Factorising expressions and quadratics

Junior Cycle · Higher Level

Understand the idea

How to start

Factorise means write an expression as a multiplication. It reverses expanding brackets.

1. Look for a factor shared by every term before trying another method.
2. For a quadratic, match the actual terms to $ax^2 + bx + c$ and identify the signed numbers.
3. For the guide-number method, the pair must pass both tests: correct product and correct sum.

Key terms & definitions

Factor
An expression multiplied by another to make a product.

Quadratic
An expression whose highest power of its variable is 2.

Common factor
A factor present in every term.

Key ideas

• Factorising writes an expression as a product. Look for a common factor first.

• A difference of squares factors as $a^2 - b^2 = (a - b)(a + b)$.

• For a quadratic $ax^2 + bx + c$, the guide number is $a \times c$. Keep the signs of a, b and c.

• Find two numbers that multiply to the guide number and add to b. Split bx using these two numbers, factor each pair, then take out the shared bracket.

• Factorising is not solving. Only set each factor to zero when the product is equal to zero.

Quick reminder

• Check for a common factor first.

• Guide number: $a \times c$. Pair: product ac, sum b.

• Split the middle term, group, then factor the shared bracket.

Worked examples

Example 01 · Start here: taking out a common factor

Factorise $18 - 6y$.

Blue follows the first term, purple follows the second term, and amber follows the common factor. The words and equations explain every step too.

Why this method?

Find the largest factor shared by both terms. Divide each whole term by it to find what belongs inside the bracket. Expand at the end to check.

$18-6y$	**Step 1** The first term is 18. The second term is $-6y$, including its minus sign.
$\text{HCF} = 6$	**Step 2** The factors of 18 are $1, 2, 3, 6, 9, 18$. The positive factors of 6 are $1, 2, 3, 6$. The largest shared factor is 6.
$\frac{18}{6} = 3$	**Step 3** Divide the first term by 6. This gives the first term inside the bracket.
$\frac{-6y}{6} = -y$	**Step 4** Divide the second whole term by 6. The minus sign stays, giving $-y$.
$6(3-y)$	**Step 5** Put the common factor outside the bracket. Keep both results inside it.
$6(3-y) = 18-6y$	**Step 6** Expand to check : 6 multiplies every term inside the bracket, giving the original expression.

Example 02 · Guide number : x^2 has coefficient 1

Factorise $x^2 + 5x + 6$.

First identify a, b and c

x^2 $+5x$ $+6$ This question

ax^2 $+bx$ $+c$ Match the form

$a = 1$ — the number multiplying x^2.

$b = 5$ — the number multiplying x, including its sign.

$c = 6$ — the number on its own, including its sign.

x^2 means $1x^2$, so $a = 1$.

Amber marks the identical whole bracket in both groups. That whole bracket is the shared factor.

Guide number : $1 \times (6) = 6$**. Target sum** : 5**.**

Chosen pair : $(2) \times (3) = 6$; $2 + (3) = 5$.

Why this method?

Check for a common factor first. The guide number is $a \times c$. Find two signed numbers that multiply to $a \times c$ and add to b, then split the middle term and group.

$GN = 1 \times 6 = 6$	**Step 1** Multiply the coefficient of x^2 by the constant term.
$2 \times 3 = 6, \quad 2 + 3 = 5$	**Step 2** Choose the factor pair whose sum is the coefficient of x.
$x^2 + 2x + 3x + 6$	**Step 3** Split $5x$ into $2x + 3x$.
$x(x + 2) + 3(x + 2)$	**Step 4** Factor the first pair and the second pair. The same bracket appears twice.
$(x + 3)(x + 2)$	**Step 5** Take out the shared bracket $(x + 2)$.
$x^2 + 2x + 3x + 6 = x^2 + 5x + 6$	**Step 6** Expand to check.

Example 03 · Guide number: coefficient greater than 1

Factorise $3x^2 + 10x + 8$.

First identify a, b and c

$3x^2$ $+10x$ $+8$ This question

ax^2 $+bx$ $+c$ Match the form

$a = 3$ — the number multiplying x^2.

$b = 10$ — the number multiplying x, including its sign.

$c = 8$ — the number on its own, including its sign.

Amber marks the identical whole bracket in both groups. That whole bracket is the shared factor.

Guide number : $3 \times (8) = 24$**. Target sum** : 10**.**

Chosen pair : $(6) \times (4) = 24$; $6 + (4) = 10$.

Why this method?

Check for a common factor first. The guide number is $a \times c$. Find two signed numbers that multiply to $a \times c$ and add to b, then split the middle term and group.

$GN = 3 \times 8 = 24$	**Step 1** The guide number is 24. The required sum is 10.
$1 \times 24 = 24, \quad 1 + 24 = 25$	**Step 2** Try factor pairs of 24. This pair has the wrong sum.
$2 \times 12 = 24, \quad 2 + 12 = 14$	**Step 3** The product is correct, but the sum is still wrong.
$6 \times 4 = 24, \quad 6 + 4 = 10$	**Step 4** Choose 6 and 4 : both tests now work.
$3x^2 + 6x + 4x + 8$	**Step 5** Split $10x$ into $6x + 4x$.
$3x(x + 2) + 4(x + 2)$	**Step 6** Factor each pair. Both groups contain $(x + 2)$.
$(3x + 4)(x + 2)$	**Step 7** Factor out the shared bracket.

$3x^2 + 6x + 4x + 8 = 3x^2 + 10x + 8$	**Step 8** Expand to check.

Example 04 · A common factor first

Factorise $6x^2 - 24$.

Why this method?

Look for a common factor first. Then choose a factorisation pattern and check it by expanding.

$6(x^2 - 4)$	**Step 1** Take out the highest common factor
$6(x - 2)(x + 2)$	**Step 2** Use difference of squares
$6(x^2 - 4) = 6x^2 - 24$	**Step 3** Expand the brackets to check that every coefficient agrees with the original expression.

Example 05 · Factor by grouping

Factorise $ax + ay + bx + by$.

Why this method?

Look for a common factor first. Then choose a factorisation pattern and check it by expanding.

$a(x + y) + b(x + y)$	**Step 1** Group pairs
$(a + b)(x + y)$	**Step 2** Take out the shared bracket

Example 06 · Factorising $ax^2 + bx + c$ with a greater than 1

Factorise $2x^2 + 11x + 15$.

First identify a, b and c

$2x^2 \quad +11x \quad +15$ This question

$ax^2 \quad +bx \quad +c$ Match the form

$a = 2$ — the number multiplying x^2.

$b = 11$ — the number multiplying x, including its sign.

$c = 15$ — the number on its own, including its sign.

Amber marks the identical whole bracket in both groups. That whole bracket is the shared factor.

Why this method?

Look for a common factor first. Then choose a factorisation pattern and check it by expanding.

$5 \times 6 = 30$ and $5 + 6 = 11$.	**Step 1** Multiply the first and last coefficients : $2 \times 15 = 30$. We need two numbers with product 30 and sum 11.
$2x^2 + 5x + 6x + 15$	**Step 2** Split $11x$ into $5x + 6x$ without changing the expression.
$x(2x + 5) + 3(2x + 5)$	**Step 3** Group the first pair and last pair. Factor each pair separately.
$(x + 3)(2x + 5)$	**Step 4** Both parts contain the whole bracket $2x + 5$. Factor that shared bracket out.
$2x^2 + 5x + 6x + 15 = 2x^2 + 11x + 15$	**Step 5** Expand to check.

Think about it: Before moving on: why does the factor of a in one bracket have to be paired with a factor of c in the other?

Example 07 · Factorising $ax^2 + bx + c$ with a greater than 1 — another example

Factorise $3x^2 + 10x + 3$.

First identify a, b and c

$3x^2 \quad +10x \quad +3$ This question

$ax^2 \quad +bx \quad +c$ Match the form

$a = 3$ — the number multiplying x^2.

$b = 10$ — the number multiplying x, including its sign.

$c = 3$ — the number on its own, including its sign.

Amber marks the identical whole bracket in both groups. That whole bracket is the shared factor.

Why this method?

Look for a common factor first. Then choose a factorisation pattern and check it by expanding.

$9 \times 1 = 9$ and $9 + 1 = 10$.	**Step 1** The first and last coefficients multiply to 9. Choose numbers with product 9 and sum 10.
$3x^2 + 9x + x + 3$	**Step 2** Split the middle term into those two parts.
$3x(x + 3) + 1(x + 3)$	**Step 3** Factor the two groups to reveal the same bracket.
$(3x + 1)(x + 3)$	**Step 4** Factor out the shared bracket.
$3x^2 + 9x + x + 3 = 3x^2 + 10x + 3$	**Step 5** Check by expanding.

Example 08 · Factorising quadratics with signed coefficients

Factorise fully : $5x^2 - 19x + 12$.

First identify a, b and c

$5x^2$ $-19x$ $+12$ This question

ax^2 $+bx$ $+c$ Match the form

$a = 5$ — the number multiplying x^2.

$b = -19$ — the number multiplying x, including its sign.

$c = 12$ — the number on its own, including its sign.

Amber marks the identical whole bracket in both groups. That whole bracket is the shared factor.

Why this method?

Look for a common factor first. Then choose a factorisation pattern and check it by expanding.

$\text{GN} = 5 \times 12 = 60$	**Step 1** Multiply the coefficient of x^2 by the constant. The pair must also add to -19.
$(-15)(-4) = 60, \quad -15 - 4 = -19$	**Step 2** Two negatives give a positive product and the required negative sum.
$5x^2 - 15x - 4x + 12$	**Step 3** Replace $-19x$ by $-15x - 4x$.
$5x(x-3) - 4(x-3)$	**Step 4** Take $5x$ from the first pair and -4 from the second pair. Taking out -4 changes $+12$ into -3 inside the bracket.
$(5x-4)(x-3)$	**Step 5** Both groups have the same bracket, so factor that bracket out.
$5x^2 - 15x - 4x + 12 = 5x^2 - 19x + 12$	**Step 6** Expand and collect the middle terms to check.

Think about it: Which two products contribute to the middle coefficient, and how do their signs combine?

Example 09 · Factorising quadratics with signed coefficients — another example

Factorise fully : $2x^2 + x - 21$.

First identify a, b and c

$2x^2 \quad +x \quad -21$ This question

$ax^2 \quad +bx \quad +c$ Match the form

$a = 2$ — the number multiplying x^2.

$b = 1$ — the number multiplying x, including its sign.

$c = -21$ — the number on its own, including its sign.

x means $1x$, so $b = 1$.

Amber marks the identical whole bracket in both groups. That whole bracket is the shared factor.

Why this method?

Look for a common factor first. Then choose a factorisation pattern and check it by expanding.

$\text{GN} = 2 \times (-21) = -42$	**Step 1** Multiply the coefficient of x^2 by the constant. The middle term is $+1x$, so the sum must be $+1$.
$(-6)(7) = -42, \quad -6 + 7 = 1$	**Step 2** Opposite signs give the negative product. The larger positive number gives the required positive sum.
$2x^2 - 6x + 7x - 21$	**Step 3** Replace $+x$ by $-6x + 7x$.
$2x(x-3)+7(x-3)$	**Step 4** Take $2x$ from the first pair and 7 from the second pair.
$(2x + 7)(x - 3)$	**Step 5** Factor out the shared bracket $(x - 3)$.
$2x^2 - 6x + 7x - 21 = 2x^2 + x - 21$	**Step 6** Expand to check that the middle coefficient is 1.

Example 10 · Factorising a difference of two squares

Factorise $16x^2 - 25y^2$.

Why this method?

Look for a common factor first. Then choose a factorisation pattern and check it by expanding.

$16x^2 - 25y^2 = (4x)^2 - (5y)^2$	**Step 1** Write each term as a square : $a^2x^2 = (\,)^2$ and $b^2y^2 = (\,)^2$
$= (4x - 5y)(4x + 5y)$	**Step 2** Difference of two squares : $u^2 - v^2 = (u - v)(u + v)$

Think about it: Before moving on: why does the middle term cancel when you multiply $(u - v)(u + v)$?

Example 11 · Factorising a difference of two squares — another example

Factorise $25x^2 - 9y^2$.

Why this method?

Look for a common factor first. Then choose a factorisation pattern and check it by expanding.

$25x^2 - 9y^2 = (5x)^2 - (3y)^2$	**Step 1** Write each term as a square : $a^2x^2 = (\,)^2$ and $b^2y^2 = (\,)^2$
$= (5x - 3y)(5x + 3y)$	**Step 2** Difference of two squares : $u^2 - v^2 = (u - v)(u + v)$

Common mistake

Choosing a pair that has the correct product but the wrong sum. For $x^2 + 5x + 6$, 1 and 6 multiply to 6 but add to 7. Use 2 and 3 : $(x + 2)(x + 3)$.

Exam technique

Expand your factors to check all three coefficients.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Factorise $5x^2 + 11x + 2$.

A. $(x + 1)(x + 2)$

B. $(5x + 1)(x + 2)$

C. $(5x - 1)(x - 2)$

D. $(5x + 2)(x + 1)$

Question 2

Factorise fully : $4x^2 - 15x - 4$.

A. $(4x + 1)(x - 4)$

B. $(4x - 1)(x + 4)$

C. $(4x - 1)(x - 4)$

D. $(x + 1)(x - 4)$

Question 3

Factorise $49x^2 - 9y^2$.

A. $(49x - 9y)(49x + 9y)$

B. $(7x + 3y)^2$

C. $(7x - 3y)^2$

D. $(7x - 3y)(7x + 3y)$

Watch an explanation

Whiteboard Math — Why factorising works

Cognito — Factorising Single Brackets

Cognito — Factorising Quadratic Equations (a = 1)

Cognito — Factorising Quadratic Equations (a > 1)

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

$(5x + 1)(x + 2)$

Question 2

$(4x + 1)(x - 4)$

Question 3

$(7x - 3y)(7x + 3y)$

Open the full chapter online