# Factors, multiples and primes

Junior Cycle · Higher Level

## Understand the idea

### How to start

A factor goes into a number exactly. A multiple is in its times table.

1. HCF asks for the biggest factor shared by the numbers.
2. LCM asks for the first positive number in both times tables.
3. Begin with lists. Use prime factors when the numbers get larger.

### Key terms & definitions

**HCF**
Highest common factor: the largest positive factor shared by the numbers.

**LCM**
Lowest common multiple: the smallest positive multiple shared by the numbers.

**Composite**
A whole number greater than 1 that is not prime.

### Key ideas

• A factor divides a number exactly. A multiple is made by multiplying by an integer.

• A prime is a whole number greater than 1 with exactly two positive factors. The number 1 is not prime.

• Write prime factors in increasing order; use powers for repeated factors.

• For HCF, take the shared prime factors with their smallest powers. For LCM, take all required primes with their largest powers.

# Quick reminder

• Prime-factor method: use shared primes with their smaller powers for the HCF.

• LCM: all required prime factors with the largest powers.

• Check your prime factorisation by multiplying back.

# Worked examples

### Example 01 · Highest common factor and lowest common multiple

Find the highest common factor (HCF) of 12 and 18.

#### Why this method?

HCF means highest common factor. Factor lists make the shared factors visible before we use prime powers.

| | |
|---|---|
| Factors of $12$ : $1, 2, 3, 4, 6, 12$. | **Step 1**<br>A factor divides $12$ with no remainder. List its factor pairs. |
| Factors of $18$ : $1, 2, 3, 6, 9, 18$. | **Step 2**<br>List the factors of $18$ in the same way. |
| Common factors : $1, 2, 3, 6$. | **Step 3**<br>Common means appearing in both lists. |
| $\text{HCF} = 6$ | **Step 4**<br>The highest common factor is the largest number in the shared list. |

Think about it: Is the HCF always less than or equal to the LCM? Explain what happens when the two original numbers are equal.

### Example 02 · Use prime powers

Find the HCF and LCM of 72 and 90.

#### Why this method?

Find the prime factors or factor lists first. Then choose the shared factors or required multiples named in the question.

| | |
|---|---|
| $72 = 2^3 \times 3^2$<br>and $90 = 2 \times 3^2 \times 5$. | **Step 1**<br>Factor both numbers |

| | |
|---|---|
| $2 \times 3^2 = 18$. | **Step 2**<br>HCF: use shared minimum powers |
| $2^3 \times 3^2 \times 5 = 360$. | **Step 3**<br>LCM: use maximum powers |
| $18 \times 360 = 72 \times 90 = 6480$. | **Step 4**<br>Multiply the factors back together to check that they make the original number. |

**Example 03 · Prime numbers and prime factors**

Write $60$ as a product of prime factors. How many prime factors are there, counting repeats?

**Why this method?**

Find the prime factors or factor lists first. Then choose the shared factors or required multiples named in the question.

| | |
|---|---|
| $60 = 2 \times 2 \times 3 \times 5$ | **Step 1**<br>Divide by the smallest prime that works, repeatedly |
| 4 prime factors | **Step 2**<br>Count every factor in the product, including repeats |

Think about it: Before moving on: is $1a$ prime number? Why?

# Common mistake

Choosing a common factor without checking whether it is the highest one. For $12$ and $18$, $3$ is a common factor, but the HCF is $6$. The LCM is $36$, *a* multiple rather than a factor.

# Exam technique

A prime factor tree must end in primes; check by multiplying them back.

# Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

**Question 1**

Find the HCF of $24$ and $36$.

**Question** 2

How many prime numbers are there from 60 to 80, inclusive?

# Watch an explanation

Corbettmaths — Number: product of primes (LCM/HCF)

Cognito — Prime & Composite Numbers

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

# Self-check answers

### **Question** 1

12

### **Question** 2

5

Open the full chapter online