with a constructed right triangle with the same two shorter sides; SSS establishes the right angle.

3. Do not reverse every statement automatically. A diagonal splitting a quadrilateral into two congruent triangles need not make it a parallelogram.

Quick reminder

• Name the angle fact before calculating.

• Line : 180°. Point : 360°. Triangle : 180°.

• Parallel-line rules require parallel lines.

Worked examples

Example 01 · Angles on a line, at a point and vertically opposite

Two angles lie on a straight line. One of them is 33°. What is the other angle?

Why this method?

Identify the angle fact and check its conditions. Use it to write an equation for the unknown angle.

$x + 33^\circ = 180^\circ$	**Step 1** Let x be the missing angle. Angles making a straight line add to 180°.
$x = 180^\circ - 33^\circ$	**Step 2** Subtract the known angle from the straight angle.
$x = 147^\circ$	**Step 3** Calculate the missing angle.
$147^\circ + 33^\circ = 180^\circ$	**Step 4** Check the two angles make a straight angle.

Think about it: Before moving on: why do vertically opposite angles have to be equal, and not just add to 180°?

Example 02 · Angles in a triangle

A triangle has angles of 61° and 24°. What is the third angle?

Why this method?

Identify the angle fact and check its conditions. Use it to write an equation for the unknown angle.

$180^\circ - 61^\circ - 24^\circ = 95^\circ$	**Step 1** The angles of a triangle add up to 180°

Think about it: Why do we subtract both known angles from 180° to find the third angle?

Example 03 · Angles with parallel lines

Two parallel lines are cut by a transversal. One of the angles formed is 42°. What is the alternate angle to it?

Why this method?

Identify the angle fact and check its conditions. Use it to write an equation for the unknown angle.

$42^\circ = 42^\circ$	**Step 1** Alternate and corresponding angles are equal for parallel lines

Think about it: Before moving on: how can you tell whether two angles are alternate, corresponding or co-interior?

Example 04 · Explaining the language of proof

Rule $R95$: through two distinct points there is exactly one line. In this deductive system $R95$ is explicitly accepted without proof as a starting statement. Explain its logical role.

Why this method?

Identify the angle fact and check its conditions. Use it to write an equation for the unknown angle.

$R95$ states that through two distinct points there is exactly one line; it is accepted without proof in this system.	**Step 1** Read the stated status
An accepted starting statement is an axiom.	**Step 2** Explain why the term applies
Later proofs can use the axiom as a premise.	**Step 3** Connect its role to later reasoning

Think about it: Can you explain the logical role of the statement, rather than simply name the term?

Example 05 · Explaining the language of proof — another example

Statement $S56$: the angles of a triangle sum to 180°. It has been established by a valid logical deduction from accepted axioms and earlier results. It was not merely measured or assumed. Explain why it is a theorem.

Why this method?

Identify the angle fact and check its conditions. Use it to write an equation for the unknown angle.

A valid deduction establishes $S56$: the angles of a triangle sum to 180°.	**Step 1** Identify what establishes the statement
The established statement is a theorem; the argument establishing it is a proof.	**Step 2** Distinguish statement and argument
It is deduced from accepted statements rather than accepted here without proof.	**Step 3** Explain the distinction from an axiom

Common mistake

Assuming angles are equal just because two lines look parallel. Corresponding and alternate-angle equalities require parallel lines. Use the given parallel marks or establish that the lines are parallel first.

Exam technique

Write the angle fact beside the calculation rather than giving an unexplained number.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Two angles lie on a straight line. One of them is 125°. What is the other angle?

Question 2

A triangle has angles of 115° and 20°. What is the third angle?

Question 3

Two parallel lines are cut by a transversal. One of the angles formed is 160°. What is the co-interior angle to it?

Question 4

Rule $R42$: through a point P off a given line l there is exactly one line parallel to l. In this deductive system $R42$ is explicitly accepted without proof as a starting statement. Explain its logical role.

A. It is a corollary: the prompt derives it directly from an earlier theorem.

B. It is a theorem: the prompt supplies a deduction establishing it.

C. It is an axiom: an accepted starting statement on which deductions can be based.

D. It is an axiom because measuring many drawings proves it for every pair of points.

Watch an explanation

Cognito — 5 Simple Angle Rules

Cognito — Alternate, Corresponding & Allied Angles

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

55

Question 2

45

Question 3

20

Question 4

It is an axiom: an accepted starting statement on which deductions can be based.

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