Fractions: all four operations

Junior Cycle · Higher Level

Understand the idea

How to start

The numerator counts parts. The denominator tells you how many equal parts make one whole.

1. For adding or subtracting, make the denominators match first.
2. For multiplying, multiply the tops and multiply the bottoms.
3. For dividing, turn the second fraction upside down, then multiply.

Key terms & definitions

Numerator
The top number: the number of equal parts.

Denominator
The bottom number: the number of equal parts in a whole; it cannot be zero.

Improper fraction
A fraction whose numerator is at least as large as its positive denominator.

Key ideas

• Equivalent fractions name the same number. Multiply or divide numerator and denominator by the same non-zero number.

• Simplify using a common factor. For addition or subtraction, use a common denominator first.

• To multiply, multiply numerators and multiply denominators. Cancel common factors before multiplying if helpful.

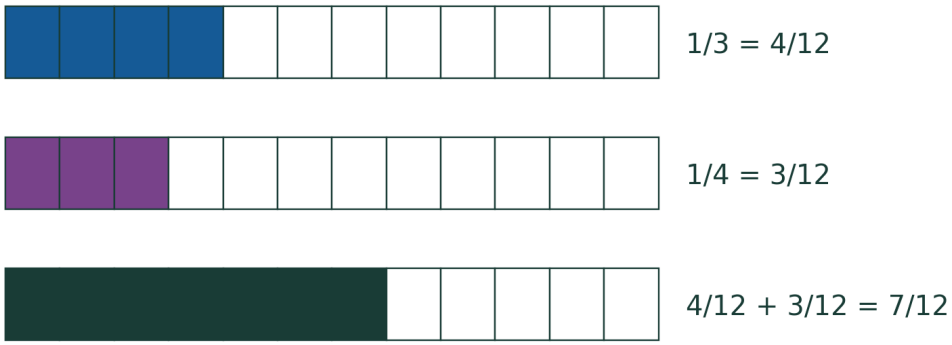
• To divide by a non-zero fraction, multiply by its reciprocal. Convert mixed numbers to improper fractions first.

• A fraction bar groups its whole numerator and denominator, like brackets.



Equal-sized wholes: three fifths is larger than three eighths because each fifth is a larger piece.

Add fractions: count the same-sized parts



Each bar is the same whole, divided into twelve equal parts. Four twelfths plus three twelfths gives seven twelfths.

Quick reminder

- Add/subtract: use the same denominator first.
- Multiply: top × top, bottom × bottom.
- Divide: multiply by the reciprocal.

Worked examples

Example 01 · Fractions as numbers

Which is greater : $\frac{3}{5}$ or $\frac{3}{8}$?

Why this method?

Identify the operation first. For addition, use equal-sized parts; for multiplication or division, change mixed numbers to improper fractions before calculating.

3 pieces each	**Step 1** Both fractions have 3 pieces
$\frac{1}{5} > \frac{1}{8}$	**Step 2** Compare the size of one piece: a fifth is bigger than an eighth
$\frac{3}{5} > \frac{3}{8}$	**Step 3** So three bigger pieces are more than three smaller pieces

Think about it: Before moving on: why does a bigger bottom number make smaller pieces?

Example 02 · Equivalent fractions

$\frac{2}{3} = \frac{?}{12}$

Why this method?

Identify the operation first. For addition, use equal-sized parts; for multiplication or division, change mixed numbers to improper fractions before calculating.

$3 \times 4 = 12$	**Step 1** What times 3 gives 12?
$2 \times 4 = 8$	**Step 2** Do the same to the top
$\frac{2}{3} = \frac{8}{12}$	**Step 3** So the missing top number is 8

Think about it: Before moving on: why is multiplying top and bottom by 4 allowed, but adding 4 to each is not?

Example 03 · Adding and subtracting fractions

$\frac{1}{3} + \frac{1}{4}$

Blue follows the first fraction as it changes into twelfths. Purple follows the second. The denominator stays 12 when they are added.

Why this method?

Identify the operation first. For addition, use equal-sized parts; for multiplication or division, change mixed numbers to improper fractions before calculating.

common denominator $= 12$	**Step 1** The pieces must be the same size before we can add them. We need a number that both 3 and 4 divide into. The smallest one is 12, so use twelfths.
$\frac{1}{3} = \frac{1 \times 4}{3 \times 4} = \frac{4}{12}$	**Step 2** Change $\frac{1}{3}$ into twelfths. The bottom 3 is multiplied by 4, so the top must also be multiplied by 4.
$\frac{1}{4} = \frac{1 \times 3}{4 \times 3} = \frac{3}{12}$	**Step 3** Change $\frac{1}{4}$ into twelfths. The bottom 4 is multiplied by 3, so the top must also be multiplied by 3.
$\frac{4}{12} + \frac{3}{12} = \frac{7}{12}$	**Step 4** Now both fractions are made of twelfths, so add the top numbers and keep the denominator 12.
$\frac{7}{12} < 1$	**Step 5** Check the size of the answer

Think about it: Why must you multiply the numerator by the same number whenever you multiply the denominator?

Example 04 · Multiply mixed numbers

Calculate $1\frac{1}{2} \times 2\frac{2}{3}$.

Why this method?

Identify the operation first. For addition, use equal-sized parts; for multiplication or division, change mixed numbers to improper fractions before calculating.

$\frac{3}{2} \times \frac{8}{3}$	**Step 1** Convert each mixed number to an improper fraction first. Then multiply the numerators and multiply the denominators.

$\frac{1 \times 8}{2 \times 1}$	**Step 2** Divide a numerator and a denominator by the same common factor. This keeps the value unchanged and makes the multiplication easier.
$\frac{8}{2} = 4$	**Step 3** Multiply and simplify

Example 05 · Divide fractions

Calculate $\frac{5}{6} \div \frac{10}{9}$.

Why this method?

Identify the operation first. For addition, use equal-sized parts; for multiplication or division, change mixed numbers to improper fractions before calculating.

$\frac{5}{6} \div \frac{10}{9} = \frac{5}{6} \times \frac{9}{10}$	**Step 1** Dividing by a fraction is multiplying by its reciprocal. Only the second fraction is turned over.
$\frac{5 \times 9}{6 \times 10} = \frac{45}{60}$	**Step 2** Multiply the numerators and multiply the denominators.
$\frac{45 \div 15}{60 \div 15} = \frac{3}{4}$	**Step 3** Divide top and bottom by the same common factor, 15.

Example 06 · Subtract a mixed number

Calculate $3\frac{1}{4} - 1\frac{5}{6}$.

Why this method?

Identify the operation first. For addition, use equal-sized parts; for multiplication or division, change mixed numbers to improper fractions before calculating.

$\frac{13}{4} - \frac{11}{6}$	**Step 1** Write each mixed number as one improper fraction so the subtraction rule can be applied.
$\frac{39}{12} - \frac{22}{12} = \frac{17}{12}$	**Step 2** Use denominator 12

$1\frac{5}{12}$	**Step 3** Write as a mixed number

Common mistake

Adding the denominators as well as the numerators. $\frac{1}{2}+\frac{1}{3}$ is not $\frac{2}{5}$. Use a common denominator: $\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$.

Exam technique

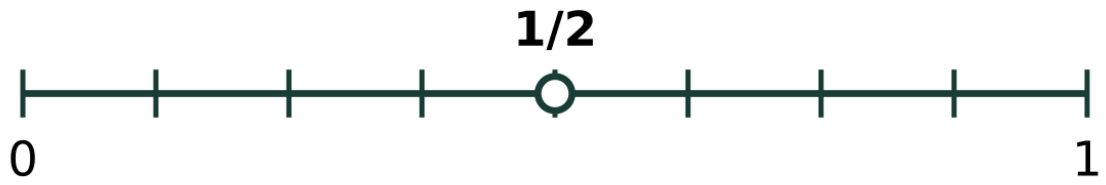
Estimate first: multiplying a positive number by a positive proper fraction makes it smaller.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Which fraction is greatest : $\frac{1}{2}, \frac{3}{4}$ or $\frac{5}{8}$? Give your answer as a fraction.



Number line from 0 to 1 split into eighths, with one half marked.

Question 2

$\frac{?}{15}=\frac{4}{5}$. Find the missing top number.

Question 3

$\frac{3}{4}-\frac{1}{8}=\frac{?}{8}$. Find the missing top number.

Watch an explanation

Cognito — Adding & Subtracting Fractions

Cognito — Multiplying & Dividing Fractions

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

$\frac{3}{4}$

Question 2

12

Question 3

5

Open the full chapter online