Simplifying, expanding and dividing

Junior Cycle · Higher Level

Understand the idea

How to start

A term includes its sign. In $4x - 3 + 2x$, the terms are $4x$, -3 and $+2x$.

1. Simplify means write an equal expression with fewer unnecessary parts.
2. Expand means multiply out brackets. Every term inside must be multiplied.
3. Keep signs with terms when moving like terms beside one another.

Key terms & definitions

Like terms
Terms with the same variable part and powers.

Expand
Remove brackets by distributing multiplication.

Polynomial
A sum of terms with non-negative whole-number powers of the variables.

Key ideas

• Collect only like terms: their variable parts and powers must match.

• Distribute a factor to every term in a bracket. A minus sign before a bracket changes every sign inside.

• For two brackets, multiply every term in the first by every term in the second. Collect like terms afterwards.

• For a linear bracket times a quadratic, all products must be included, including the cubic term.

• To divide a polynomial by a linear factor, find terms whose product reconstructs the dividend. Check by multiplying quotient and divisor.

Quick reminder

• Collect only like terms.

• Multiply every term when expanding brackets.

• Check division by multiplying the quotient and divisor.

Worked examples

Example 01 · Collecting like terms

Simplify $9x + 12 + 9x - 10$. What is the constant term?

Blue follows the x terms. Purple follows the constant terms, including their signs.

Why this method?

Identify the terms, brackets or divisor first. Each written line should show how the expression changes without changing its value.

$9x+12+9x-10$	**Step 1** Start with the whole expression, keeping each sign with its term.
$9x + 9x+12 - 10$	**Step 2** Put the x terms together and the number terms together.
$18x+2$	**Step 3** Nine x plus nine x is eighteen x. Twelve minus ten is two.
The constant term is 2.	**Step 4** The question also asks for the term with no x.

Think about it: Before moving on: why can you add the x terms together but not add an x term to a number?

Example 02 · Expanding one bracket

Expand $6(x + 9)$. What is the constant term?

Amber follows the outside multiplier. It must multiply both terms inside the bracket.

Why this method?

Identify the terms, brackets or divisor first. Each written line should show how the expression changes without changing its value.

$6(x + 9)$	**Step 1** Six multiplies everything inside the bracket.
$6 \times x + 6 \times 9$	**Step 2** Multiply each term by 6, including the number 9.
$6x + 54$	**Step 3** Write the complete expanded expression.
The constant term is 54.	**Step 4** The number term has no variable.

Think about it: Before moving on: why does the number outside have to multiply the number inside the bracket as well as the x?

Example 03 · Start here: expand two simple brackets

Expand $(x + 2)(x + 3)$.

Blue follows the products made by x. Purple follows the products made by $+2$. Collect only the like terms.

Why this method?

Every term in one bracket must multiply every term in the other. Write the four products so none is missed.

$x(x + 3) = x^2 + 3x$	**Step 1** The first x multiplies every term in the second bracket.
$2(x + 3) = 2x + 6$	**Step 2** The $+2$ also multiplies every term in the second bracket.
$x^2 + 3x + 2x + 6$	**Step 3** Put all four products together before collecting terms.

$x^2 + 5x + 6$	**Step 4** Only $3x$ and $2x$ are like terms, so combine them.

Example 04 · Expanding a linear bracket times a quadratic bracket

Expand $(3x + 2)(2x^2 + 3x + 2)$.

Why this method?

Identify the terms, brackets or divisor first. Each written line should show how the expression changes without changing its value.

$3x \times (2x^2 + 3x + 2)$ and $2 \times (2x^2 + 3x + 2)$	**Step 1** Multiply each term in the first bracket by each term in the second
$6x^3 + 13x^2 + 12x + 4$	**Step 2** Combine terms with exactly the same variable part and power. Keep unlike terms separate.

Think about it: Before moving on: how many products make up the x^2 term, and what are they?

Example 05 · Expanding a linear bracket times a quadratic bracket — another example

Expand $(3x + 1)(3x^2 + 4x + 2)$.

Why this method?

Identify the terms, brackets or divisor first. Each written line should show how the expression changes without changing its value.

$3x \times (3x^2 + 4x + 2)$ and $1 \times (3x^2 + 4x + 2)$	**Step 1** Multiply each term in the first bracket by each term in the second
$9x^3 + 15x^2 + 10x + 2$	**Step 2** Combine terms with exactly the same variable part and power. Keep unlike terms separate.

Example 06 · Dividing a quadratic by a linear expression

Divide $x^2 + 9x + 20$ by $x + 5$.

Why this method?

Identify the terms, brackets or divisor first. Each written line should show how the expression changes without changing its value.

$x^2 + 9x + 20 = (x + 5)(x + 4)$	**Step 1** Factorise the quadratic: find two numbers that add to 9 and multiply to 20
$(x + 5)(x + 4) \div (x + 5) = x + 4$	**Step 2** Divide out the common factor representing the divisor. Check by multiplying the quotient by the divisor.

Think about it: Before moving on: why does the divisor cancel, and why is the sum of the constants not the answer?

Example 07 · Dividing a quadratic by a linear expression — another example

Divide $x^2 + 10x + 24$ by $x + 6$.

Why this method?

Identify the terms, brackets or divisor first. Each written line should show how the expression changes without changing its value.

$x^2 + 10x + 24 = (x + 6)(x + 4)$	**Step 1** Factorise the quadratic: find two numbers that add to 10 and multiply to 24
$(x + 6)(x + 4) \div (x + 6) = x + 4$	**Step 2** Divide out the common factor representing the divisor. Check by multiplying the quotient by the divisor.

Example 08 · Dividing a cubic by a linear expression

Divide $x^3 + 13x^2 + 54x + 72$ by $x + 3$.

Why this method?

Build the quotient one term at a time. Each time, divide the leading term, multiply the whole divisor, then subtract the whole product.

$\frac{x^3}{x} = x^2$	**Step 1** Divide the leading term of the cubic by x. This gives the first term of the quotient.

$(x+3)x^2 = x^3 + 3x^2$	**Step 2** Multiply the whole divisor by x^2, then subtract that whole product from the dividend.
$(13-3)x^2 + 54x + 72 = 10x^2 + 54x + 72$	**Step 3** The cubic terms cancel. Keep the remaining terms in descending powers.
$\frac{10x^2}{x} = 10x$	**Step 4** Divide the new leading term by x to get the next quotient term.
$(x+3)(10x) = 10x^2 + 30x$	**Step 5** Multiply the whole divisor by this new term.
$(54-30)x + 72 = 24x + 72$	**Step 6** Subtract that whole product. The x^2 terms cancel.
$\frac{24x}{x} = 24;$ $(x+3)(24) = 24x + 72$	**Step 7** Divide the leading term by x once more, then multiply the divisor.
$x^2 + 10x + 24$	**Step 8** Subtract. The remainder is zero, so collect the three quotient terms.
$(x+3)(x^2 + 10x + 24) = x^3 + 13x^2 + 54x + 72$	**Step 9** Check by multiplying the divisor by the quotient.

Think about it: Before moving on: why is the answer a quadratic, and why is the constant term the product of only two of the factors?

Example 09 · Dividing a cubic by a linear expression — another example

Divide $x^3 + 15x^2 + 66x + 80$ by $x + 2$.

Why this method?

Build the quotient one term at a time. Each time, divide the leading term, multiply the whole divisor, then subtract the whole product.

$\frac{x^3}{x} = x^2$	**Step 1** Divide the leading term of the cubic by x. This gives the first term of the quotient.

$(x+2)x^2 = x^3 + 2x^2$	**Step 2** Multiply the whole divisor by x^2, then subtract that whole product from the dividend.
$(15-2)x^2 + 66x + 80 = 13x^2 + 66x + 80$	**Step 3** The cubic terms cancel. Keep the remaining terms in descending powers.
$\frac{13x^2}{x} = 13x$	**Step 4** Divide the new leading term by x to get the next quotient term.
$(x+2)(13x) = 13x^2 + 26x$	**Step 5** Multiply the whole divisor by this new term.
$(66-26)x + 80 = 40x + 80$	**Step 6** Subtract that whole product. The x^2 terms cancel.
$\frac{40x}{x} = 40;$ $(x+2)(40) = 40x + 80$	**Step 7** Divide the leading term by x once more, then multiply the divisor.
$x^2 + 13x + 40$	**Step 8** Subtract. The remainder is zero, so collect the three quotient terms.
$(x+2)(x^2 + 13x + 40) = x^3 + 15x^2 + 66x + 80$	**Step 9** Check by multiplying the divisor by the quotient.

Common mistake

Combining terms with different letter parts. $3x + 2x^2$ cannot become $5x^3$. Only like terms combine: $3x + 2x = 5x$.

Exam technique

Set out long division in powers of x and include a zero coefficient for any missing power.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Simplify $2x + 9 + 4x - 3$. What is the constant term?

Question 2

Expand $9(x + 6)$. What is the constant term?

Question 3

Expand $(2x + 3)(4x^2 + 5x + 1)$.

A. $8x^3 + 22x^2 + 17x + 3$

B. $8x^3 + 22x^2 + 17x - 3$

C. $8x^3 + 12x^2 + 17x + 3$

D. $8x^3 + 22x^2 + 2x + 3$

Question 4

Divide $x^2 + 15x + 56$ by $x + 7$.

A. $x + 7$

B. $x + 15$

C. $x + 8$

D. $x + 56$

Question 5

Divide $x^3 + 10x^2 + 31x + 30$ by $x + 2$.

A. $x^2 + 10x + 15$

B. $x + 8$

C. $x^2 + 8x + 30$

D. $x^2 + 8x + 15$

Watch an explanation

Cognito — Collecting Like Terms

Cognito — Expanding Double Brackets

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

6

Question 2

54

Question 3

$8x^3 + 22x^2 + 17x + 3$

Question 4

$x + 8$

Question 5

$x^2 + 8x + 15$

Open the full chapter online