Inequalities and solution sets

Junior Cycle · Higher Level

Understand the idea

How to start

The sign $<$ means less than. The sign $\leq$ means less than or equal to.

1. Read whether the endpoint is included.

2. Use the same operations on both sides. Reverse the sign if you multiply or divide by a negative number.

3. Check whether the answer needs all real numbers or only whole numbers.

Key terms & definitions

Strict inequality
A comparison using $<$ or $>$, excluding equality.

Interval
A continuous set of numbers between boundaries.

Boundary
An endpoint at which the truth of an inequality can change.

Key ideas

• Solve an inequality much like an equation, but reverse its sign when multiplying or dividing by a negative number.

• Use an open dot for a strict endpoint and a filled dot when the endpoint is included.

• State the domain. Integer solutions and real solutions are different sets.

• On a graph, $f(x) > g(x)$ where the graph of f is above the graph of g.

• For a strict inequality, crossings and touching points are excluded. For an inclusive inequality, include points where the graphs agree.

Quick reminder

• Reverse the inequality when multiplying or dividing by a negative.

• Open endpoint: equality excluded. Filled endpoint: equality included.

• Check whether the domain is integers or real numbers.

Worked examples

Example 01 · Linear inequalities: the largest whole solution

What is the largest whole number n that satisfies $7n + 1 < 85$?

Purple marks the same subtraction on both sides. Subtracting 1 leaves the inequality sign unchanged.

Why this method?

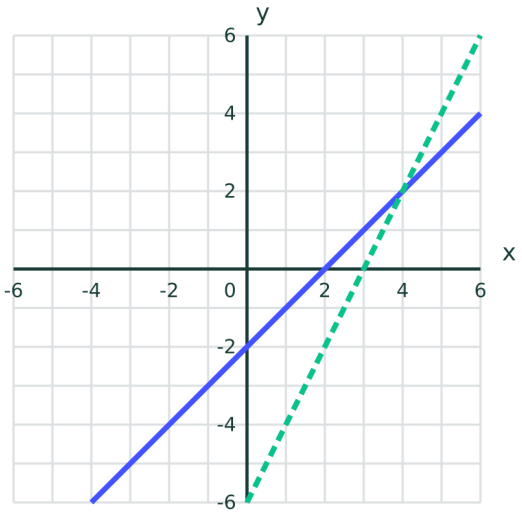
Isolate the variable or compare the heights of the graphs. Keep track of whether equality is included at the boundary.

$7n + 1 < 85$	**Step 1** We want n on its own. The sign means strictly less than.
$7n + 1 - 1 < 85 - 1$	**Step 2** Subtract 1 from both sides.
$7n < 84$	**Step 3** Simplify the subtraction.
$n < 12$	**Step 4** Divide both sides by positive 7. The inequality sign stays the same.
$n = 11$	**Step 5** The question asks for the largest whole number below 12. Twelve itself is excluded.
$7(11) + 1 = 78 < 85$	**Step 6** Check 11. The next whole number gives $7(12) + 1 = 85$, which is not less than 85.

Think about it: Before moving on: why is the bound itself not a solution when the inequality is strict?

Example 02 · Solution sets of inequalities from graphs

Graph 1 : The blue line is f(x) and the green line is g(x). Using the graph, find the solution set of $f(x) < g(x)$.



f: solid blue g: dashed green

Two straight lines drawn on axes marked from -6 to 6 on each side. The lines cross at one point.

Why this method?

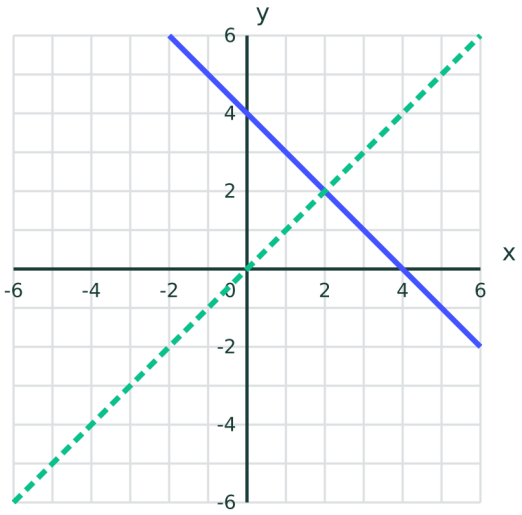
Isolate the variable or compare the heights of the graphs. Keep track of whether equality is included at the boundary.

The lines cross at $x = 4$	**Step 1** Read where the lines cross; the inequality holds on one side of that x-value
$x > 4$	**Step 2** Check the side: take a value of x on each side and compare f(x) with g(x)

Think about it: Before moving on: why is the crossing point not included when the inequality is $f(x) < g(x)$?

Example 03 · Solution sets of inequalities from graphs — another example

Graph 2 : The blue line is f(x) and the green line is g(x). Using the graph, find the solution set of $f(x) < g(x)$.



f: solid blue g: dashed green

Two straight lines drawn on axes marked from -6 to 6 on each side. The lines cross at one point.

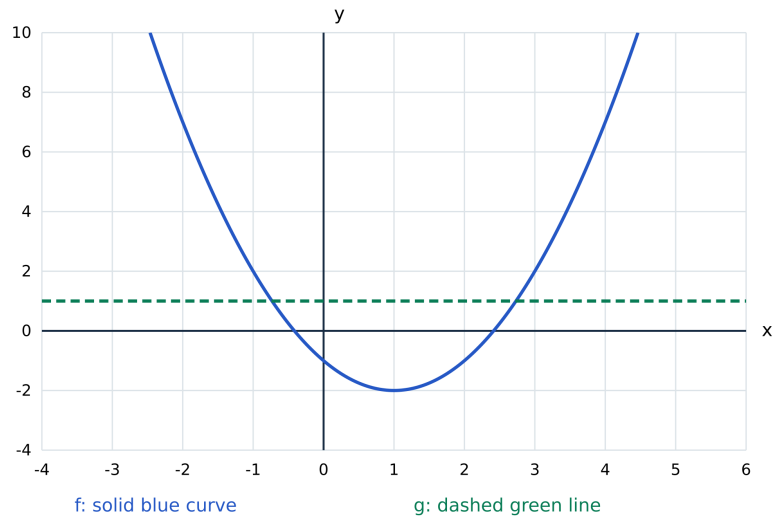
Why this method?

Isolate the variable or compare the heights of the graphs. Keep track of whether equality is included at the boundary.

The lines cross at $x = 2$	**Step 1** Read where the lines cross; the inequality holds on one side of that x-value
$x > 2$	**Step 2** Check the side: take a value of x on each side and compare f(x) with g(x)

Example 04 · Approximate inequality regions from graphs

The graph shows $f(x) = (x - 1)^2 - 2$ and $g(x) = 1$. Use the graph to estimate the solution region of $f(x) < g(x)$. Round both crossing x-values to 1 decimal place and state whether the actual crossings are included.



Graph of $f(x) = (x - 1)^2 - 2$ and $g(x) = 1$. Axes have numeric scales; f is a solid blue curve and g is a dashed green horizontal line.

Why this method?

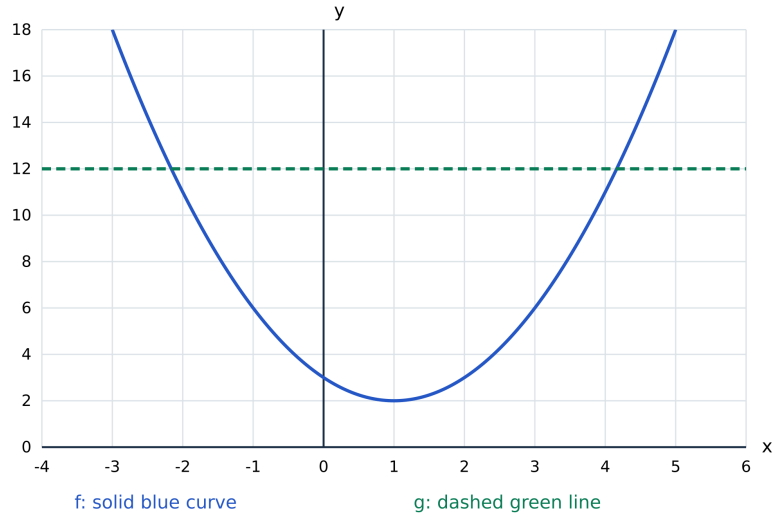
Isolate the variable or compare the heights of the graphs. Keep track of whether equality is included at the boundary.

The crossings are at approximately $x = -0.7$ and $x = 2.7$.	**Step 1** Estimate the two crossing x-values
At $x = 1$, $f(x) = -2 < 1 = g(x)$. At $x = 5$, $f(x) = 14 > 1 = g(x)$. Thus f is below g between the crossings and above g outside.	**Step 2** Compare vertical positions in each region
Between the crossings at $x \approx -0.7$ and $x \approx 2.7$; exclude both actual crossings.	**Step 3** Apply the requested comparison and boundary rule

Think about it: Why do you need to compare the graphs between and outside the crossings, rather than report only the crossing coordinates?

Example 05 · Approximate inequality regions from graphs — another example

The graph shows $f(x) = (x - 1)^2 + 2$ and $g(x) = 12$. Use the graph to estimate the solution region of $f(x) > g(x)$. Round both crossing x-values to 1 decimal place and state whether the actual crossings are included.



Graph of $f(x) = (x - 1)^2 + 2$ and $g(x) = 12$. Axes have numeric scales; f is a solid blue curve and g is a dashed green horizontal line.

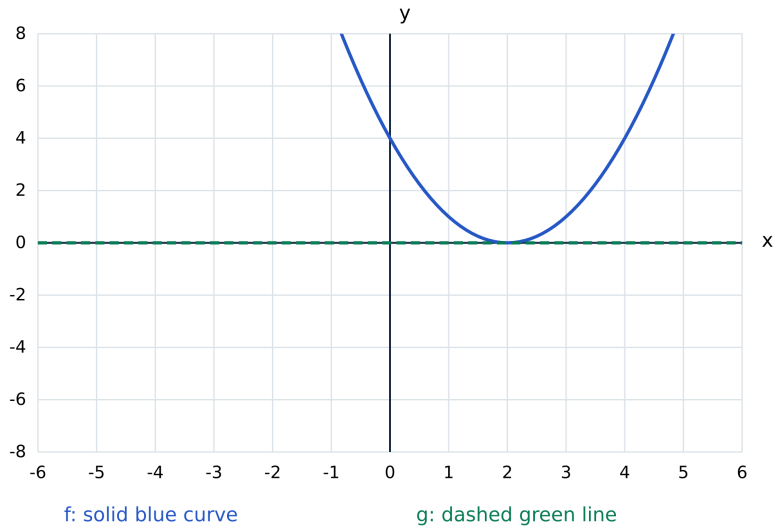
Why this method?

Isolate the variable or compare the heights of the graphs. Keep track of whether equality is included at the boundary.

The crossings are at approximately $x = -2.2$ and $x = 4.2$.	**Step 1** Estimate the two crossing x-values
At $x = 1$, $f(x) = 2 < 12 = g(x)$. At $x = 5$, $f(x) = 18 > 12 = g(x)$. Thus f is below g between the crossings and above g outside.	**Step 2** Compare vertical positions in each region
Outside the crossings at $x \approx -2.2$ and $x \approx 4.2$; exclude both actual crossings.	**Step 3** Apply the requested comparison and boundary rule

Example 06 · Graph inequalities: touching or non-intersecting curves

The graph shows $f(x) = 1(x - (2))^2 + (0)$ and $g(x) = 0$. Use the graph to solve $f(x) < g(x)$ for all real x. The formulas describe the curves beyond the displayed window.



Graph of $f(x) = 1(x - (2))^2 + (0)$ and $g(x) = 0$; labelled x and y scales, solid blue f, dashed green g.

Why this method?

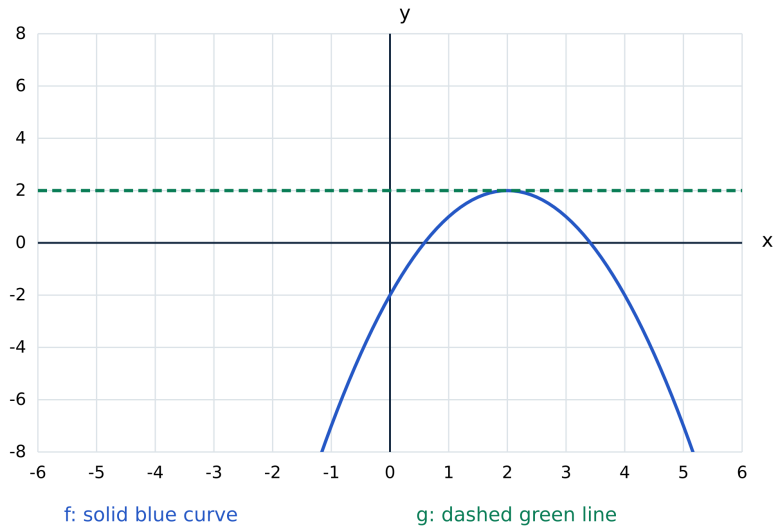
Isolate the variable or compare the heights of the graphs. Keep track of whether equality is included at the boundary.

The curves meet only at $(2, 0)$. They touch without crossing.	**Step 1** Inspect the turning point and the horizontal line
For every x other than 2, f is above g.	**Step 2** Compare heights on both sides of the turning point
No real x.	**Step 3** Exclude the equality point

Think about it: Why can a strict inequality hold on both sides of a contact point but fail at the point itself?

Example 07 · Graph inequalities: touching or non-intersecting curves — another example

The graph shows $f(x) = -1(x - (2))^2 + (2)$ and $g(x) = 2$. Use the graph to solve $f(x) < g(x)$ for all real x. The formulas describe the curves beyond the displayed window.



Graph of $f(x) = -1(x - (2))^2 + (2)$ and $g(x) = 2$; labelled x and y scales, solid blue f, dashed green g.

Why this method?

Isolate the variable or compare the heights of the graphs. Keep track of whether equality is included at the boundary.

The curves meet only at $(2, 2)$. They touch without crossing.	**Step 1** Inspect the turning point and the horizontal line
For every x other than 2, f is below g.	**Step 2** Compare heights on both sides of the turning point
All real x except $x = 2$.	**Step 3** Exclude the equality point

Common mistake

Forgetting to reverse the sign when multiplying or dividing both sides by a negative number. For example, $-2x < 6$ gives $x > -3$, not $x < -3$. Adding or subtracting does not reverse the sign.

Exam technique

Test one point in each region when reading a graphical inequality.

Try it yourself

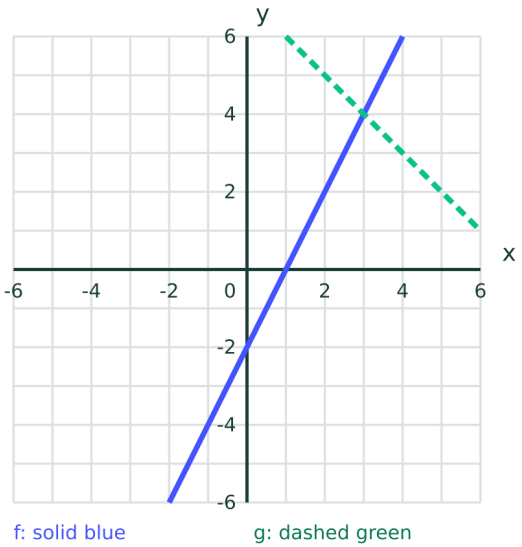
Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

What is the largest whole number n that satisfies $4n + 3 < 35$?

Question 2

The blue line is f(x) and the green line is g(x). Using the graph, find the solution set of $f(x) < g(x)$.



f: solid blue g: dashed green

Two straight lines drawn on axes marked from -6 to 6 on each side. The lines cross at one point.

A. $x < 3$

B. $y < 4$

C. $x \leq 3$

D. $x > 3$

Question 3

The graph shows $f(x) = (x - 2)^2 + 3$ and $g(x) = 5$. Use the graph to estimate the solution region of $f(x) < g(x)$. Round both crossing x-values to 1 decimal place and state whether the actual crossings are included.

A. Between the crossings at x ≈ -1.4 and x ≈ 1.4; exclude both actual crossings.

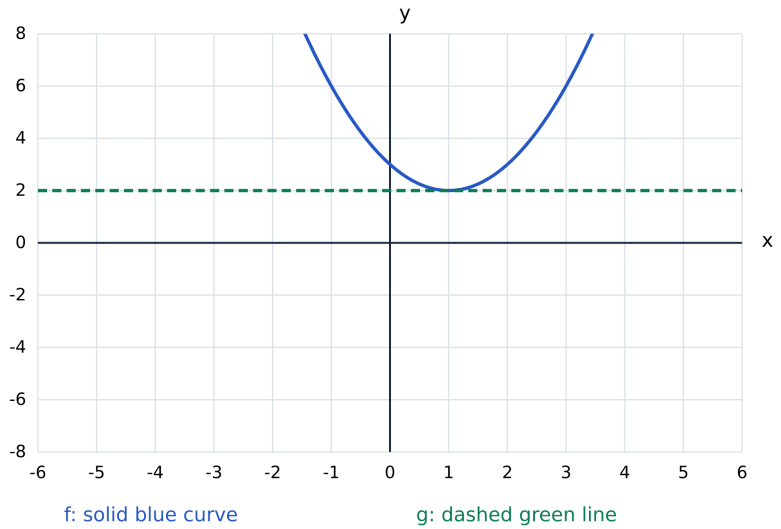
B. Outside the crossings at x ≈ 0.6 and x ≈ 3.4; exclude both actual crossings.

C. Between the crossings at x ≈ 0.6 and x ≈ 3.4; exclude both actual crossings.

D. Between the crossings at x ≈ 0.6 and x ≈ 3.4; include both actual crossings.

Question 4

The graph shows $f(x) = 1(x - (1))^2 + (2)$ and $g(x) = 2$. Use the graph to solve $f(x) > g(x)$ for all real x. The formulas describe the curves beyond the displayed window.



Graph of $f(x) = 1(x - (1))^2 + (2)$ and $g(x) = 2$; labelled x and y scales, solid blue f, dashed green g.

A. No real x.

B. All real x except $x = 1$.

C. All real x.

D. Only $x = 1$.

Watch an explanation

Cognito — Algebra with Inequalities 1

Cognito — Introduction to Inequalities

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

7

Question 2

$x < 3$

Question 3

Between the crossings at x ≈ 0.6 and x ≈ 3.4; exclude both actual crossings.

Question 4

All real x except $x = 1$.

Open the full chapter online