Averages, spread and grouped data

Junior Cycle · Higher Level

Understand the idea

How to start

Mean is a shared-out total. Median is the middle of an ordered list. Mode is the most common value.

1. Read which measure the question asks for.
2. For the mean, add the values and divide by the number of values.
3. For the median, order the list first. Range is largest minus smallest.

Key terms & definitions

Mean
The total shared equally across all observations.

Median
The middle of the ordered data.

Mode
The most frequent value.

Range
The largest value minus the smallest value.

Key ideas

• The mean is $\frac{\text{sum of values}}{\text{number of values}}$. For frequencies, use $\frac{\sum fx}{\sum f}$.

• Find the median after ordering the data. With an even number of values, average the two middle values.

• The mode is the most frequent value; data can have more than one mode or none. The range is maximum minus minimum.

• An outlier can change the mean strongly. The median is often more useful for very skewed data.

• For grouped data, use class midpoints to estimate the mean. It is an estimate because the individual values are not known.

Quick reminder

• Mean: add and divide by the number of values.

• Median: order first, then find the middle.

• Grouped mean uses midpoints and is an estimate.

Worked examples

Example 01 · Mean, median, mode and range

The data set is $5, 18, 6, 8, 18$. Find the mean.

Why this method?

For a mean, add and count. Ordering is needed for the median, but it is not needed to calculate this mean.

$5 + 18 + 6 + 8 + 18 = 55$	**Step 1** Add every value, including the repeated 18.
Number of values $= 5$.	**Step 2** Count observations, including repeats. There are five values, not four different values.
$\text{mean} = \frac{55}{5}$	**Step 3** The mean shares the total equally among the five observations.
$\text{mean} = 11$	**Step 4** Divide 55 by 5. The mean lies between the smallest and largest values.

Think about it: Before moving on: which of these four measures is not an average, and why?

Example 02 · Reading a stem-and-leaf plot

Using the stem-and-leaf plot, what is the range of the data?

Stem	Leaves
1	117
2	146
3	33578

Why this method?

Identify the measure requested. Order values for a median; add and count for a mean; use the extremes for a range.

11 and 38	**Step 1** Find the smallest and largest values
$38 - 11 = 27$	**Step 2** Subtract the smallest from the largest

Think about it: Before moving on: which value is the smallest, and how do you read it from the plot?

Example 03 · Mean from grouped data

Intervals $0 - < 10, 10 - < 20$ and $20 - < 30$ have frequencies $2, 5$ and 3. Estimate the mean.

Why this method?

Identify the measure requested. Order values for a median; add and count for a mean; use the extremes for a range.

$5, 15$ and 25.	**Step 1** The individual values are unknown. Use the middle of each class as a representative value, so the final mean will be an estimate.
$\sum fx = 2(5) + 5(15) + 3(25) = 160$	**Step 2** Multiply each representative value by its frequency because it stands for that many observations.
$\bar{x} \approx \frac{160}{10} = 16$	**Step 3** Divide by the number of observations, not the number of classes, to share the total equally.

This is an estimate because each class is represented by its midpoint.	**Step 4** State why this is an estimate: we used a midpoint to represent all the unknown individual values in each class.

Example 04 · Mean from a grouped frequency table

A frequency table has classes with midpoints 5, 15 and 25, with frequencies 3, 1 and 1. Estimate the mean.

Why this method?

Identify the measure requested. Order values for a median; add and count for a mean; use the extremes for a range.

$3 \times 5 + 1 \times 15 + 1 \times 25 = 55$	**Step 1** Multiply each midpoint by its frequency and add
$55 \div 5 = 11$	**Step 2** Divide by the number of observations, not the number of classes, to share the total equally.

Think about it: Before moving on: why are the frequencies used as weights rather than averaging the midpoints directly?

Example 05 · Mean from a grouped frequency table — another example

A frequency table has classes with midpoints 10, 20 and 30, with frequencies 3, 1 and 6. Estimate the mean.

Why this method?

Identify the measure requested. Order values for a median; add and count for a mean; use the extremes for a range.

$3 \times 10 + 1 \times 20 + 6 \times 30 = 230$	**Step 1** Multiply each midpoint by its frequency and add
$230 \div 10 = 23$	**Step 2** Divide by the number of observations, not the number of classes, to share the total equally.

Common mistake

Finding the middle value before sorting the data. In $8, 2, 5$, the median is 5 after ordering as $2, 5, 8$; it is not the middle entry 2 in the original list.

Exam technique

State whether a calculated mean is exact or estimated from grouped data.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

The data set is $2, 5, 5, 9, 14$. Find the median.

Question 2

Using the same stem-and-leaf plot, how many values are 25 or more?

Stem	Leaves
1	117
2	146
3	33578

Question 3

A frequency table has classes with midpoints 10, 20 and 30, with frequencies 1, 4 and 5. Estimate the mean.

A. 10

B. 240

C. 24

D. 20

Watch an explanation

Cognito — Averages - Mean, Median, Mode & Range

Cognito — Grouped Frequency Tables

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

5

Question 2

6

Question 3

24

Open the full chapter online