Combined events, trees and tables

Junior Cycle · Higher Level

Understand the idea

How to start

For two events, keep track of what happens first and what happens second.

1. List the complete paths that give the event you want.
2. For independent events, multiply the probabilities along each path.
3. Add different successful paths when they cannot happen together.

Key terms & definitions

Independent events
Knowing one outcome does not change the probability of the other.

Mutually exclusive
Events that cannot happen together.

Tree diagram
A branching display of successive outcomes and their probabilities.

Key ideas

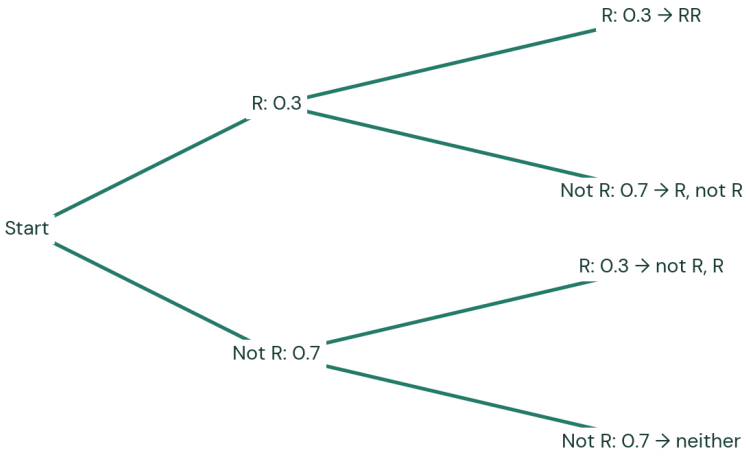
• The counting principle multiplies the number of choices at successive stages when each stage has the stated choices.

• For independent events, multiply probabilities along a tree branch.

• For mutually exclusive complete paths to the required event, add the path probabilities.

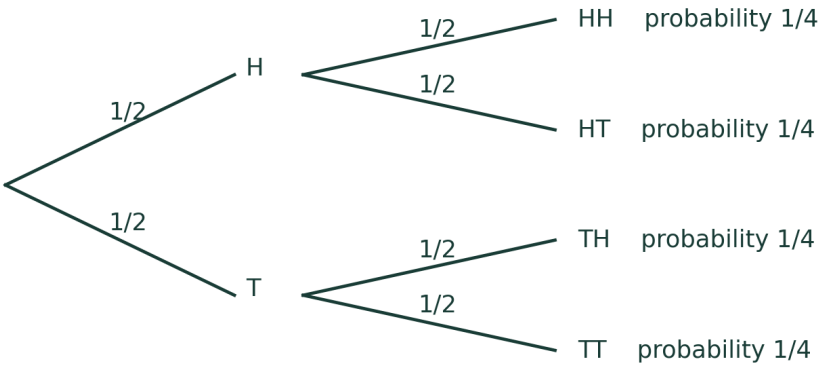
• With replacement can make repeated draws independent; without replacement changes the available outcomes.

• A two-way table shows overlaps. Use the correct total for the group named in the question.



Two independent spins. Multiply along a path; add the two disjoint paths for exactly one red.

Two fair coin tosses: four equally likely paths



Two independent fair coin tosses give HH, HT, TH and TT. Each complete path has probability one half times one half, or one quarter.

Quick reminder

- Multiply along an independent sequence of events.
- Add mutually exclusive successful paths.
- Use the total of the group actually selected.

Worked examples

Example 01 · Two independent spins

A spinner has 10 equal-sized sections, with 3 red sections. The probability of red is 0.3. It is spun twice independently. Find the probability of exactly one red.

Blue follows red then not red. Purple follows not red then red. Add the two complete paths for exactly one red.

Why this method?

List the possible paths or groups first. Decide when we are multiplying along a path and when we are adding different successful paths.

$P(\text{notred}) = 1 - 0.3 = 0.7$	**Step 1** Red and not red cover all possible outcomes of one spin.
Successful paths: red then not red, or not red then red.	**Step 2** Exactly one red means one red spin and one non-red spin. Both orders count.
$P(\text{redthennotred}) = 0.3 \times 0.7 = 0.21$	**Step 3** The spins are independent, so multiply along this path.
$P(\text{notredthenred}) = 0.7 \times 0.3 = 0.21$	**Step 4** Calculate the other successful order.
$P(\text{exactlyonered}) = 0.21 + 0.21 = 0.42$	**Step 5** Add these paths because the same pair of spins cannot follow both orders.

Example 02 · Two dice

Two fair six-sided dice are rolled. Find the probability of a total of 8.

Why this method?

List the possible paths or groups first. Decide when we are multiplying along a path and when we are adding different successful paths.

$6 \times 6 = 36$	**Step 1** Count equally likely ordered outcomes
$(2, 6), (3, 5), (4, 4), (5, 3), (6, 2).$	**Step 2** List favourable pairs

$P(8) = \frac{5}{36}$	**Step 3** Form the probability

Example 03 · A two-way table

Of 24 students, 14 cycle. Of those 14, 6 wear glasses. If a cyclist is selected, what is the probability they wear glasses?

Why this method?

List the possible paths or groups first. Decide when we are multiplying along a path and when we are adding different successful paths.

Choose from the 14 cyclists.	**Step 1** Only cyclists can be selected here, so use 14 as the total, not 24.
$P(\text{glasses} \mid \text{cyclists}) = \frac{6}{14}$	**Step 2** Six of the fourteen cyclists wear glasses.
$\frac{6}{14} = \frac{3}{7}$	**Step 3** Divide numerator and denominator by 2. The denominator still represents the cyclist group.

Common mistake

Confusing independent with mutually exclusive. Two coin tosses are independent. Getting heads and tails on the same single toss is mutually exclusive: both cannot happen on that toss.

Exam technique

Check that the probabilities leaving each tree point add to one.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

A spinner has probability 0.2 of red. It is spun twice independently. Find the probability of exactly one red.

Question 2

Two fair six-sided dice are rolled. Find the probability of a total of 9.

Question 3

Of 30 students, 18 walk to school. Of those 18, 8 wear glasses. If a walker is selected, find the probability they wear glasses.

Watch an explanation

Cognito — AND / OR Rules

Cognito — Tree Diagrams

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

$0.2(0.8) + 0.8(0.2) = 0.32$.

Question 2

$\frac{4}{36} = \frac{1}{9}$.

Question 3

$\frac{8}{18} = \frac{4}{9}$.

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