Functions and graphs

Junior Cycle · Higher Level

Understand the idea

How to start

$f(3)$ means the output of the rule f when the input is 3. It does not mean f multiplied by 3.

1. Decide whether you are finding an input, an output or a point on a graph.

2. For an output, replace x by the input and calculate.

3. On a graph, read the horizontal coordinate first, then the vertical coordinate.

Key terms & definitions

Domain
The set of allowed inputs.

Range
The set of outputs actually produced.

Intercept
A point at which a graph meets an axis.

Turning point
A point at which a curve changes from increasing to decreasing, or vice versa.

Key ideas

• A function assigns exactly one output to each input in its domain. $f(x)$ means the output for input x.

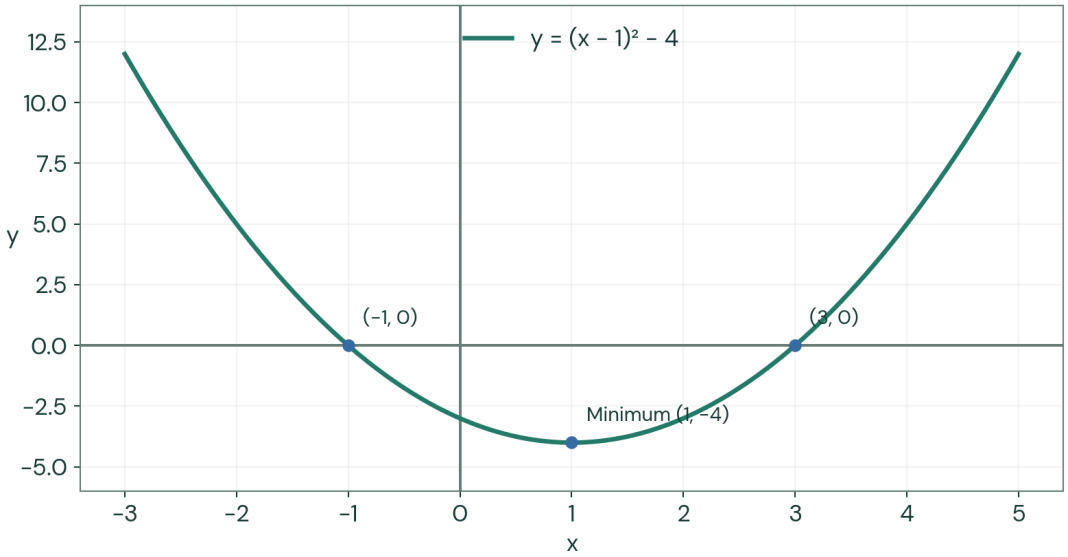
• The domain contains allowed inputs; the codomain is the stated set of possible outputs, and the range contains outputs actually reached.

• Make a table of values, label axes and choose consistent scales before plotting.

• A root is an x-value where the graph crosses or touches the x-axis. The y-intercept occurs when x is zero.

• An intersection solves $f(x) = g(x)$. A maximum or minimum describes an extreme output within the given domain.

• For contextual graphs, interpret coordinates with units. A model may only make sense for non-negative or whole-number inputs.



The quadratic has roots -1 and 3 and a minimum at $(1, -4)$.

Quick reminder

• Input x first, output y second.

• Roots: where $y = 0$. Y-intercept: where $x = 0$.

• An intersection has the same coordinates on both graphs.

Worked examples

Example 01 · Evaluating a function

The function f is given by $f(x) = 3x + 4$. What is $f(3)$?

Purple follows the input 3 into the rule, then the multiplication result 9. The coefficient 3 stays in place.

Why this method?

Decide whether the input or the output is known. For graphs, label the axes and read the relevant coordinate in the right order.

$f(x) = 3x + 4$	**Step 1** The rule says multiply the input by 3, then add 4.
$f(3) = 3(3)+4$	**Step 2** Replace x by the input 3. The 3 outside the brackets is the coefficient.

$f(3) = 9+4$	**Step 3** Multiply before adding.
$f(3) = 13$	**Step 4** Thirteen is the output when the input is 3.

Think about it: Before moving on: what does $f(3)$ mean, and how is it different from $3 \times f$?

Example 02 · Finding the input for a given output

The function f is given by $f(x) = 8x + 7$. For what value of x is $f(x) = 87$?

Why this method?

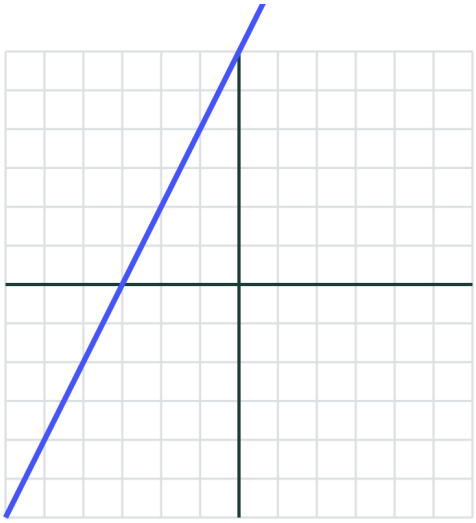
Decide whether the input or the output is known. For graphs, label the axes and read the relevant coordinate in the right order.

$8x + 7 = 87$	**Step 1** Write the equation
$8x = 87 - 7 = 80$	**Step 2** Take the constant away from both sides
$x = 80 \div 8 = 10$	**Step 3** Divide every term on both sides by the coefficient so that the coefficient of the variable becomes 1.

Think about it: Before moving on: how can you check your answer for x by evaluating the function?

Example 03 · Reading the y-intercept from a graph

Graph 1 : The straight line in the graph crosses the y-axis. What is the y-value where it crosses?



Straight line rising from left to right on axes marked from -6 to 6 on each side. The line is drawn across the whole grid.

Why this method?

Decide whether the input or the output is known. For graphs, label the axes and read the relevant coordinate in the right order.

The line crosses the y-axis at $y = 6$

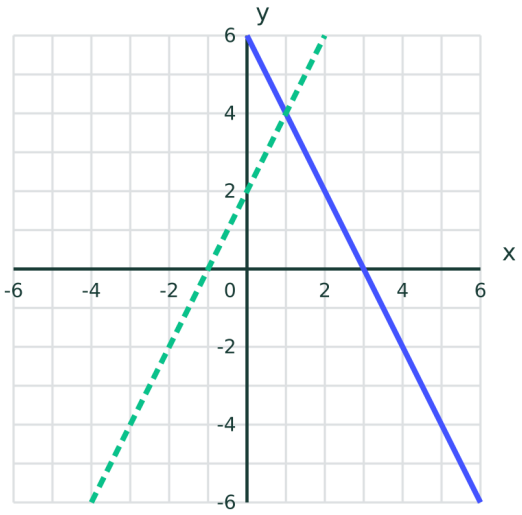
Step 1

The y-intercept is where the line crosses the y-axis, at $x = 0$

Think about it: Before moving on: which point on the graph gives the y-intercept, and why is it the only one you need?

Example 04 · Reading where two graphs cross

Graph 1 : The two straight lines in the graph cross at one point. What is the x-value of the point where they cross?



f: solid blue g: dashed green

Two straight lines drawn on axes marked from -6 to 6 on each side. The lines cross at one point.

Why this method?

Decide whether the input or the output is known. For graphs, label the axes and read the relevant coordinate in the right order.

The lines cross at $x = 1$

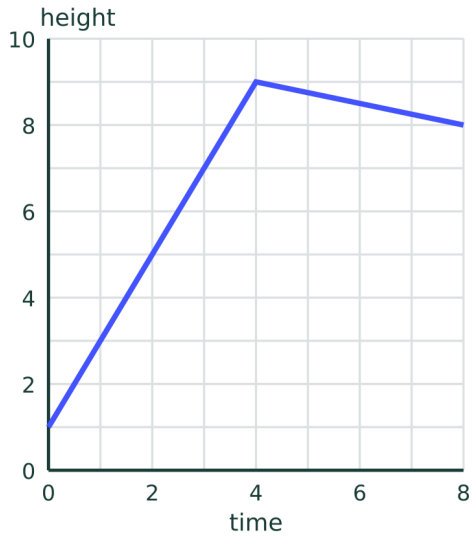
Step 1

The lines cross where their y-values are equal. Read the x-value of that point from the horizontal axis

Think about it: Before moving on: why is the x-value where the lines cross the solution, and not the y-value?

Example 05 · Greatest height and time on a graph

Graph 1 : What is the greatest height shown on the graph?



Graph of height against time from 0 to 8. The line rises to a high point and then falls.

Why this method?

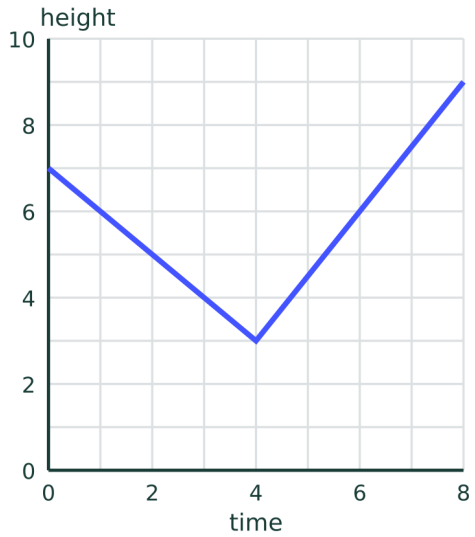
Decide whether the input or the output is known. For graphs, label the axes and read the relevant coordinate in the right order.

It is at (4, 9)	**Step 1** Find the highest point on the graph (the turning point)
The greatest height is 9	**Step 2** Read the height from the vertical axis

Think about it: Before moving on: why do we read the height from the vertical axis, and not the time, when asked for the greatest height?

Example 06 · Least height and time on a graph

Graph 1 : What is the least height shown on the graph?



Graph of height against time from 0 to 8. The line falls to a low point and then rises.

Why this method?

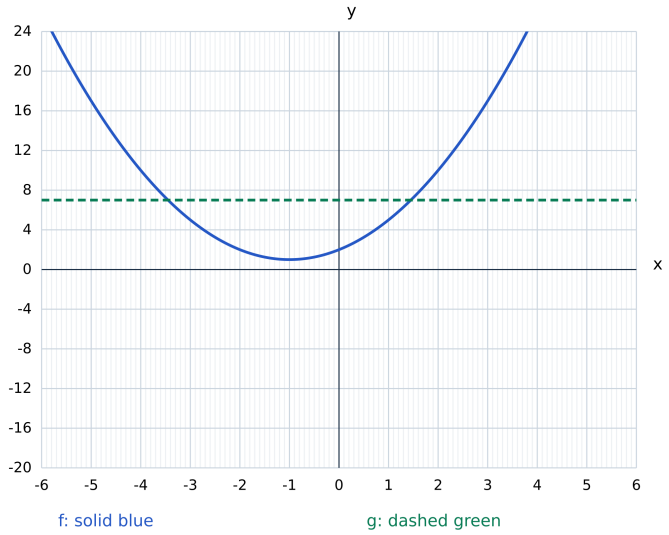
Decide whether the input or the output is known. For graphs, label the axes and read the relevant coordinate in the right order.

It is at (4, 3)	**Step 1** Find the lowest point on the graph (the turning point)
The least height is 3	**Step 2** Read the height from the vertical axis

Think about it: Before moving on: how can you tell the lowest point from the start and finish of the graph?

Example 07 · Approximate equation crossings

The graph shows $f(x) = (1)(x - (-1))^2 + (0)x + (1)$ and $g(x) = (0)(x - (-1))^2 + (0)x + (7)$. Use the graph to solve $f(x) = g(x)$, giving both x-values to one decimal place.



Graph of the two formulas in the question, x from -6 to 6 in tenths; y from -20 to 24. f is solid blue; g is dashed green.

Why this method?

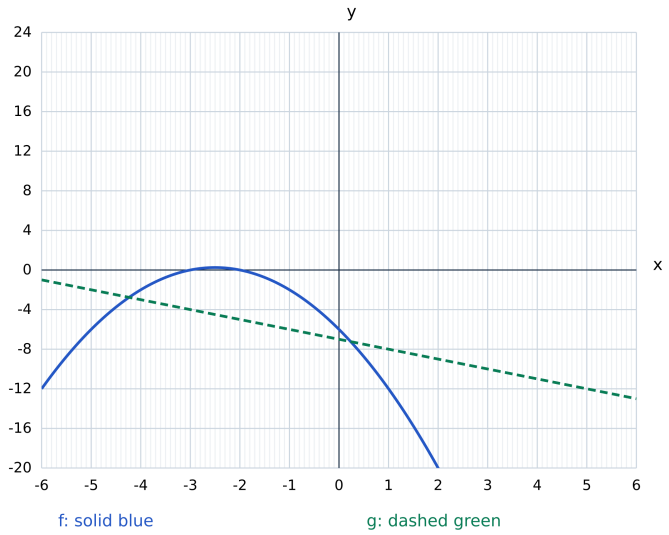
Decide whether the input or the output is known. For graphs, label the axes and read the relevant coordinate in the right order.

Crossings: $x \approx -3.4, 1.4$.	**Step 1** Locate both crossings and read the x-axis
$x \approx -3.4$ or $x \approx 1.4$	**Step 2** State both approximate solutions

Think about it: Why must you check all crossings rather than stopping after the first?

Example 08 · Approximate equation crossings — another example

The graph shows $f(x) = (-1)(x-(-2))^2 + (-1)x + (-2)$ and $g(x) = (0)(x-(-2))^2 + (-1)x + (-7)$. Use the graph to solve $f(x) = g(x)$, giving both x-values to one decimal place.



Graph of the two formulas in the question, x from -6 to 6 in tenths; y from -20 to 24. f is solid blue; g is dashed green.

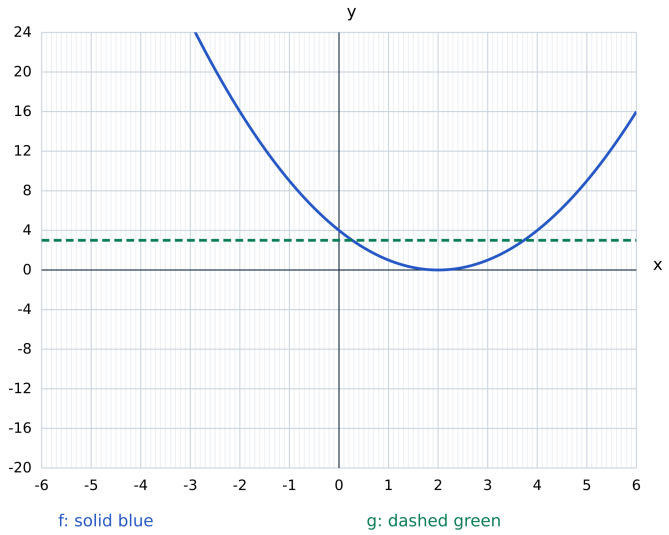
Why this method?

Decide whether the input or the output is known. For graphs, label the axes and read the relevant coordinate in the right order.

Crossings: $x \approx -4.2, 0.2$.	**Step 1** Locate both crossings and read the x-axis
$x \approx -4.2$ or $x \approx 0.2$	**Step 2** State both approximate solutions

Example 09 · Graphical comparisons of lines and curves

The graph shows $f(x) = (1)(x - (2))^2 + (0)x + (0)$ and $g(x) = (0)(x - (2))^2 + (0)x + (3)$. Use the graph to solve $f(x) < g(x)$ for all real x. Report boundary x-values to one decimal place; the formulas continue beyond the window.



Graph of the two formulas in the question, x from -6 to 6 in tenths; y from -20 to 24. f is solid blue; g is dashed green.

Why this method?

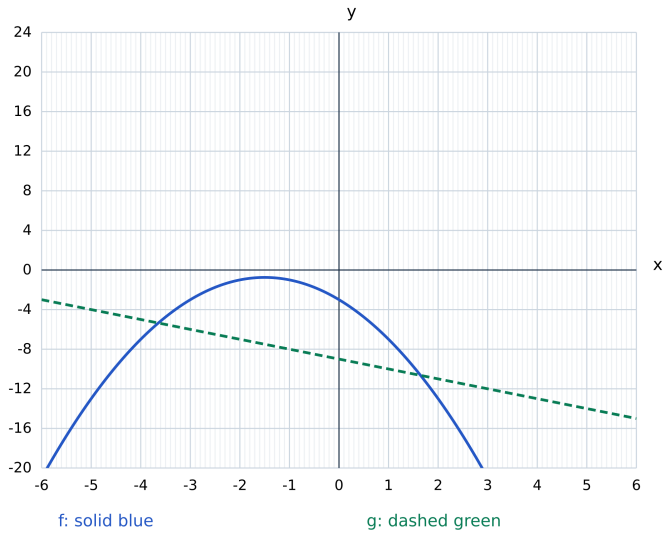
Decide whether the input or the output is known. For graphs, label the axes and read the relevant coordinate in the right order.

Crossings: x ≈ 0.3, 3.7.	**Step 1** Locate the two crossings
The requested relation holds between the crossings.	**Step 2** Compare curve heights between and outside the crossings
Between x ≈ 0.3 and x ≈ 3.7; endpoints excluded.	**Step 3** Treat boundary equality according to the inequality sign

Think about it: Why must you check all crossings rather than stopping after the first?

Example 10 · Graphical comparisons of lines and curves — another example

The graph shows $f(x) = (-1)(x - (-1))^2 + (-1)x + (-2)$ and $g(x) = (0)(x - (-1))^2 + (-1)x + (-9)$. Use the graph to solve $f(x) < g(x)$ for all real x. Report boundary x-values to one decimal place; the formulas continue beyond the window.



Graph of the two formulas in the question, x from -6 to 6 in tenths; y from -20 to 24. f is solid blue; g is dashed green.

Why this method?

Decide whether the input or the output is known. For graphs, label the axes and read the relevant coordinate in the right order.

Crossings: x ≈ $-3.6, 1.6$.	**Step 1** Locate the two crossings
The requested relation holds outside the crossings.	**Step 2** Compare curve heights between and outside the crossings
Outside x ≈ -3.6 and x ≈ 1.6; endpoints excluded.	**Step 3** Treat boundary equality according to the inequality sign

Common mistake

Swapping input and output coordinates. The point $(2, 5)$ means input 2 gives output 5. It does not mean that input 5 gives output 2.

Exam technique

Read the scale before estimating a crossing and label a graphical estimate as approximate.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

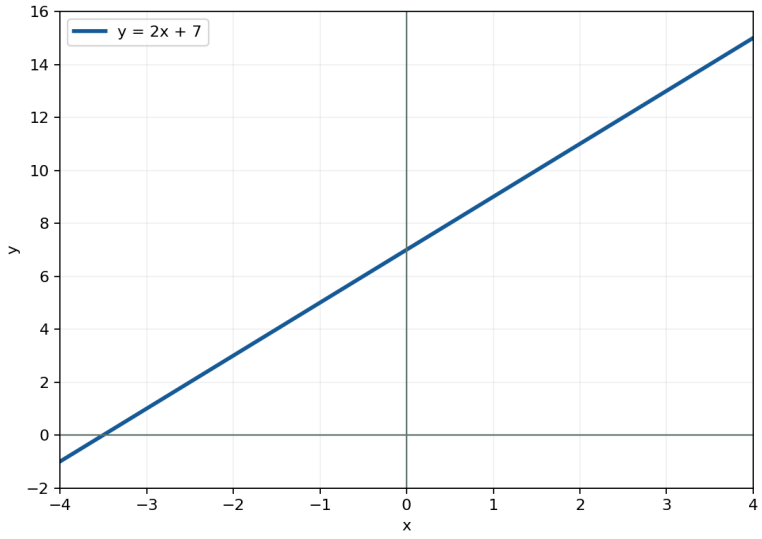
The function f is given by $f(x) = 6x + 11$. What is $f(7)$?

Question 2

The function f is given by $f(x) = 5x + 3$. For what value of x is $f(x) = 38$?

Question 3

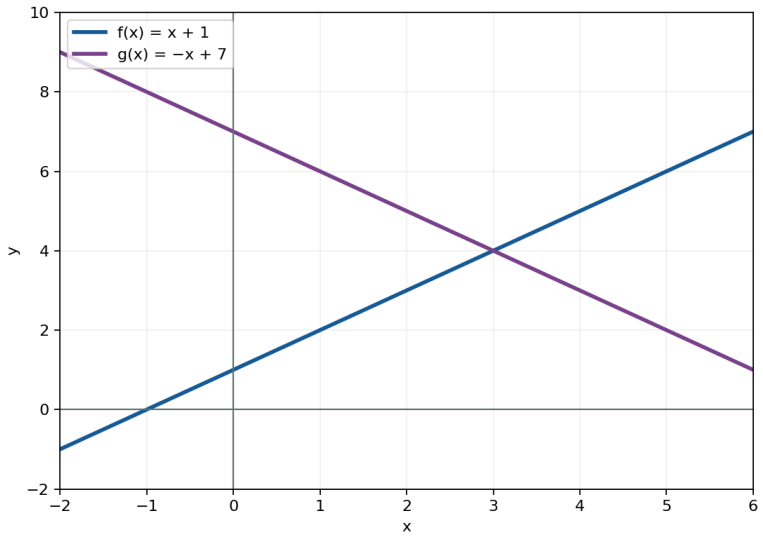
The graph shows $y = 2x + 7$. Read the y-intercept.



Graph of $y = 2x + 7$ with numbered x and y axes. The y-axis extends above 7.

Question 4

Use the graph of $f(x) = x + 1$ and $g(x) = -x + 7$. What is the x-coordinate where the lines meet?



Two labelled lines $f(x) = x + 1$ and $g(x) = -x + 7$ on numbered axes.

A. 2

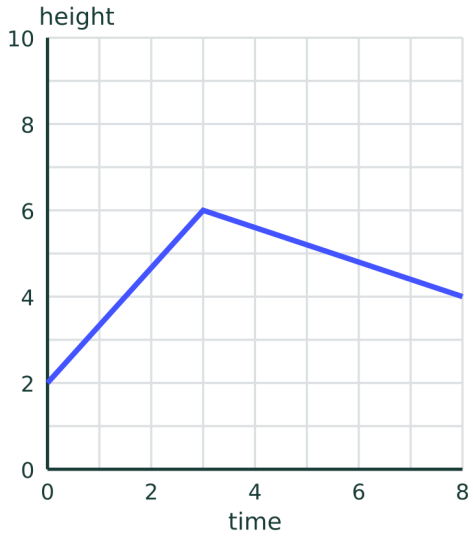
B. 3

C. -2

D. 4

Question 5

What is the greatest height shown on the graph?



Graph of height against time from 0 to 8. The line rises to a high point and then falls.

A. 4

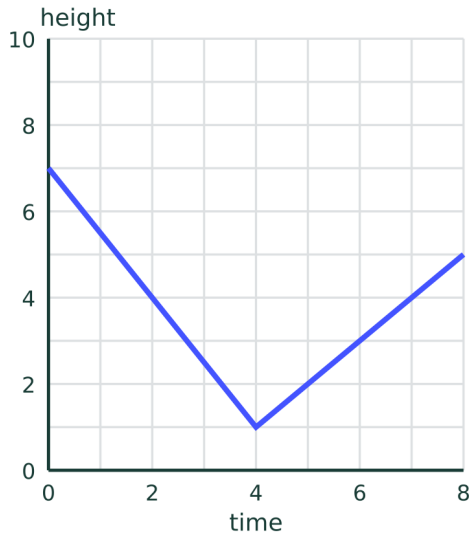
B. 2

C. 6

D. 3

Question 6

At what time is the least height reached?



Graph of height against time from 0 to 8. The line falls to a low point and then rises.

A. 1

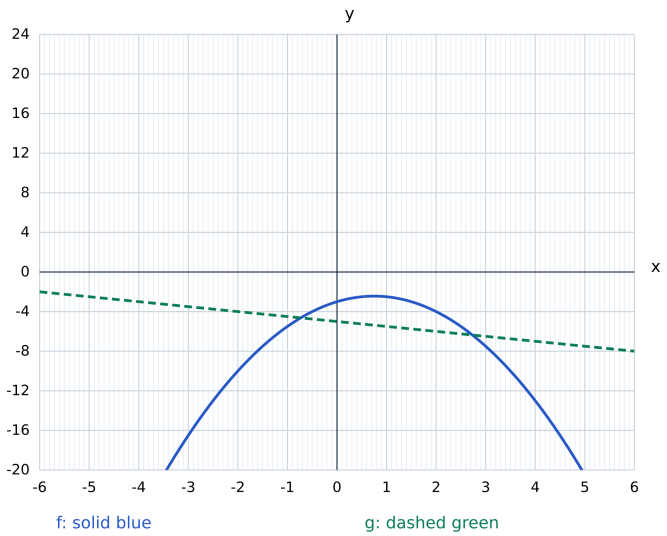
B. 5

C. 4

D. 7

Question 7

The graph shows $f(x) = (-1)(x-(1))^2 + (-0.5)x + (-2)$ and $g(x) = (0)(x-(1))^2 + (-0.5)x + (-5)$. Use the graph to solve $f(x) = g(x)$, giving both x-values to one decimal place.



Graph of the two formulas in the question, x from -6 to 6 in tenths; y from -20 to 24. f is solid blue; g is dashed green.

A. $x \approx -0.7$ or $x \approx 2.7$

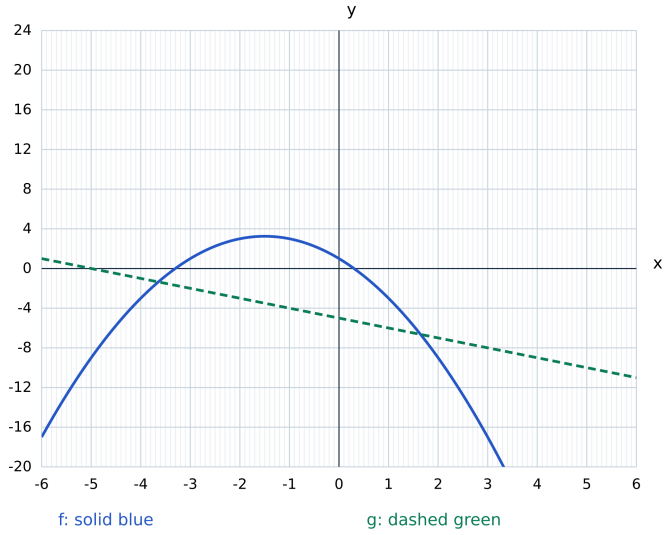
B. $x \approx -4.6$ or $x \approx -6.4$

C. $x \approx -0.7$

D. $x \approx -1.7$ or $x \approx 1.7$

Question 8

The graph shows $f(x) = (-1)(x-(-1))^2 + (-1)x + (2)$ and $g(x) = (0)(x-(-1))^2 + (-1)x + (-5)$. Use the graph to solve $f(x) < g(x)$ for all real x. Report boundary x-values to one decimal place; the formulas continue beyond the window.



Graph of the two formulas in the question, x from -6 to 6 in tenths; y from -20 to 24. f is solid blue; g is dashed green.

A. Outside $x \approx -2.6$ and $x \approx 2.6$; endpoints excluded.

B. Between $x \approx -3.6$ and $x \approx 1.6$; endpoints excluded.

C. Outside $x \approx -3.6$ and $x \approx 1.6$; endpoints excluded.

D. Outside $x \approx -3.6$ and $x \approx 1.6$; endpoints included.

Watch an explanation

Corbettmaths — Function Machines

Cognito — What are Quadratic Graphs

Tulla Maths — Calculator: make a table of values

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

53

Question 2

7

Question 3

7

Question 4

3

Question 5

6

Question 6

4

Question 7

$x \approx -0.7$ or $x \approx 2.7$

Question 8

Outside $x \approx -3.6$ and $x \approx 1.6$; endpoints excluded.

Open the full chapter online