Changing the subject of a formula

Junior Cycle · Higher Level

Understand the idea

How to start

The subject is the letter on its own. Making t the subject means finishing with $t = \ldots$.

1. Circle the letter you need to isolate.
2. Undo operations in reverse order, doing the same to both sides.
3. Keep a whole sum in brackets or above the same fraction bar when dividing it.

Key terms & definitions

Subject
The variable written alone on one side of a formula.

Rearrange
Write an equivalent equation with a different structure.

Restriction
A condition needed for a division or other operation to be valid.

Key ideas

• The subject is the variable isolated on one side of a formula.

• Use inverse operations on both sides. Keep letters as quantities rather than trying to guess values.

• Clear denominators by multiplying every term. Use brackets when dividing a whole sum.

• If the required variable occurs more than once, collect those terms and factor it out.

• Check restrictions before dividing by an expression that could be zero.

Quick reminder

• Identify the variable that must be the subject.

• Undo addition/subtraction, then multiplication/division as needed.

• Keep a whole numerator together with a fraction bar or brackets.

Worked examples

Example 01 · Start here: undo two operations

Make t the subject of $v = u + at$. Assume $a \neq 0$.

Why this method?

Work backwards through the operations around t: undo adding u, then undo multiplying by a.

$v - u = at$	**Step 1** Our goal is t on its own. Subtract u from both sides to remove the added term.
$\frac{v-u}{a} = t$	**Step 2** The remaining t is multiplied by a. Divide the whole left-hand side and the right-hand side by a.
$t = \frac{v-u}{a}$	**Step 3** Write the subject on the left if preferred. The numerator must stay together.

Example 02 · A variable on both sides

Make x the subject of $y = \frac{x+2}{x-1}$.

Why this method?

Identify the required subject. Undo the operations around it in reverse order, doing the same to both sides.

$x \neq 1$	**Step 1** The original denominator $x - 1$ cannot be zero.
$y(x-1) = x + 2$	**Step 2** Multiply both sides by $x - 1$ to remove the denominator.

$yx - y = x + 2$	**Step 3** Expand the bracket on the left.
$yx - x = y + 2$	**Step 4** Subtract x from both sides, then add y to both sides. The x terms are now together.
$x(y - 1) = y + 2$	**Step 5** Factor x from both terms on the left.
$x = \frac{y+2}{y-1}, \quad y \neq 1$	**Step 6** Divide both sides by $y - 1$. It cannot be zero; the original relationship never gives $y = 1$.

Example 03 · Changing the subject of $y = a +$ bx

The formula is $y = a +$ bx. Find x when $y = 48$, $a = 4$ and $b = 4$.

Why this method?

Identify the required subject. Undo the operations around it in reverse order, doing the same to both sides.

$y - a =$ bx, so $48 - 4 = 4x$	**Step 1** Subtract a from both sides to remove that term while keeping the equation balanced.
$x = (48 - 4) \div 4 = 11$	**Step 2** Divide the whole of $(y - a)$ by b

Think about it: Before moving on: why must the whole of $y - a$ be divided by b, and not just the y?

Example 04 · Changing the subject of $A = h(a + b) \div 2$

The formula is $A = h(a + b) \div 2$. Find h when $A = 66$, $a = 5$ and $b = 6$.

Why this method?

Identify the required subject. Undo the operations around it in reverse order, doing the same to both sides.

$2A = h(a + b),$ so $2 \times 66 = h(5 + 6)$	**Step 1** Multiply both sides by 2 to clear the division
$h = 2A \div (a + b) = 132 \div 11 = 12$	**Step 2** Divide every term on both sides by $(a + b)$ so that the coefficient of the variable becomes 1.

Think about it: Before moving on: why is the bracket divided as a whole, and what happens if only one of its terms is divided?

Example 05 · Changing the subject of a formula

$y = a + bx \rightarrow x = ?$

Why this method?

Identify the required subject. Undo the operations around it in reverse order, doing the same to both sides.

$y = a + bx$	**Step 1** Write the formula
$y - a = bx$	**Step 2** Subtract a from both sides to remove that term while keeping the equation balanced.
$(y - a) \div b = x$	**Step 3** Divide every term on both sides by b so that the coefficient of the variable becomes 1.
$x = \frac{(y - a)}{b}$	**Step 4** Write with x as the subject

Common mistake

Dividing only one part of a sum when changing the subject. From $y = 3x + 4$, subtract 4 first, then divide the whole difference: $x = \frac{y-4}{3}$, not $y - \frac{4}{3}$.

Exam technique

Check the rearranged formula using a simple numerical example.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

The formula is $y = a + bx$. Find x when $y = 31$, $a = 7$ and $b = 4$.

Question 2

The formula is $A = h(a + b) \div 2$. Find h when $A = 30$, $a = 4$ and $b = 8$.

Question 3

Make x the subject of $y = c + mx$.

A.
$$x = \frac{(y - c)}{m}$$

B.
$$x = \frac{m}{(y - c)}$$

C.
$$x = \frac{(y + c)}{m}$$

D. $x = y - \frac{c}{m}$

Watch an explanation

Cognito — Rearranging Formulas (Subject Appears Once)

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

6

Question 2

5

Question 3

$$x = \frac{(y - c)}{m}$$

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