Indices and scientific notation

Junior Cycle · Higher Level

Understand the idea

How to start

A power counts repeated multiplication: $x^3 = x \times x \times x$.

1. Identify the base: the number or expression being multiplied repeatedly.
2. Choose the rule for the operation shown. Adding powers and multiplying powers are different.
3. Keep brackets around a negative base, because they change what the power applies to.

Key terms & definitions

Base
The number or expression being raised to a power.

Index
The exponent indicating a power.

Scientific notation
A number written as a coefficient between 1 and 10 multiplied by an integer power of 10.

Key ideas

• For the same non-zero base, $a^m a^n = a^{m+n}$ and $\frac{a^m}{a^n} = a^{m-n}$. Also $(a^m)^n = a^{mn}$.

• A zero power is 1 for a non-zero base. A negative index means reciprocal: $a^{-n} = \frac{1}{a^n}$.

• For positive real bases, $a^{p/q} = \sqrt[q]{a^p}$. Check the real domain before evaluating roots of negative numbers.

• An even root of a negative number is not real. An odd root can be negative. Zero cannot have a negative power.

• Scientific notation is $a \times 10^n$ with $1 \le a < 10$ for positive values and integer n. Small positive numbers use negative powers.

Quick reminder

- $a^m a^n = a^{m+n}$: multiply same bases → add indices.
- $\frac{a^m}{a^n} = a^{m-n}$: divide same non-zero bases → subtract.
- $(a^m)^n = a^{mn}$: power of a power → multiply indices.

Worked examples

Example 01 · Start here: multiply powers of the same base

Simplify $x^3 \times x^2$.

Why this method?

The bases are both x and the operation is multiplication. Count the factors to see why their indices are added.

$(x \times x \times x)(x \times x)$	**Step 1** Write out what each power means. There are three x factors and then two more.
x^5	**Step 2** Count the five factors. Multiplication keeps them all in the product.
$x^{3+2} = x^5$	**Step 3** This is why we add the indices for multiplication with the same base.

Example 02 · Scientific notation calculation

Calculate $(3 \times 10^{-4})(2 \times 10^7)$ in scientific notation.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

$3 \times 2 = 6$	**Step 1** Multiply coefficients
$10^{-4} \times 10^7 = 10^3$	**Step 2** Add indices of the common base

6×10^3	**Step 3** Put the coefficient and the power of ten together, then check that the coefficient is between 1 and 10.

Example 03 · Powers with negative and zero integer bases

Evaluate $(-6)^9$. The minus sign is inside the brackets.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

9 factors, each equal to -6.	**Step 1** Read the brackets before multiplying
Result : -10077696.	**Step 2** An odd number of negative factors gives a negative product. Calculate 6 to the ninth power, then apply that sign.

Think about it: How does a bracket change whether a minus sign is raised to the power?

Example 04 · Powers with negative and zero integer bases — another example

Evaluate -7^8. The exponent is applied before the leading minus sign.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

First evaluate 7^8, then negate it.	**Step 1** Read the brackets before multiplying
Result : -5764801.	**Step 2** The power applies only to 7. Calculate 7 to the eighth power, then apply the leading negative sign.

Example 05 · Negative indices

Evaluate 3^{-3}. Give your answer as a fraction.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

$3^{-3} = 1 \div 3^3$	**Step 1** A negative index means the reciprocal of the positive power
$3^3 = 27,$ so the answer is $\frac{1}{27}$	**Step 2** Work out the power

Think about it: Before moving on: why does a negative index give a fraction, and not a negative number?

Example 06 · Negative indices — another example

Evaluate 5^{-3}. Give your answer as a fraction.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

$5^{-3} = 1 \div 5^3$	**Step 1** A negative index means the reciprocal of the positive power
$5^3 = 125,$ so the answer is $\frac{1}{125}$	**Step 2** Work out the power

Example 07 · Fractional indices

Evaluate $125^{\frac{2}{3}}$.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

$125^{\frac{2}{3}} = (\sqrt[3]{125})^2$	**Step 1** The denominator 3 tells us to take a cube root. The numerator 2 tells us to square.
$\sqrt[3]{125} = 5$	**Step 2** Five cubed is $5 \times 5 \times 5 = 125$.

$125^{\frac{2}{3}} = 5^2 = 25$	**Step 3** Square the cube root to finish.

Think about it: Before moving on: why does the q in the denominator cancel with the q in the base written as k^q?

Example 08 · Fractional indices — another example

Evaluate $16^{\frac{2}{4}}$.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

$\frac{2}{4} = \frac{1}{2}$	**Step 1** Simplify the index first. One half means square root.
$16^{\frac{2}{4}} = 16^{\frac{1}{2}} = \sqrt{16}$	**Step 2** Rewrite the power as a root.
$\sqrt{16} = 4$	**Step 3** Four squared is 16. Use the non-negative square root.

Example 09 · A power of zero is one

Evaluate 9^0.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

$9^0 = 1$	**Step 1** Any non-zero number to the power 0 is 1
$9 \div 9 = 1$	**Step 2** Check with the division rule $: a^m \div a^m = a^0,$ and a number divided by itself is 1

Think about it: Before moving on: why is the answer $1,$ and not 0 or the base?

Example 10 · A power of zero is one — another example

Evaluate 13^0.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

$13^0 = 1$	**Step 1** Any non-zero number to the power 0 is 1
$13 \div 13 = 1$	**Step 2** Check with the division rule $: a^m \div a^m = a^0,$ and a number divided by itself is 1

Example 11 · Powers of a product

Simplify $(2x)^4$.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

$(2x)^4 = 2^4 \times x^4$	**Step 1** Raise each factor to the power
$2^4 = 16,$ so the answer is $16x^4$	**Step 2** Work out the power of the number

Think about it: Before moving on: which factors does the power apply to, and why does it not multiply the index by the number?

Example 12 · Powers of a product — another example

Simplify $(5x)^2$.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

$(5x)^2 = 5^2 \times x^2$	**Step 1** Raise each factor to the power
$5^2 = 25,$ so the answer is $25x^2$	**Step 2** Work out the power of the number

Example 13 · Powers of a quotient

Simplify $\left(\frac{x}{7}\right)^4$.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

$\left(\frac{x}{7}\right)^4 = \frac{x^4}{7^4}$	**Step 1** Raise the numerator and the denominator to the power
$7^4 = 2401$, so the answer is $\frac{x^4}{2401}$	**Step 2** Work out the power of the denominator

Think about it: Before moving on: why is the denominator raised to the power, and not left as k?

Example 14 · Powers of a quotient — another example

Simplify $\left(\frac{x}{6}\right)^3$.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

$\left(\frac{x}{6}\right)^3 = \frac{x^3}{6^3}$	**Step 1** Raise the numerator and the denominator to the power
$6^3 = 216$, so the answer is $\frac{x^3}{216}$	**Step 2** Work out the power of the denominator

Example 15 · Multiplying powers with fractional indices

Simplify
$3^{\frac{4}{3}} \times 3^{\frac{2}{3}}$
to a single power of 3.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

$3^{\frac{4}{3}} \times 3^{\frac{2}{3}} = 3^{\frac{(4+2)}{3}}$	**Step 1** Same base: add the indices

$\frac{(4+2)}{3} = 2,$

so the answer is 3^2

Step 2

Simplify the index, which is a whole number

Think about it: Before moving on: why are the indices added, and not multiplied, when the base is the same?

Example 16 · Multiplying powers with fractional indices — another example

Simplify

$3^{\frac{4}{2}} \times 3^{\frac{2}{2}}$

to a single power of 3.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

$3^{\frac{4}{2}} \times 3^{\frac{2}{2}} = 3^{\frac{(4+2)}{2}}$

Step 1

Same base: add the indices

$\frac{(4+2)}{2} = 3,$

so the answer is 3^3

Step 2

Simplify the index, which is a whole number

Example 17 · A power of a power with a fractional index

Simplify

$(5^{\frac{4}{2}})^2$

to a single power of 5.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

$(5^{\frac{4}{2}})^2 = 5^{(\frac{4}{2}) \times 2}$

Step 1

Power of a power: multiply the indices

$(\frac{4}{2}) \times 2 = 4,$

so the answer is 5^4

Step 2

The denominator cancels with the power

Think about it: Before moving on: why does the denominator of the fractional index cancel when the power is the same number?

Example 18 · A power of a power with a fractional index — another example

Simplify
$(3^{\frac{3}{3}})^3$
to a single power of 3.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

$(3^{\frac{3}{3}})^3 = 3^{(\frac{3}{3}) \times 3}$	**Step 1** Power of a power: multiply the indices
$(\frac{3}{3}) \times 3 = 3,$ so the answer is 3^3	**Step 2** The denominator cancels with the power

Example 19 · Index quotients with roots and rational bases

Simplify exactly
$: 2^{-\frac{3}{2}} \div 2^{-\frac{2}{2}}$
. The base is positive; use the positive square root. Give a fraction or simplified surd, not a decimal.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

The new index is $(-\frac{3}{2}) - (-\frac{2}{2}) = -\frac{1}{2}$.	**Step 1** Divide powers with the same non-zero base by subtracting indices
$2^{-\frac{1}{2}} = (\sqrt{2})^{-1}$.	**Step 2** Interpret the half-index using the positive square root
$(\sqrt{2})^{-1} = \frac{\sqrt{2}}{2}$.	**Step 3** A negative index means take the reciprocal, then simplify

Think about it: Why must you keep the square root or half-index, and what changes when the resulting index is negative?

Example 20 · Index quotients with roots and rational bases — another example

Simplify exactly : $(\sqrt{2})^{-1} \div (\sqrt{2})^{-2}$. The base is positive; use the positive square root. Give a fraction or simplified surd, not a decimal.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

The new index is $(-1) - (-2) = 1$.	**Step 1** Divide powers with the same non-zero base by subtracting indices
$(\sqrt{2})^1 = (\sqrt{2})^1$.	**Step 2** Keep the square-root base
$(\sqrt{2})^1 = \sqrt{2}$.	**Step 3** Pair square-root factors, or evaluate the fraction power

Example 21 · Real powers and their domains

Evaluate

$$(-32)^{-\frac{1}{5}}$$

over the real numbers. Give an exact simplified value.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

The 5th root of -32 is -2.	**Step 1** Take the real odd root
Result : $-\frac{1}{2}$.	**Step 2** Raise to the numerator power, then invert if the index is negative

Think about it: Why must a rational exponent be reduced before deciding whether a negative base is allowed?

Example 22 · Real powers and their domains — another example

For which real values of a is $a^{\frac{1}{6}}$ defined as a real number? The fraction is in lowest terms.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

Even denominator: negative bases are excluded.	**Step 1** Check the denominator parity
Positive exponent: zero is permitted.	**Step 2** Check zero separately
$a \geq 0$.	**Step 3** Combine the restrictions

Example 23 · Standard form for numbers between 0 and 1

Write 0.002 in standard form.

Why this method?

Look at the structure: a product, quotient or power of a power. Choose the index rule for that structure before calculating.

$0.002 \to 2,$ moved 3 places	**Step 1** Move the decimal point to the right of the first non-zero digit. Count the places moved
$0.002 = 2 \times 10^{-3}$	**Step 2** A number between 0 and 1 has a negative power of $10,$ equal to the number of places moved

Think about it: Before moving on: why is the power of 10 negative for a number less than 1?

Common mistake

Adding powers when adding terms. $x^2 + x^3$ is not x^5: these are unlike terms. For multiplication with the same base, $x^2 \times x^3 = x^5$.

Exam technique

Use brackets to distinguish $(-3)^2$ from -3^2.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Evaluate -7^3. The exponent is applied before the leading minus sign.

A. -343

B. 343

C. -4

D. -21

Question 2

Evaluate 2^{-5}. Give your answer as a fraction.

A. $\frac{1}{32}$

B. 32

C. -32

D. $\frac{1}{10}$

Question 3

Evaluate $32^{\frac{2}{5}}$.

A. 1024

B. 32

C. 4

D. 128

Question 4

Evaluate 17^0.

A. 289

B. 1

C. 0

D. 17

Question 5

Simplify $(9x)^2$.

A. $18x^2$

B. $81x^2$

C. $9x^2$

D. $81x$

Question 6

Simplify $\left(\frac{x}{9}\right)^2$.

A. $x^2 \times 9$

B. $\frac{x^2}{81}$

C. $\frac{x}{81}$

D. $\frac{x^2}{9}$

Question 7

Simplify
$2^{\frac{1}{3}} \times 2^{\frac{5}{3}}$
to a single power of 2.

A. 2^2

B. 2^6

C. 2^3

D. 2^5

Question 8

Simplify
$\left(7^{\frac{3}{2}}\right)^2$
to a single power of 7.

A. 7^3

B. 7^5

C. 7^6

D. $7^{\frac{3}{2}}$

Question 9

Simplify exactly
$: 7^{\frac{2}{2}} \div 7^{-\frac{1}{2}}$
. The base is positive; use the positive square root. Give a fraction or simplified surd, not a decimal.

A. 343

B. $\frac{\sqrt{7}}{49}$

C. $\sqrt{7}$

D. $7\sqrt{7}$

Question 10

For which real values of a is $a^{\frac{3}{5}}$ defined as a real number? The fraction is in lowest terms.

A. All real a.

B. $a \geq 0$.

C. $a > 0$.

D. $a \neq 0$.

Question 11

Write 0.000067 in standard form.

A. 6.7×10^{-5}

B. 6.7×10^{5}

C. 67×10^{-6}

D. 6.7×10^{-4}

Watch an explanation

Cognito — Multiplying & Dividing Powers

Cognito — Negative Powers

Cognito — Fractional Powers

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

-343

Question 2

$\frac{1}{32}$

Question 3

4

Question 4

1

Question 5

$81x^2$

Question 6

$\frac{x^2}{81}$

Question 7

2^2

Question 8

7^3

Question 9

$7\sqrt{7}$

Question 10

All real a.

Question 11

6.7×10^{-5}

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