JUNIOR CYCLE HIGHER LEVEL MATHEMATICS

Course revision notes

Worked examples, diagrams and step-by-step methods.

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How to revise: cover the next line, try it, then check the reason on the right.

Curriculum reference: NCCA Junior Cycle Mathematics specification.

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Curriculum reference: NCCA Junior Cycle Mathematics specification.

Integers and order of operations

Use brackets and powers first. Then multiply or divide, then add or subtract.

Work out −6 + 3 × (8 − 12).

$-6+3\times(-4)$	Start inside the brackets.
$-6+(-12)$	Multiply before adding.
-18	

Work out 18 ÷ 3 × 2.

6×2	Division and multiplication have the same priority. Work left to right.
12	

Watch the signs

Subtracting a negative is the same as adding: 4 − (−3) = 7.

Number sets and ordering

Classify a number by what it is, not just how it is written.

Set	Meaning	Example
Natural ($\mathbb{N}$)	Counting numbers	5
Integers ($\mathbb{Z}$)	Positive and negative whole numbers, and zero	-4
Rational ($\mathbb{Q}$)	Can be written as a fraction of integers with a non-zero denominator	0.75
Irrational ($\mathbb{R} \setminus \mathbb{Q}$)	Cannot be written as a fraction of integers	$\sqrt{2}$
Real ($\mathbb{R}$)	All rational and irrational numbers	π

Which is greater, −7 or −3?

$-3 > -7$ — -3 lies farther right on the number line.

-3

A number can belong to more than one set

For example, -4 is an integer, a rational number, and a real number.

Numbers

Try the questions before opening the separate answer key. Show one step per line.

1.1 · Fluency

Write −8, 3/5 and √7 in the smallest applicable set from integers, rational and irrational numbers.

1.2 · Variation

Order −0.7, −0.07 and −0.77 from least to greatest.

1.3 · Choose a method

A balance starts at −€18, rises by €25 and falls by €12. Find the balance.

Check your work: are the sign, units and level of accuracy appropriate?

Prime factors

A factor divides exactly. A multiple is produced by multiplying.

Write 84 as a product of prime factors.

$84 = 2 \times 42$	Split off a factor of 2.
$84 = 2 \times 2 \times 21$	Split 42 again.
$84 = 2 \times 2 \times 3 \times 7$	Stop when every factor is prime.

$84 = 2^2 \times 3 \times 7$

Prime numbers

A prime has exactly two positive factors: 1 and itself. The number 1 is not prime.

Highest common factor and lowest common multiple

Use the prime powers to see what the two numbers share.

Find the HCF and LCM of 24 and 36.	
$24 = 2^3 \times 3$	Factorise 24.
$36 = 2^2 \times 3^2$	Factorise 36.
$2^2 \times 3 = 12$	HCF: take the smaller power of each common prime.
$2^3 \times 3^2 = 72$	LCM: take the largest power of each prime present.
$\text{HCF} = 12, \quad \text{LCM} = 72$	

Factors and Multiples

Try the questions before opening the separate answer key. Show one step per line.

2.1 · Fluency

Write 90 as a product of prime factors.

2.2 · Variation

Find the HCF and LCM of 24 and 36.

2.3 · Choose a method

Two buses leave together, then every 18 and 24 minutes. When do they next leave together?

Check your work: are the sign, units and level of accuracy appropriate?

Equivalent fractions and addition

Change the denominator by multiplying or dividing the top and bottom by the same non-zero number.

Calculate 3/4 − 1/6.

$\frac{3}{4}=\frac{9}{12}$	Multiply top and bottom by 3.
$\frac{1}{6}=\frac{2}{12}$	Multiply top and bottom by 2.
$\frac{9}{12}-\frac{2}{12}=\frac{9-2}{12}$	Subtract the numerators and keep the common denominator.

$\frac{7}{12}$

Mixed numbers

To convert 2¾ to an improper fraction, write 2 as 8 quarters, then add 3 quarters to get 11 quarters.

Multiplication and division

For multiplication, cancel common factors. For division, flip only the second fraction.

Calculate 3/10 × 5/9.

$\frac{3\times5}{10\times9}$	Multiply the numerators and denominators.
$\frac{1}{2\times3}$	Cancel 5 with 10 and 3 with 9.

$\frac{1}{6}$

Calculate 3/5 ÷ 9/10.

$\frac{3}{5}\times\frac{10}{9}$	Multiply by the reciprocal of 9/10.
$\frac{3\times10}{5\times9}$	Cancel common factors: 10 ÷ 5 = 2 and 9 ÷ 3 = 3.

$\frac{2}{3}$

Fractions

Try the questions before opening the separate answer key. Show one step per line.

3.1 · Fluency

Simplify 42/56.

3.2 · Variation

Calculate 5/6 − 1/4.

3.3 · Choose a method

A tank is 3/5 full. One quarter of the water currently in it is used. What fraction of the full tank remains?

Check your work: are the sign, units and level of accuracy appropriate?

Place value and rounding

Use the next digit to decide whether to round up. Keep zeros needed to show the requested precision.

Round 0.04786 to two significant figures.

0.04786	The leading zeros do not count. The first two significant digits are 4 and 7.
$0.04786 \approx 0.048$	The next digit is 8, so round the 7 up.
0.048	

Round 3.996 to two decimal places.

$3.996 \approx 4.00$	The third decimal digit is 6. The rounding carries through both 9s.
4.00	

Multiplying decimals and converting to fractions

Use place value to check that the decimal point is in the right place.

Calculate 0.24 × 0.5.

$24\times5=120$	Multiply as whole numbers first.
$0.24\times0.5=0.120$	The factors have three decimal places altogether.
0.12	

Write 0.375 as a fraction in simplest form.

$0.375=\frac{375}{1000}$	Three decimal places means thousandths.
$\frac{375}{1000}=\frac{3}{8}$	Divide top and bottom by 125.
$\frac{3}{8}$	

Decimals

Try the questions before opening the separate answer key. Show one step per line.

4.1 · Fluency

Write 0.375 as a fraction in simplest form.

4.2 · Variation

Round 0.04687 to three decimal places.

4.3 · Choose a method

A 4.8 m ribbon is cut into 0.35 m lengths. How many complete lengths and how much remains?

Check your work: are the sign, units and level of accuracy appropriate?

Finding a percentage and percentage change

A percentage is a fraction out of 100.

A price rises from €80 to €92. Find the percentage increase.

$92-80=12$	Find the increase.
$\frac{12}{80}\times 100=15$	Divide by the original price, then multiply by 100.

15%

Choose the right base

For percentage change, divide by the starting amount, not the final amount.

Multipliers and reverse percentages

A 20% reduction leaves 80% of the original amount.

A jacket costs €72 after a 20% discount. Find its original price.

$100\% - 20\% = 80\%$	The sale price is 80% of the original.
$0.8x = 72$	Write the percentage as a multiplier.
$x = \frac{72}{0.8} = 90$	Divide by the multiplier to undo the discount.

Original price: €90

Successive changes

Multiply the two multipliers. A 20% increase followed by a 20% decrease gives 1.2 × 0.8 = 0.96, so the overall change is a 4% decrease.

Percentages

Try the questions before opening the separate answer key. Show one step per line.

5.1 · Fluency

Find 15% of €240.

5.2 · Variation

A price rises from €75 to €87. Find the percentage increase.

5.3 · Choose a method

After a 20% reduction, a bike costs €144. What was its original price?

Check your work: are the sign, units and level of accuracy appropriate?

Simplifying and sharing a ratio

Put quantities in the same units before simplifying the ratio.

Share €84 in the ratio 3 : 4.

$3+4=7$	There are seven equal parts altogether.
$84\div 7=12$	Find the value of one part.
$3(12)=36,\quad 4(12)=48$	Multiply by the number of parts in each share.

€36 and €48

Check

The shares must add back to €84. The larger share goes with the larger ratio number.

Direct proportion and unit rates

Find the amount for one unit, then scale to the number of units you need.

Five identical notebooks cost €12.50. What do eight cost?

$12.50 \div 5 = 2.50$	Find the price of one notebook.
$8 \times 2.50 = 20$	Multiply the unit price by eight.
€20	

A map scale is 1 : 25,000. What does 6 cm represent?

$6 \times 25000 = 150000$	The result is in centimetres.
$150000 \div 100000 = 1.5$	Convert centimetres to kilometres.
$1.5\,\text{km}$	

Ratio and Proportion

Try the questions before opening the separate answer key. Show one step per line.

6.1 · Fluency

Simplify 48 : 72.

6.2 · Variation

Share €105 in the ratio 2 : 5.

6.3 · Choose a method

Six notebooks cost €10.50. What would 14 cost at the same unit price?

Check your work: are the sign, units and level of accuracy appropriate?

Index laws

With the same base, powers can be combined by working with their indices.

Simplify (2³ × 2⁵) ÷ 2⁴.

$2^3 \times 2^5 = 2^8$	Add indices when multiplying.
$2^8 \div 2^4 = 2^4$	Subtract when dividing.

16

Remember

For a power of a power, multiply indices. A power outside brackets applies to each factor: (ab)³ = a³b³.

Zero, negative and fractional indices

A negative index gives a reciprocal. A fractional index involves a root.

Evaluate 27 raised to the power −⅔.

$27^{-2/3} = \frac{1}{27^{2/3}}$	A negative index means take the reciprocal.
$27^{2/3} = (\sqrt[3]{27})^2 = 3^2$	The denominator 3 tells you to take the cube root.

$\frac{1}{9}$

Remember

For a non-zero base, $a^0 = 1$. For positive a and b, $(a/b)^n = a^n/b^n$; the same rule works for fractional indices.

Surds

Take out perfect-square factors before collecting like surds.

Simplify √72 + √8.

$\sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2}$	The square root of 36 is 6.
$\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$	Simplify the other surd.
$6\sqrt{2} + 2\sqrt{2} = (6+2)\sqrt{2}$	Both terms contain the same surd.

$8\sqrt{2}$

Remember

Unlike surds cannot be added as like terms. For products, √3 × √6 = √18 = 3√2.

Standard form and accuracy

In standard form, the first factor is at least 1 and less than 10.

Write 0.0005078 in standard form, correct to three significant figures.

$0.0005078 = 5.078 \times 10^{-4}$	Move the decimal point four places right to make 5.078.
$5.078 \approx 5.08$	The first three significant digits are 5, 0 and 7; the next digit is 8.

5.08×10^{-4}

Remember

Leading zeros are not significant. Decimal places instead count digits to the right of the decimal point.

Number operations and estimation

Use brackets and indices first, then multiply or divide, then add or subtract.

Find 18 − (−3)² × 2, and explain how to check.

$(-3)^2 = 9$	Square the whole bracket.
$18 - 9 \times 2 = 18 - 18$	Multiply before subtracting.

0

Remember

Independent check: 9 × 2 is exactly 18. Subtraction is not commutative. Multiplication is distributive: 7 × 19 = 7 × (20 − 1).

Number Extensions

Try the questions before opening the separate answer key. Show one step per line.

7.1 · Fluency

Simplify $(3^5 \times 3^4) \div 3^6$.

7.2 · Variation

Simplify $\sqrt{50} - \sqrt{8}$.

7.3 · Choose a method

Write 0.0004837 in standard form to three significant figures.

Check your work: are the sign, units and level of accuracy appropriate?

VAT, discounts and bills

A discount is based on the original price. VAT is based on the price before VAT.

A €200 item is reduced by 10%, then 23% VAT is added. Find the bill.

$200 \times 0.90 = 180$	After the discount, 90% remains.
$180 \times 1.23 = 221.40$	Include the price plus the VAT.

€221.40

Remember

Use the rate given in the question. For a utility bill add the fixed charge to units × rate before applying any tax specified.

Profit, mark-up and margin

Mark-up and margin use different starting amounts.

An item costs €80 and sells for €100. Find the mark-up and margin.

$100-80=20$	Profit = selling price minus cost price.
$\frac{20}{80}\times 100=25$	Mark-up divides by cost price.
$\frac{20}{100}\times 100=20$	Margin divides by selling price.

25% mark-up; 20% margin

Remember

For a loss percentage, divide the amount lost by the cost price.

Compound interest

Each year, apply the multiplier to the new balance.

Find the value of €500 after 3 years at 4% compound interest.

$1+0.04=1.04$	Add the interest rate to 1.
$500(1.04)^3=562.432$	Apply the multiplier three times.

€562.43

Remember

Interest earned is final balance minus original amount: €62.43.

Income tax and net pay

Use the single rate and the deductions stated in the question.

Gross pay is €500. Tax is 20%, with a €30 tax credit. Other deductions are €22. Find net pay.

$0.2(500)=100$	Calculate tax before the credit.
$100-30=70$	The tax credit reduces the tax.
$500-70-22=408$	Subtract tax and other deductions from gross pay.

€408

Remember

These are practice figures, not current tax rules.

Currency and value for money

Write the direction of the exchange rate. Compare prices for equal quantities.

At €1 = £0.85, how many euro does a £68 item cost?

$0.85x = 68$	Each of the x euro buys £0.85.
$x = 68 \div 0.85$	Divide to convert pounds back to euro.

€80

Remember

For unit value, €3.30 for 750 g costs €4.40 per kg; €4.20 for 1 kg is cheaper.

Euro change versus percentage change

The larger euro change need not be the larger percentage change.

Plan A rises from €30 to €36. Plan B rises from €60 to €69. Which rises more by each measure?

$36-30=6,\quad 69-60=9$	B rises by more euro.
$\frac{6}{30}\times 100=20,\quad \frac{9}{60}\times 100=15$	Divide each change by its own starting price.

B by amount; A by percentage.

Remember

Common mistake: comparing the percentage changes using the final prices as the denominators.

Financial Maths

Try the questions before opening the separate answer key. Show one step per line.

8.1 · Fluency

A €160 item has 23% VAT added. Find the total.

8.2 · Variation

A shop pays €60 and sells for €80. Find the mark-up and margin.

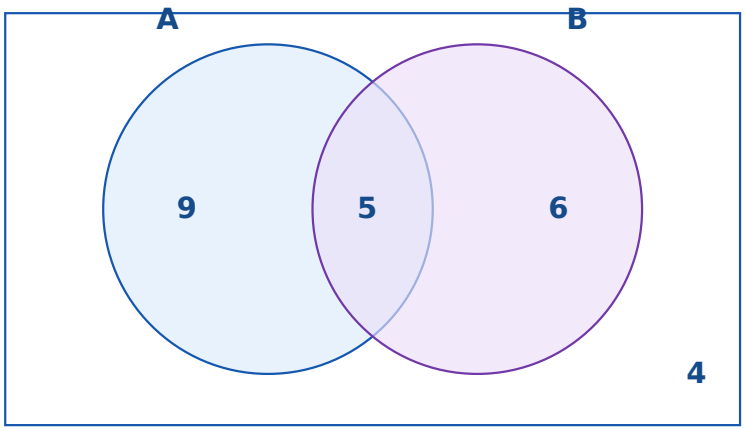
8.3 · Choose a method

A saver puts €900 into an account at 3% compound interest. Find the balance after two years.

Check your work: are the sign, units and level of accuracy appropriate?

Two-set Venn diagrams

Start by filling in the overlap. “Or” includes people in both sets.



The middle belongs to both sets. The 4 is outside both, but inside the universal set.

In a class of 24, 14 play A, 11 play B and 5 play both. How many play neither?

$14-5=9$	These play only A.
$11-5=6$	These play only B.
$9+5+6=20$	The union counts each person once.

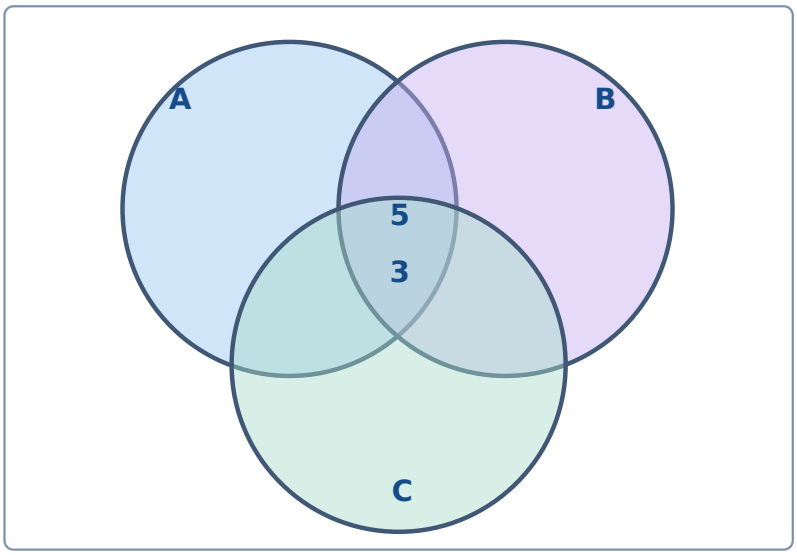
$24-20=4$

Remember

Intersection = common members; union = all members of either set; complement = outside the set but inside the universal set.

Three sets and notation

Fill the centre first, then the pair-only regions, then the single-only regions.



The 5 is in A and B only. The 3 in the centre is in all three sets.

A and B have 8 members in common. Of these, 3 also belong to C. Find the number in A and B but not C.

$\#(A \cap B) = 8$	The pair intersection includes the centre.
$\#(A \cap B \cap C) = 3$	Subtract the triple intersection.

$8 - 3 = 5$

Remember

∈ means “is a member of”; ⊂ means “is a subset of”; ∅ means empty set. ∩ and ∪ are commutative and associative; set difference is neither.

Does order matter in a set operation?

Commutative means swapping operands. Associative means moving brackets without swapping operands.

Use A = {1, 2}, B = {2, 3}, C = {2, 4}. Compare the two bracketings of intersection, and compare the two orders of set difference.

$(A \cap B) \cap C = \{2\}$	Intersect A and B first.
$A \cap (B \cap C) = \{2\}$	Intersect B and C first.
$A \setminus B = \{1\}, \quad B \setminus A = \{3\}$	Keep only members of the first set that are not in the second.

Intersection matches; difference does not.

Remember

This verifies the associative rule for this example. A single counterexample is enough to show that difference is not commutative. Try the same bracket test for union.

Sets

Try the questions before opening the separate answer key. Show one step per line.

9.1 · Fluency

If A = {1, 2, 4} and B = {2, 3}, find A ∩ B and A ∪ B.

9.2 · Variation

With those same sets and C = {2, 4, 5}, find (A ∪ B) ∩ C.

9.3 · Choose a method

There are 30 students. 18 play music, 13 play sport and 6 do both. How many do neither?

Check your work: are the sign, units and level of accuracy appropriate?

Expressions and like terms

An expression has no equals sign. An equation says two expressions have the same value.

Simplify 3x² + 4x − 5 + 2x² − x + 7.

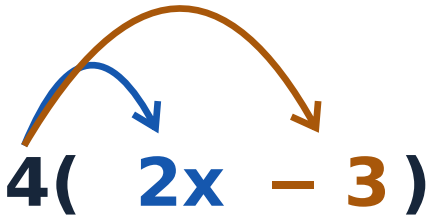
$(3x^2+2x^2)+(4x-x)+(-5+7)$	Group x² terms, x terms and numbers separately.
$5x^2+3x+2$	Add the coefficients within each group.
$5x^2+3x+2$	

Remember

Substitute x = 2 by replacing each x with 2; the original and simplified expressions both give 28.

Expanding one bracket

Multiply the outside factor by each term, including its sign.



Follow each arrow from the outside 4 to one signed term.

Expand 4(2x − 3).

$4(2x) = 8x$	Multiply the first term by 4.
$4(-3) = -12$	Multiply the second term by 4.

$8x - 12$

Remember

The minus belongs to the 3. Keep it when multiplying.

Expanding two brackets

Every term in one bracket multiplies every term in the other.

×	x	−3
x	x^2	−3x
4	4x	−12

Expand (x + 4)(x − 3).

$x^2 - 3x + 4x - 12$	Read off all four products from the box.
$-3x + 4x = x$	Collect the middle terms.
$x^2 + x - 12$	

Remember

For a linear bracket times a quadratic bracket, use the same method: draw two rows and three product columns.

Linear equations with fractions

Multiply both complete sides by the lowest common denominator.

Solve (2x + 1)/5 = (x − 2)/3.

$15\left(\frac{2x+1}{5}\right)=15\left(\frac{x-2}{3}\right)$	Multiply both sides by 15.
$3(2x+1)=5(x-2)$	15 ÷ 5 = 3 and 15 ÷ 3 = 5.
$6x+3=5x-10$	Expand both brackets.
$x+3=-10$	Subtract 5x from both sides.

$x=-13$

Remember

Check: both original fractions equal −5 when x = −13.

Inequalities: real numbers

For real numbers, draw a continuous line. An open circle excludes the endpoint; a filled circle includes it.



Filled circle: 3 is included.

Solve 2x ≤ 6 for real numbers.

$2x \div 2 \leq 6 \div 2$	Divide both sides by positive 2. Do not reverse the sign.

$x \leq 3$

Remember

For x < 3 the circle at 3 is open. For x ≤ 3 it is filled.

Inequalities: integers and natural numbers

Use individual dots for allowed whole-number values. The number set tells you which dots to draw.

Solve −2x > −6. Give the natural-number solutions using N = {1, 2, 3, …}.

$x < 3$	Dividing by −2 reverses the inequality sign.
$x \in \{1, 2\}$	Only these natural numbers are strictly less than 3.

$\{1, 2\}$

Remember

For integers use …, −2, −1, 0, 1, 2. Check whether a question includes 0 in its natural-number convention.

Algebra 1

Try the questions before opening the separate answer key. Show one step per line.

10.1 · Fluency

Expand −3(2x − 5).

10.2 · Variation

Solve (x + 2)/3 = (x − 1)/2. Show the line that multiplies both sides by 6.

10.3 · Choose a method

Solve −3x ≥ −9 for positive natural numbers, then show the answer on a number line.

Check your work: are the sign, units and level of accuracy appropriate?

ALGEBRA 2

Choosing a factorising method

Recognise the structure first. Then choose the matching method.

FIRST CHECK
Is there a common factor in every term? If yes, take out the HCF before doing anything else.

2 TERMS	3 TERMS	4 TERMS
Difference of two squares?	In the form $ax^2 + bx + c$?	Can the terms be paired?
Perfect square - perfect square.	Use the guide number ac.	Factorise each pair.
Use (a - b)(a + b).	Split the middle term.	Take out the repeated bracket.

Highest common factor

Worked example — ONE CHANGE PER LINE

$18x^3y^2 - 12x^2y$	Start with the full expression.
$\text{HCF} = 6x^2y$	6 is the largest number dividing both coefficients; x^2y uses the smallest shared powers.
$18x^3y^2 \div 6x^2y = 3xy$	Divide the first term by the HCF.
$-12x^2y \div 6x^2y = -2$	Divide the second term by the HCF.

FINAL ANSWER
$6x^2y(3xy - 2)$

CHECK $6x^2y(3xy - 2) = 18x^3y^2 - 12x^2y$

ALGEBRA 2

Factorising special structures

Use a pattern only when every required feature is present.

Difference of two squares

PATTERN

Two perfect squares separated by a minus sign: $a^2 - b^2 = (a - b)(a + b)$.

Worked example — ONE CHANGE PER LINE

$25x^2 - 49$	Both terms are perfect squares and there is a minus sign.
$(5x)^2 - 7^2$	Write each term as a square.

FINAL ANSWER

$(5x - 7)(5x + 7)$

CHECK $(5x - 7)(5x + 7) = 25x^2 - 49$

WATCH OUT $25x^2 + 49$ is a sum of squares. This identity does not apply.

SECOND EXAMPLE

$9a^2 - 16 = (3a)^2 - 4^2 = (3a - 4)(3a + 4)$.

ALGEBRA 2

Factorising by grouping

Pair four terms, factorise each pair, and take out the bracket that repeats.

METHOD

Pair the first two terms and the last two terms. Factorise each pair. Then take out the repeated bracket.

Worked example — ONE CHANGE PER LINE

$2xy+6x+5y+15$	Start: there are four terms.
$(2xy+6x)+(5y+15)$	Pair the first two terms and the last two terms.
$2x(y+3)+5(y+3)$	Take 2x from the first pair and 5 from the second.

FINAL ANSWER

$(2x+5)(y+3)$

CHECK $(2x+5)(y+3)=2xy+6x+5y+15$

ALGEBRA 2

Quadratic factorising

Write a, b and c first. Then use the guide number ac.

GUIDE-NUMBER METHOD

For $ax^2 + bx + c$: calculate ac, find two numbers that multiply to ac and add to b, then split the middle term.

$6x^2+11x+3$	$a=6$	$b=11$	$c=3$

Guide-number example — ONE CHANGE PER LINE

$ac=6\times3=18$	Multiply a by c. This is the guide number.
$9\times2=18$ and $9+2=11$	Find two numbers that multiply to 18 and add to b = 11.
$6x^2+9x+2x+3$	Replace 11x with 9x + 2x.
$(6x^2+9x)+(2x+3)$	Pair the four terms.
$3x(2x+3)+1(2x+3)$	Factorise each pair.

FINAL ANSWER

$(3x+1)(2x+3)$

CHECK $(3x+1)(2x+3)=6x^2+11x+3$

EASIER CASE WHEN A = 1

For $x^2 + 5x + 6$, find two numbers that multiply to 6 and add to 5: 2 and 3. Therefore $x^2 + 5x + 6 = (x + 2)(x + 3)$.

ALGEBRA 2

Simplifying algebraic fractions

Factorise completely before cancelling. Restrictions come from the original denominator.

RULE

You may cancel complete factors joined by multiplication. You may not cancel separate terms joined by + or -.

Worked example

ONE CHANGE PER LINE

$\frac{x^2-16}{x^2+7x+12}$	Start with the original fraction.
$x^2-16=(x-4)(x+4)$	Factorise the numerator using difference of two squares.
$x^2+7x+12=(x+3)(x+4)$	Factorise the denominator.
$\frac{(x-4)(x+4)}{(x+3)(x+4)}$	Now the complete factor (x + 4) can be cancelled.
FINAL ANSWER $\frac{x-4}{x+3}$	Write the simplified fraction.

RESTRICTIONS

From the original denominator: x ≠ -3 and x ≠ -4. The restriction x ≠ -4 remains even though (x + 4) cancels.

YOU MAY CANCEL FACTORS	YOU MAY NOT CANCEL TERMS
$\frac{3x(x+2)}{x(x-1)}=\frac{3(x+2)}{x-1}$	$\frac{x+3}{x}\neq 3$

ALGEBRA 2

Adding and subtracting algebraic fractions

Use the lowest common denominator. Multiply the numerator and denominator by the same missing factor.

Addition

ONE CHANGE PER LINE

$\frac{2}{x}+\frac{3}{x+1}$	The denominators are different.
$\text{LCD}=x(x+1)$	The lowest common denominator contains both factors.
$\frac{2(x+1)}{x(x+1)}+\frac{3x}{x(x+1)}$	Give both fractions the common denominator.
$\frac{2x+2+3x}{x(x+1)}$	Combine the numerators only.

FINAL ANSWER

$$\frac{5x+2}{x(x+1)}$$

RESTRICTIONS $x \neq 0$ and $x \neq -1$. The denominators must be identical before the numerators are combined.

Subtraction

ONE CHANGE PER LINE

$\frac{4}{x}-\frac{1}{x+2}$	The denominators are different.
$\frac{4(x+2)}{x(x+2)}-\frac{x}{x(x+2)}$	Use the common denominator $x(x+2)$.
$\frac{4x+8-x}{x(x+2)}$	Expand the numerator. Keep the subtraction sign, then collect like terms.

FINAL ANSWER

$$\frac{3x+8}{x(x+2)}$$

RESTRICTIONS $x \neq 0$ and $x \neq -2$.

SIGN CHECK Use brackets so the minus applies to every term: $-(x+2) = -x-2$, not $-x+2$.

ALGEBRA 2

Multiplying and dividing algebraic fractions

Factorise first. Cancel complete factors before multiplying or dividing.

Multiplication

ONE CHANGE PER LINE

$\frac{x^2-9}{x} \times \frac{2x}{x+3}$	Factorise x^2 - 9 before cancelling.
$\frac{(x-3)(x+3)}{x} \times \frac{2x}{x+3}$	Write x^2 - 9 as (x - 3)(x + 3).
$(x-3) \times 2$	Cancel the complete factors x and (x + 3).

FINAL ANSWER

$2(x-3)$

DIVIDING FRACTIONS

Keep the first fraction, change ÷ to ×, and flip the second fraction. Here x ≠ 0 and x ≠ -3; the fraction being divided by must also not equal zero.

Division

ONE CHANGE PER LINE

$\frac{x^2-9}{x^2} \div \frac{x+3}{2x}$	Start with the division.
$\frac{x^2-9}{x^2} \times \frac{2x}{x+3}$	Change ÷ to × and flip the second fraction.
$\frac{(x-3)(x+3)}{x^2} \times \frac{2x}{x+3}$	Factorise x^2 - 9.
$\frac{2(x-3)}{x}$	Cancel x and the complete factor (x + 3).

FINAL ANSWER

$\frac{2(x-3)}{x}$

ALGEBRA 2

Changing the subject of a formula

Undo operations in reverse order and perform the same operation on both sides.

METHOD ORDER

Undo addition or subtraction, then multiplication or division, then powers or roots.

Example 1 · Make r the subject

ONE CHANGE PER LINE

$A = \pi r^2$	Start with the area formula.
$\frac{A}{\pi} = r^2$	Divide both sides by π.
FINAL ANSWER $r = \sqrt{\frac{A}{\pi}}$	Take the positive square root because a radius is positive.

Example 2 · Make h the subject

ONE CHANGE PER LINE

$V = \frac{\pi r^2 h}{3}$	Start with the volume formula.
$3V = \pi r^2 h$	Multiply both sides by 3.
FINAL ANSWER $h = \frac{3V}{\pi r^2}$	Divide both sides by πr^2.

ALGEBRA 2

When the variable appears twice

Collect the terms containing the required variable, then factorise that variable.

Example 3 · Make x the subject — ONE CHANGE PER LINE

$y=\frac{2x-1}{x+3}$	Start with the original formula.
$y(x+3)=2x-1$	Multiply both sides by (x + 3).
$xy+3y=2x-1$	Expand the left-hand side.
$xy-2x=-1-3y$	Take 2x from both sides and take 3y from both sides.
$x(y-2)=-1-3y$	Factorise x on the left-hand side.
$x=\frac{-1-3y}{y-2}$	Divide both sides by (y - 2).
FINAL ANSWER $x=\frac{1+3y}{2-y}$	Multiply numerator and denominator by -1 to simplify the signs.

FINAL CHECK Test a simple value: if y = 0, the rearranged formula gives x = 1/2, and (2 × 1/2 - 1)/(1/2 + 3) = 0. Also check that each operation was applied to the entire side.

Algebra 2: factorising, fractions and formulae

Try the questions before opening the separate answer key. Show one step per line.

11.1 · Fluency

Factorise $15x^2 - 10x$ completely.

11.2 · Variation

Simplify $2/(x + 1) - 1/(x - 1)$, stating restrictions.

11.3 · Choose a method

Make r the subject of $V = \pi r^2 h$, assuming $h > 0$ and r is a radius.

Check your work: are the sign, units and level of accuracy appropriate?

Simultaneous equations

Line up matching terms and add vertically when their coefficients are opposites.

Row	x-term	y-term	Right side
A	x	+y	7
+ B	x	−y	3
A + B	2x	0	10

Solve x + y = 7 and x − y = 3.

$(x+x)+(y-y)=7+3$	Add the x, y and right-hand columns separately.
$2x=10$	The y-terms cancel.
$x=5$	Divide both sides by 2.
$5+y=7$	Substitute x = 5 into equation A.

$x=5,\quad y=2$

Remember

If coefficients do not cancel, label the originals and write each row operation explicitly, e.g. A × (−2) then add B.

Quadratics by factorising

Rearrange to make one side zero before using the zero-product rule.

Solve x² − x − 12 = 0.

$(x-4)(x+3)=0$	The numbers −4 and 3 multiply to −12 and add to −1.
$x-4=0 \quad \text{or} \quad x+3=0$	At least one factor must be zero.

$x=4 \quad \text{or} \quad x=-3$

Remember

Working backwards, roots 4 and −3 give the monic equation (x − 4)(x + 3) = 0.

The quadratic formula

Write the equation as $ax^2 + bx + c = 0$ and keep the signs with the coefficients.

Solve $2x^2 + x - 4 = 0$.

$a=2,\quad b=1,\quad c=-4$	Read the three coefficients.
$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$	Use the quadratic formula.
$x=\dfrac{-1\pm\sqrt{1-4(2)(-4)}}{4}$	Substitute, with brackets around negative values.
$x=\dfrac{-1\pm\sqrt{33}}{4}$	

Remember

The discriminant $b^2 - 4ac$ determines the real roots: positive gives two distinct roots, zero gives one repeated root, and negative gives no real roots.

Dividing algebraic expressions

Divide the leading terms, multiply back, subtract and bring down.

Divide x³ + 2x² − 5x − 6 by x + 1.

$x^3 \div x = x^2$	Subtract x²(x + 1). The remaining expression is x² − 5x − 6.
$x^2 \div x = x$	Subtract x(x + 1). The remaining expression is −6x − 6.
$(-6x) \div x = -6$	Subtract −6(x + 1). The remainder is zero.

$x^2 + x - 6$

Remember

Check by multiplying the quotient by x + 1. The same method works for a quadratic dividend.

Doubling and tripling patterns

A constant multiplier is different from a constant difference.

Term number	1	2	3	4	5
Term value	4	12	36	108	324

The sequence starts 4, 12, 36, 108. Find the next term.

$12 \div 4 = 3, \quad 36 \div 12 = 3$	Each term is three times the previous term.
$108 \times 3 = 324$	Apply the same rule once more.

324

Remember

Linear: constant first difference. Quadratic: constant non-zero second difference. Doubling and tripling: multiply by 2 or 3 each step.

Build a quadratic from its roots

Each root r gives a factor x − r. Use both factors, then expand if asked.

Write a monic quadratic equation with roots −2 and 5.

$x-(-2)=x+2$	The negative root gives a plus sign.
$(x+2)(x-5)=0$	Both factors belong in the equation.
$x^2-5x+2x-10=0$	Expand the brackets.
$x^2-3x-10=0$	

Remember

Monic means the coefficient of x² is 1. Check by substituting each root. Non-zero multiples of this equation have the same roots.

Equations and Extensions

Try the questions before opening the separate answer key. Show one step per line.

12.1 · Fluency

Write a monic quadratic equation with roots 3 and −4.

12.2 · Variation

Solve $2x + y = 7$ and $x - y = 2$.

12.3 · Choose a method

Does $3x^2 - 2x + 2 = 0$ have real solutions? Justify without finding roots.

Check your work: are the sign, units and level of accuracy appropriate?

PATTERNS

Reading a pattern

Separate the position of a term from its value before doing any calculation.

THE ESSENTIAL DISTINCTION

n is the position number. T_n is the value at that position. In the sequence 5, 8, 11, 14, ... the number 3 is a position, while 11 is the value at that position.

Position n	1	2	3	4	5
Term value	5	8	11	14	17
1st difference		3	3	3	3

POSITION QUESTION

"Find the 20th term" gives n = 20. You must find T_{20}.

VALUE QUESTION

"Which term has value 62?" gives $T_n = 62$. You must find n.

BEFORE SOLVING

Write: I know _____. I need to find _____. This prevents a term value from being substituted as a position number.

How patterns are classified

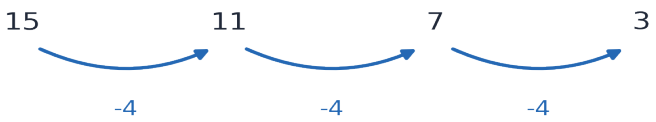
LINEAR	QUADRATIC	MULTIPLICATIVE
Constant first difference. Example: 5, 8, 11, 14, ...	Constant second difference. Example: 3, 8, 15, 24, ...	Constant ratio. Example: 3, 6, 12, 24, ...

PATTERNS

Finding the general term of a linear sequence

Use the common difference and $T_n = a + (n - 1)d$.

Worked example · 15, 11, 7, 3, ...



Common difference d = -4

STEP 1 · IDENTIFY A
The first term is a = 15.

STEP 2 · IDENTIFY D
Each term decreases by 4, so d = -4.

LINEAR-PATTERN FORMULA	MEANING OF THE LETTERS
$T_n = a + (n - 1)d$	a = first term d = common difference n = position number T_n = value at position n

Step 3 · Substitute and simplify — ONE CHANGE PER LINE

$T_n = a + (n - 1)d$	Start with the formula.
$T_n = 15 + (n - 1)(-4)$	Substitute a = 15 and d = -4.
$T_n = 15 - 4n + 4$	Expand the bracket carefully.
FINAL ANSWER $T_n = -4n + 19$	This is the general term.

WHY N - 1?
The first term is already present. Reaching term n requires n - 1 changes.

STEP 4 · CHECK
$T_1 = -4(1) + 19 = 15$ and $T_4 = -4(4) + 19 = 3$.

PATTERNS

Three different question types

Classify the task before substituting into the formula.

QUESTION TYPE	WHAT IS GIVEN?	WHAT DO YOU DO?
Find the rule	The sequence	Keep n. Simplify T_n.
Find a term value	A position n	Substitute the position for n.
Find the position	A value T_n	Set the rule equal to the value and solve for n.

A · Find the 20th term of 5, 8, 11, ... — ONE CHANGE PER LINE

I know: n = 20 I need: T_{20}	The given number is a position.
$T_n = 3n + 2$	Use the rule for this sequence.
$T_{20} = 3(20) + 2$	Substitute 20 for n.
FINAL ANSWER $T_{20} = 62$	The answer is a term value.

B · Which term has value 62? — ONE CHANGE PER LINE

I know: $T_n = 62$ I need: n	The given number is a value.
$62 = 3n + 2$	Set the rule equal to 62.
$60 = 3n$	Subtract 2 from both sides.
FINAL ANSWER $n = 20$	The answer is a position: 62 is the 20th term.

DO NOT DO THIS

T_{62} means the term at position 62. It does not mean "the term whose value is 62".

PATTERNS

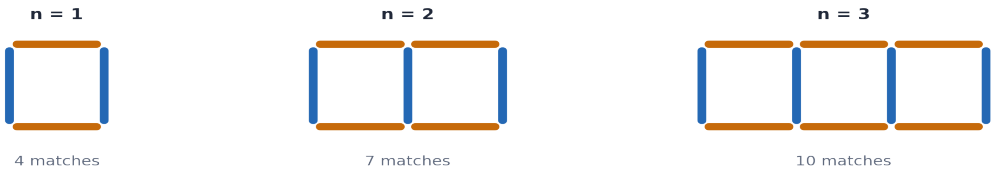
Patterns shown with diagrams or tables

Count each diagram, make a value row, and then use the same linear method.

RELIABLE METHOD

1. Label the diagrams n = 1, 2, 3, ... 2. Count the required objects. 3. Find the difference. 4. Build and check the rule.

Worked example · Matchsticks



Position n	1	2	3	4
Term value	4	7	10	13
1st difference		3	3	3

Build the rule — ONE CHANGE PER LINE

$a=4 \quad d=3$	Identify the first term and common difference.
$T_n=4+(n-1)3$	Use the linear-pattern formula.
$T_n=4+3n-3$	Expand.

FINAL ANSWER

$T_n=3n+1$

The 3n counts three matches for each repeated section; the extra 1 completes the first end. For 10 squares, $T_{10} = 31$.

When the table starts at n = 0

Position n	0	1	2	3
Term value	2	6	10	14
1st difference		4	4	4

Use the supplied positions — ONE CHANGE PER LINE

$T_n=4n+2$	At n = 0, the 4n part is zero, leaving 2.

FINAL ANSWER

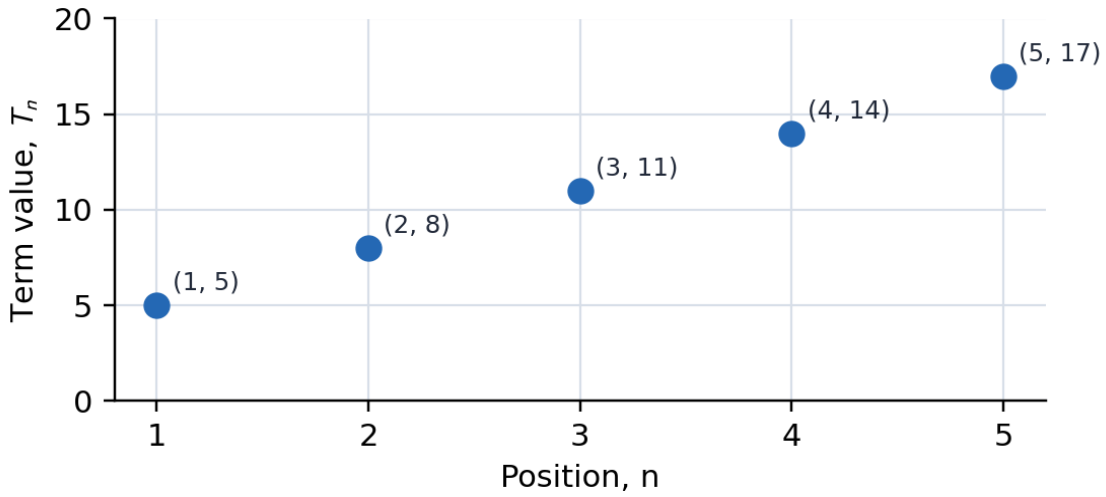
$T_n=4n+2$

Always use the position labels supplied in the question.

PATTERNS

Graphing a sequence

A sequence is discrete: plot separate points for whole-number positions.



HORIZONTAL AXIS
Position n. For an ordinary sequence, n = 1, 2, 3, ...

VERTICAL AXIS
Term value T_n. The point (3, 11) means the third term has value 11.

DO NOT JOIN THE POINTS
Values between positions are not terms of the sequence. A line may show the linear trend, but the sequence itself consists of the plotted points.

Reading a graph	ONE CHANGE PER LINE
$(n, T_n) = (4, 14)$	The x-coordinate is the position.
$T_4 = 14$	The y-coordinate is the term value.
$T_n = 3n + 2$	The plotted points follow this rule.
FINAL ANSWER $T_5 = 17$	At n = 5, the next plotted point is (5, 17).

PATTERNS

Recognising quadratic patterns

A quadratic pattern has changing first differences but constant second differences.

Position n	1	2	3	4
Term value	13	17	29	49
1st difference		4	12	20
2nd difference			8	8

FIRST DIFFERENCES
4, 12, 20 are not constant, so the pattern is not linear.

SECOND DIFFERENCES
8, 8 are constant, so a quadratic rule is appropriate.

KEY FACT
For $T_n = an^2 + bn + c$, the constant second difference is 2a. Therefore a is half the second difference.

Find the coefficient of n^2 — ONE CHANGE PER LINE

$\text{second difference} = 8$	Read the constant second difference.
$2a = 8$	The second difference equals 2a.
FINAL ANSWER $a = 4$	Now use two terms to find b and c.

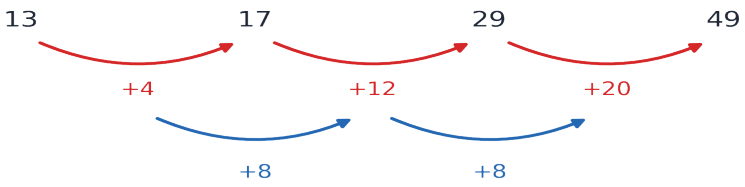
COMMON MISTAKE
Do not use the whole second difference as the coefficient of n^2. Divide it by 2 first.

PATTERNS

Finding a quadratic nth-term rule

Use $T_n = an^2 + bn + c$. Find a first, then solve two equations for b and c.

Step 1 · Find the second difference



Second difference = 8

Step 2 · Find a

Use second difference = 2a	ONE CHANGE PER LINE
$2a = 8$	The second difference is 8.
FINAL ANSWER $a = 4$	Substitute a = 4 into $T_n = an^2 + bn + c$.

Step 3 · Form two equations

USE THE FIRST TERM	USE THE SECOND TERM
$T_1 = 4(1)^2 + b(1) + c = 13$	$T_2 = 4(2)^2 + b(2) + c = 17$
$4 + b + c = 13$	$16 + 2b + c = 17$
$b + c = 9$	$2b + c = 1$

Step 4 · Solve the simultaneous equations

Eliminate c, then substitute	ONE CHANGE PER LINE
$(b + c) - (2b + c) = 9 - 1$	Subtract the second equation from the first.
$-b = 8$	The c terms cancel.
$b = -8$	Multiply both sides by -1.
$-8 + c = 9$	Substitute b = -8 into b + c = 9.
$c = 17$	Add 8 to both sides.
FINAL ANSWER $T_n = 4n^2 - 8n + 17$	Check: $T_3 = 4(3^2) - 8(3) + 17 = 29$.

PATTERNS

Choosing the correct method

Use the structure of the pattern and the wording of the question.

WHAT YOU SEE	WHAT IT MEANS	NEXT STEP
Constant first difference	Linear pattern	Use $T_n = a + (n - 1)d$.
Constant second difference	Quadratic pattern	Use 2a, then form two equations for b and c.
Constant ratio	Multiplicative pattern	Continue by multiplying by the ratio.
"Find the 30th term"	A position is given	Substitute n = 30.
"Which term is 143?"	A value is given	Set $T_n = 143$ and solve for n.

Final paired example · $T_n = 4n - 5$ — ONE CHANGE PER LINE

$T_{12} = 4(12) - 5$	"Find the 12th term" gives n = 12.
$T_{12} = 43$	The answer is a value.
$143 = 4n - 5$	"Which term has value 143?" gives $T_n = 143$.
$148 = 4n$	Add 5 to both sides.
FINAL ANSWER $n = 37$	143 is the 37th term, not the 143rd term.

FINAL CHECK

1. Did I distinguish position from value? 2. Did I use the correct difference? 3. Does my answer have the right form: rule, value or position? 4. Does it reproduce the original terms?

Patterns

Try the questions before opening the separate answer key. Show one step per line.

13.1 · Fluency

Find the general term of 12, 17, 22, 27, ...

13.2 · Variation

Which term of that sequence has value 72?

13.3 · Choose a method

Classify 3, 9, 27, 81, ... and 2, 5, 10, 17, ..., giving a reason for each.

Check your work: are the sign, units and level of accuracy appropriate?

FUNCTIONS AND GRAPHS

What is a function?

A function gives exactly one output for every allowed input.



READ F(X) CORRECTLY

f(x) means the output of function f when the input is x. It does not mean f multiplied by x.

Worked example · f(x) = 2x + 3 — ONE CHANGE PER LINE

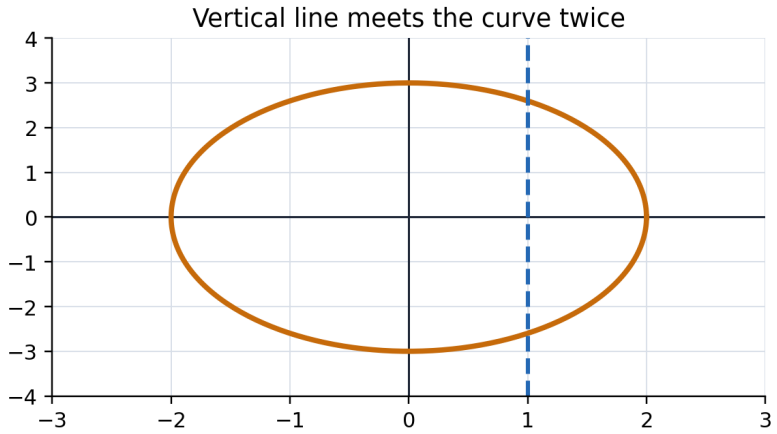
$f(4) = 2(4) + 3$	Replace x by 4.
$f(4) = 8 + 3$	Multiply before adding.
FINAL ANSWER $f(4) = 11$	Input 4 produces output 11.

FUNCTION

Each input has exactly one output.

NOT A FUNCTION

At least one input has more than one output.



VERTICAL-LINE TEST

If a vertical line can meet a graph more than once, the graph is not a function.

Finding outputs and inputs

First decide which quantity is known: the input x or the output f(x).

FIND AN OUTPUT
A value of x is given. Substitute it into the rule.

FIND AN INPUT
A value of f(x) is given. Form an equation and solve for x.

A · Find f(-3) when f(x) = x² - 2x + 1 — ONE CHANGE PER LINE

$f(-3) = (-3)^2 - 2(-3) + 1$	Use brackets around the negative input.
$f(-3) = 9 + 6 + 1$	Square first, then multiply.
FINAL ANSWER $f(-3) = 16$	The output is 16.

B · For what x is f(x) = 16? — ONE CHANGE PER LINE

$x^2 - 2x + 1 = 16$	Set the rule equal to the given output.
$x^2 - 2x - 15 = 0$	Bring all terms to one side.
$(x - 5)(x + 3) = 0$	Factorise.
FINAL ANSWER $x = 5 \quad \text{or} \quad x = -3$	Two different inputs can have the same output.

IMPORTANT
A function may give the same output for different inputs. It is still a function because each individual input has only one output.

FUNCTIONS AND GRAPHS

Domain and range

The domain contains allowed inputs. The range contains outputs actually produced.

DOMAIN
The set of possible x-values.

RANGE
The set of resulting f(x)-values.

INPUT x	RULE $f(x)=x^2$	OUTPUT f(x)
-2	$(-2)^2$	4
-1	$(-1)^2$	1
0	0^2	0
1	1^2	1
2	2^2	4

State the domain and range — ONE CHANGE PER LINE

$\text{Domain} = \{-2, -1, 0, 1, 2\}$	List every allowed input once.
$\text{Range} = \{0, 1, 4\}$	List each output once; do not repeat 1 or 4.

RESTRICTIONS
A denominator cannot be zero. A square root cannot contain a negative number when working with real numbers.

Example · g(x) = 1/(x - 3) — ONE CHANGE PER LINE

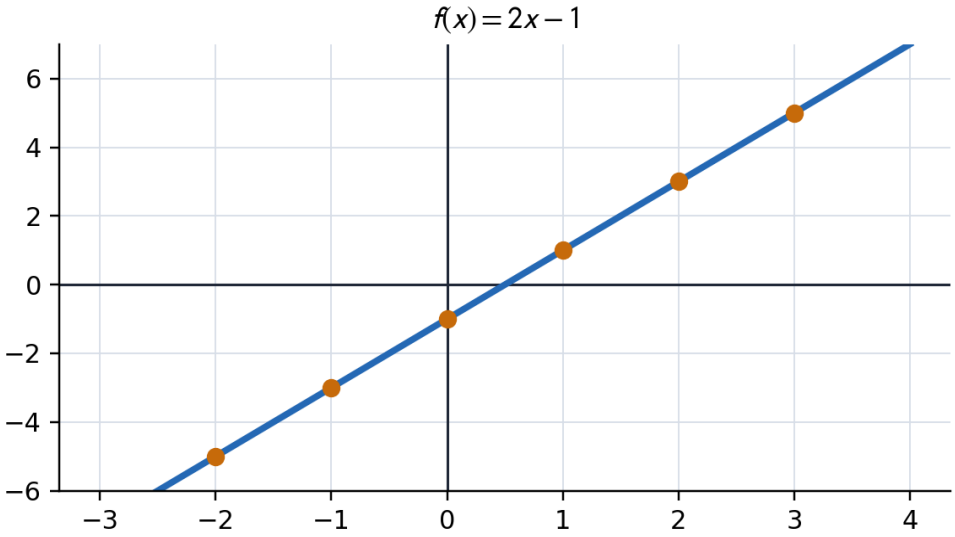
$x - 3 \neq 0$	The denominator cannot equal zero.
$x \neq 3$	Solve the restriction.
FINAL ANSWER $\text{Domain}: \ x \in \mathbb{R}, \ x \neq 3$	Every real input except 3 is allowed.

FUNCTIONS AND GRAPHS

Drawing a graph from a function

Choose x-values, calculate y=f(x), plot the coordinates and draw the graph.

x	-2	-1	0	1	2	3
f(x)=2x-1	-5	-3	-1	1	3	5
(x,y)	(-2,-5)	(-1,-3)	(0,-1)	(1,1)	(2,3)	(3,5)



GRAPHING CHECKLIST

Label both axes. Use equal, sensible scales. Plot each point accurately. A straight-line function may be joined with a ruler. State the equation beside the graph.

Read a point	ONE CHANGE PER LINE
$(2, 3)$	The horizontal coordinate is the input x=2.
$f(2) = 3$	The vertical coordinate is the output.
FINAL ANSWER $2(2) - 1 = 3$	The point satisfies the rule.

FUNCTIONS AND GRAPHS

Reading intercepts and roots

Intercepts occur where a graph meets an axis.

Y-INTERCEPT
Set x=0. The point lies on the vertical axis.

ROOT OR X-INTERCEPT
Set f(x)=0. The point lies on the horizontal axis.

For f(x)=2x-6 — ONE CHANGE PER LINE

$f(0)=2(0)-6=-6$	Set x=0 for the y-intercept.
$2x-6=0$	Set f(x)=0 for the root.
$2x=6$	Add 6 to both sides.
FINAL ANSWER $y\text{–intercept}=(0,\ -6),\quad \text{root}=3$	The x-intercept is (3,0).

FROM A GRAPH
Read a root from the x-axis and a y-intercept from the y-axis. Always give a coordinate if the question asks for a point.

Sign of a function

ABOVE THE X-AXIS
f(x)>0.

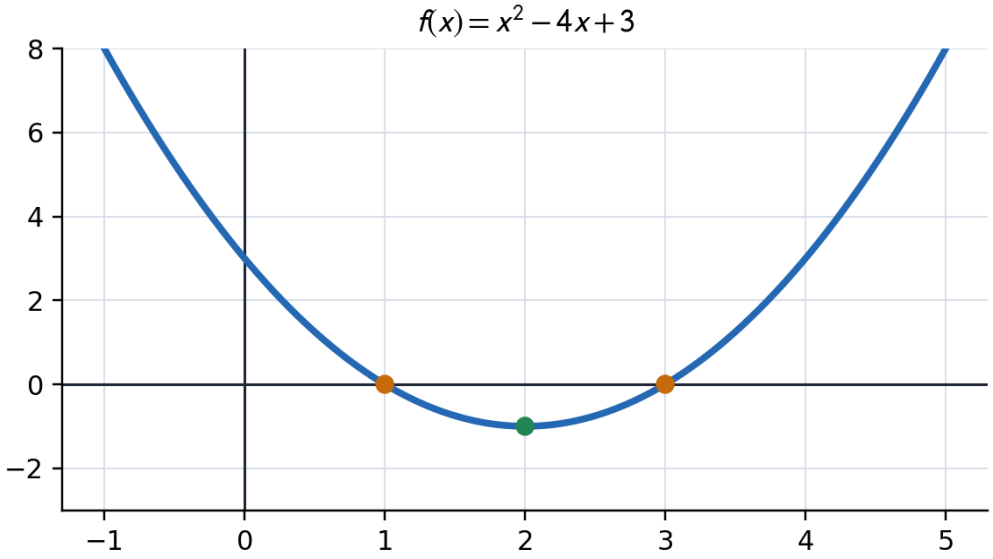
BELOW THE X-AXIS
f(x)<0.

AT THE X-AXIS
f(x)=0 exactly at a root.

FUNCTIONS AND GRAPHS

Quadratic graphs

A quadratic graph is a parabola with roots, a turning point and an axis of symmetry.



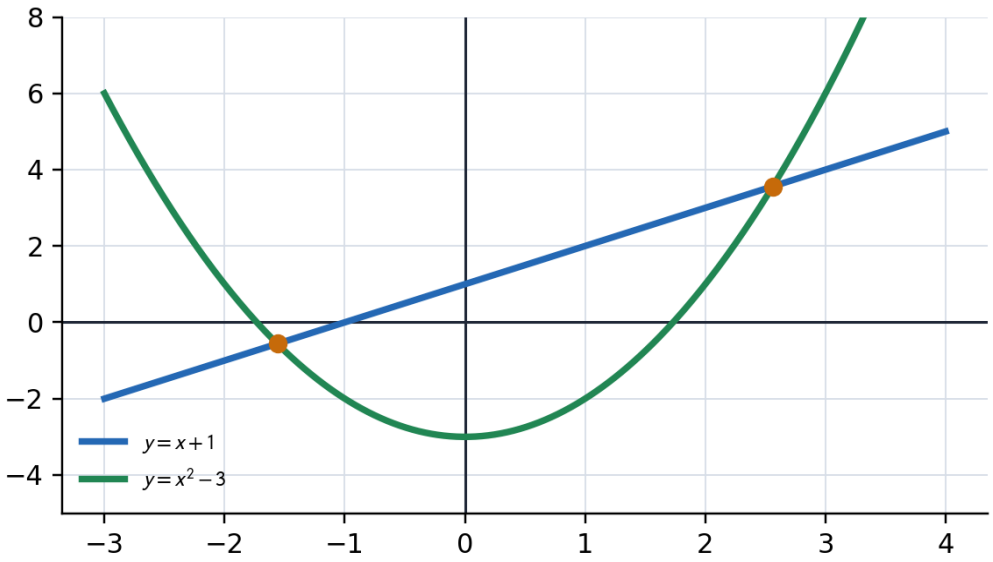
FEATURE	HOW TO READ IT	FOR THIS GRAPH
Roots	Where f(x)=0	x=1 and x=3
y-intercept	Where x=0	(0,3)
Minimum	Lowest point	(2,-1)
Axis of symmetry	Vertical line through turning point	x=2

SHAPE

If the coefficient of x^2 is positive, the graph opens upward and has a minimum. If it is negative, it opens downward and has a maximum.

Where two graphs intersect

At an intersection, both functions have the same x-value and the same y-value.



GRAPHICAL SOLUTION

The x-coordinates of the crossing points solve f(x)=g(x). Read every crossing, not just the first.

Solve x+1 = x²-3 algebraically — ONE CHANGE PER LINE

$x+1=x^2-3$	Set the two rules equal.
$x^2-x-4=0$	Bring all terms to one side.
$x=\frac{1\pm\sqrt{17}}{2}$	Use the quadratic formula.

FINAL ANSWER

$x\approx -1.56$ or $x\approx 2.56$

These match the two crossings on the graph.

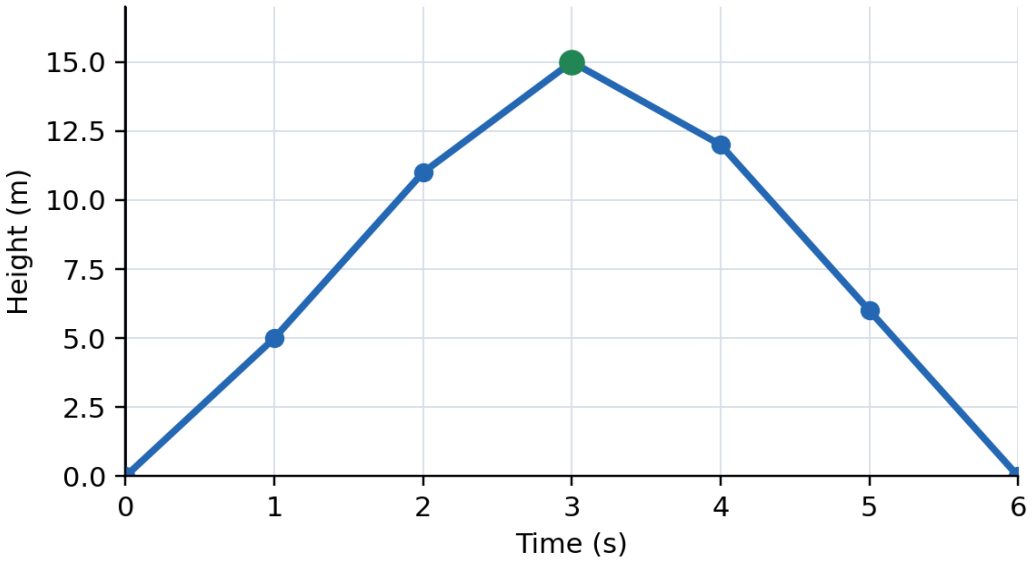
APPROXIMATE ANSWERS

When reading a graph, use the scale carefully and state the requested accuracy, for example x is approximately 2.6 to one decimal place.

FUNCTIONS AND GRAPHS

Functions in context

Interpret both coordinates and include units.



Reading the height-time graph	ONE CHANGE PER LINE
$(3, 15)$	At 3 seconds, the height is 15 metres.
maximum height $= 15$ m	Read the vertical coordinate of the highest point.
$h(0) = 0, \quad h(6) = 0$	The object is at ground level at 0 s and 6 s.
FINAL ANSWER maximum at $t = 3$ s	The turning point gives both the time and the greatest height.

QUESTION	LOOK FOR
Find f(4)	Substitute x=4.
Solve f(x)=7	Set the rule equal to 7 and solve.
Find a root	Set f(x)=0 or read the x-axis.
Find the maximum	Read the highest turning point.
Solve f(x)=g(x)	Find every intersection.

Functions and graphs

Try the questions before opening the separate answer key. Show one step per line.

14.1 · Fluency

For f(x) = 3x − 6, find f(4).

14.2 · Variation

Find the two intercepts of y = 2x − 8.

14.3 · Choose a method

Sketch f(x) = x + 3 and g(x) = 2x. Use the sketch to identify where f(x) is above g(x).

Check your work: are the sign, units and level of accuracy appropriate?

Units, time and scale

Square the conversion factor for areas and cube it for volumes.

A trip of 54 km takes 45 minutes. Find the average speed in km/h.

$45 \div 60 = 0.75$	Convert minutes to hours.
$54 \div 0.75 = 72$	Average speed is total distance divided by total time.

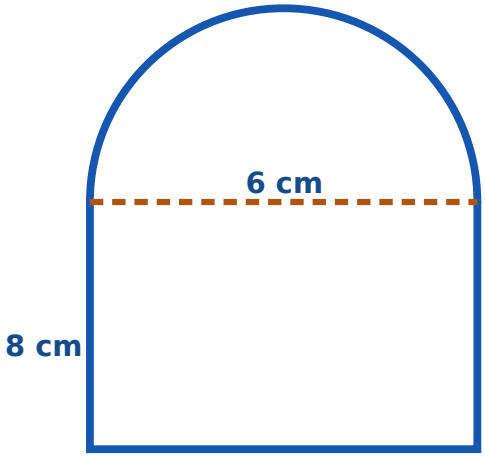
$72\,\text{km/h}$

Remember

1 m² = 10,000 cm²; 1 cm³ = 1 mL. At 1 : 500, a 4 cm line represents 2,000 cm, or 20 m.

Composite areas and perimeters

Split the shape into simpler pieces. Count only the outside edges for perimeter.



The dashed diameter is a dividing line, not an outside edge.

A semicircle of diameter 6 cm sits on the short side of an 8 cm by 6 cm rectangle. Find its area and perimeter.

$A = 8(6) + \frac{\pi(3)^2}{2}$	Add rectangle and semicircle.
$P = 8 + 8 + 6 + 3\pi$	The shared diameter is inside the figure.

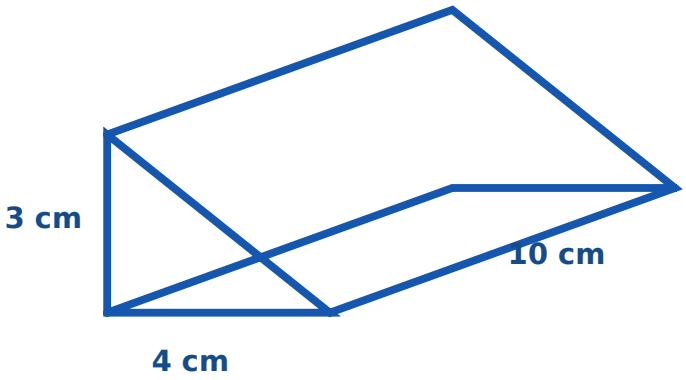
$A = 48 + 4.5\pi, \quad P = 22 + 3\pi$

Remember

The area is in cm² and the perimeter is in cm.

Prisms and nets

The volume of a prism is cross-section area × prism length.



The blue outline shows a triangular end and the length of the prism.

A right triangular prism has a 3–4–5 cm end and is 10 cm long. Find volume and total surface area.

$A=\frac{1}{2}(3)(4)=6$	Find one triangular end.
$V=6(10)=60$	The same cross-section runs along the prism.
$S=2(6)+10(3+4+5)=132$	Two triangle ends and three rectangular faces.

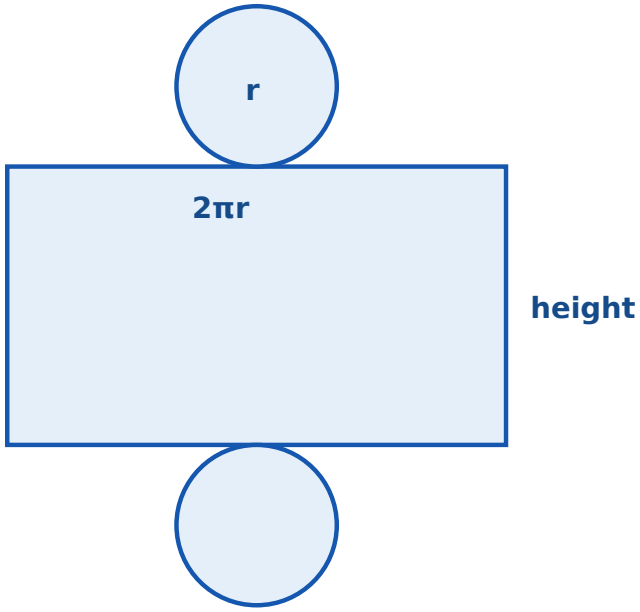
$60\,\text{cm}^3$ and $132\,\text{cm}^2$

Remember

A net has three rectangles, side by side, with the two triangular ends attached to opposite edges.

Cylinders and spheres

A cylinder net has two discs and one rectangle. A sphere has no flat face.



Unrolling the curved surface gives a rectangle whose long edge is one circumference.

Find a closed cylinder’s volume and surface area if r = 3 cm and h = 5 cm.

$V=\pi(3)^2(5)=45\pi$	Base area times height.
$S=2\pi(3)^2+2\pi(3)(5)=48\pi$	Two circular ends plus the curved surface.

$45\pi\,\text{cm}^3$ and $48\pi\,\text{cm}^2$

Remember

Cylinder net rectangle: 2πr by h. For a sphere: volume = 4πr³/3; surface area = 4πr². Remove hidden contact faces for composite solids.

Capacity and spherical solids

Convert cubic centimetres to millilitres before converting to litres.

Find the volume and surface area of a sphere of radius 6 cm.

$V=\frac{4}{3}\pi(6)^3=288\pi$	Use the sphere volume formula.
$S=4\pi(6)^2=144\pi$	Use the sphere surface area formula.

$288\pi\,\text{cm}^3$ and $144\pi\,\text{cm}^2$

Remember

A hemisphere has half the volume; for its total surface area add the circular base to half the sphere’s surface area.

Measurement

Try the questions before opening the separate answer key. Show one step per line.

15.1 · Fluency

Convert 0.75 m² to cm².

15.2 · Variation

A triangular prism has a cross-section area of 12 cm² and length 8 cm. Find its volume.

15.3 · Choose a method

A closed cylinder has radius 4 cm and height 9 cm. Find its surface area in terms of π.

Check your work: are the sign, units and level of accuracy appropriate?

Angles and parallel lines

Mark the parallel lines and trace the transversal before naming an angle fact.

Two alternate interior angles are 3x + 10° and 70°. Find x.

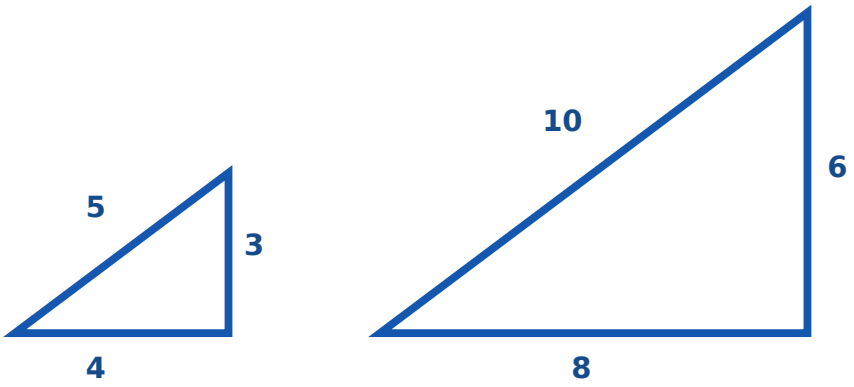
$3x + 10 = 70$	Alternate angles are equal on parallel lines.
$3x = 60$	Subtract 10.
$x = 20$	

Remember

Corresponding angles are equal. Co-interior angles sum to 180°. Three interior triangle angles sum to 180°.

Congruent and similar triangles

Congruent triangles have the same shape and size. Similar triangles have the same shape.



The larger triangle’s corresponding sides are twice as long.

One triangle has sides 3, 4, 5. Another has sides 6, 8, 10. Compare them.

$\frac{6}{3} = \frac{8}{4} = \frac{10}{5} = 2$

Match shortest to shortest, and so on.

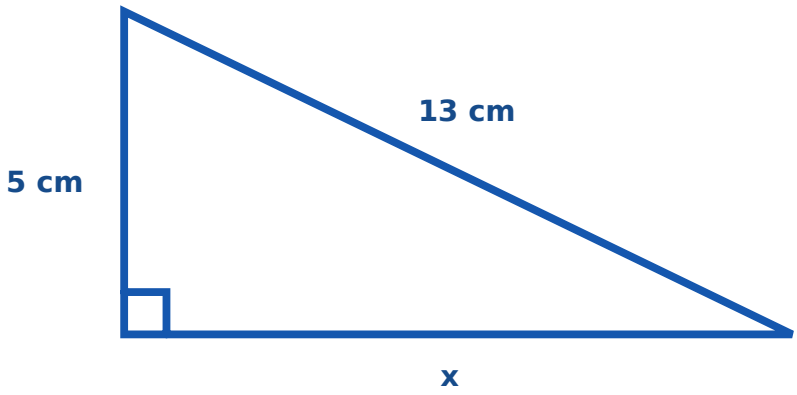
Similar, not congruent

Remember

For congruence look for SSS, SAS, ASA or RHS. Equal angles give similarity. Lengths scale by k; areas scale by k².

Pythagoras: finding a missing side

Only use this theorem in a right-angled triangle. The hypotenuse is opposite the right angle.



The little square marks the 90° angle.

The hypotenuse is 13 cm and one shorter side is 5 cm. Find the other shorter side.

$x^2 + 5^2 = 13^2$	Put the hypotenuse on its own side.
$x^2 = 169 - 25 = 144$	Subtract the square of the known shorter side.
$x = \sqrt{144}$	Take the positive square root for a length.

$12\,\text{cm}$

Remember

To check the converse, test the longest side: if its square equals the sum of the other squares, the triangle is right-angled.

Proof, converses and midpoints

State facts, show how they connect, then state what you proved.

M and N are the midpoints of AB and AC. If BC = 14 cm, find MN.

$AM = MB, \quad AN = NC$	This is what midpoint means.
$MN = \frac{1}{2}BC$	The segment joining the midpoints is parallel to BC and half as long.
$MN = 7\,\text{cm}$	

Remember

An axiom is a starting assumption; a theorem is proved; a corollary follows from another result. A converse swaps the condition and conclusion and needs its own justification.

Exterior angles and isosceles triangles

Equal sides have equal opposite angles. Vertically opposite angles are equal.

An isosceles triangle has a 44° angle between its equal sides. Find the exterior angle at a base vertex.

$180 - 44 = 136$	The triangle angles sum to 180°.
$136 \div 2 = 68$	Both base angles are equal.
$180 - 68 = 112$	The base angle and the exterior angle make a straight line.

112°

Remember

Check using the exterior angle theorem: 44° + 68° = 112°. Equal base angles also imply that the opposite sides are equal.

Proving a parallelogram fact

Use a diagonal to turn a quadrilateral problem into a pair of triangle problems.

Explain why opposite sides of parallelogram ABCD are equal.

$\angle BAC = \angle DCA$	AB is parallel to CD; AC is a transversal.
$\angle BCA = \angle DAC$	BC is parallel to AD; AC is a transversal.
$AC = CA$	The triangles share their included side.

$\triangle ABC \cong \triangle CDA$

Remember

By ASA the triangles are congruent, so AB = CD and BC = DA. The diagonals of a parallelogram bisect each other. Both opposite side pairs being equal also gives a parallelogram.

Triangles and Proof

Try the questions before opening the separate answer key. Show one step per line.

16.1 · Fluency

Two triangle angles are 43° and 79°. Find the third.

16.2 · Variation

Check whether sides 7, 24 and 25 form a right triangle.

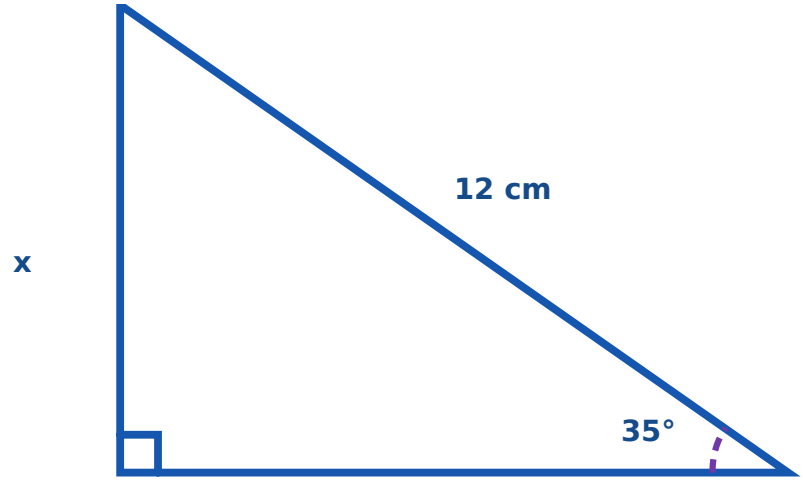
16.3 · Choose a method

Two similar triangles have corresponding sides 6 cm and 9 cm. The smaller area is 20 cm². Find the larger area.

Check your work: are the sign, units and level of accuracy appropriate?

Choosing sin, cos or tan

Choose the angle first. The opposite side is across from that angle.



The 35° angle is at the right-hand tip; the square marks the separate right angle.

Find the vertical side when the hypotenuse is 12 cm and the angle at the right-hand tip is 35°.

$\sin 35^\circ = \frac{x}{12}$	Use sine: opposite over hypotenuse.
$x = 12\sin 35^\circ$	Multiply by 12.
$x \approx 6.88\,\text{cm}$	

Remember

SOH CAH TOA: sin = opposite/hypotenuse; cos = adjacent/hypotenuse; tan = opposite/adjacent. Use degree mode.

Finding an angle

Use inverse trig when the sides are known and the angle is unknown.

Find θ if the opposite side is 7 cm and the adjacent side is 11 cm.

$\tan\theta = \frac{7}{11}$	Both known sides are legs.
$\theta = \tan^{-1}\left(\frac{7}{11}\right)$	In degree mode, use inverse tan.

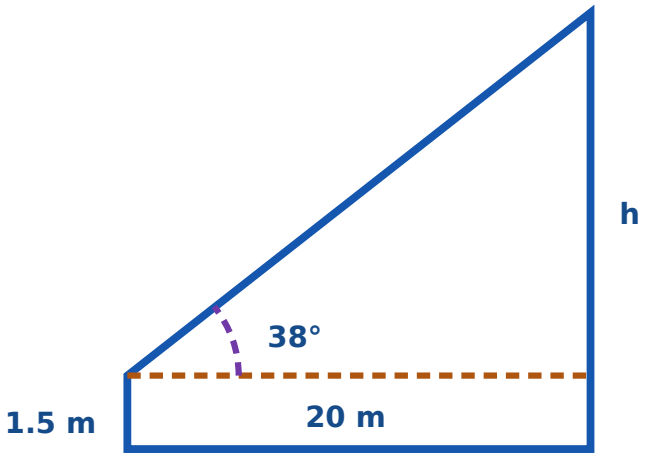
$\theta \approx 32.47^\circ$

Remember

An angle of elevation is measured up from the horizontal, not from the vertical.

Angles of elevation and depression

Draw a horizontal from the observer before adding the sightline.



The dashed horizontal is at the observer’s eye level.

A student is 20 m from a tower. The angle of elevation is 38°. Their eye is 1.5 m above the ground. Find tower height.

$\tan 38^\circ = \frac{h}{20}$	h is the height above eye level.
$h = 20\tan 38^\circ \approx 15.63$	Find the rise from the horizontal.
$15.63 + 1.5 = 17.13$	Add eye height.

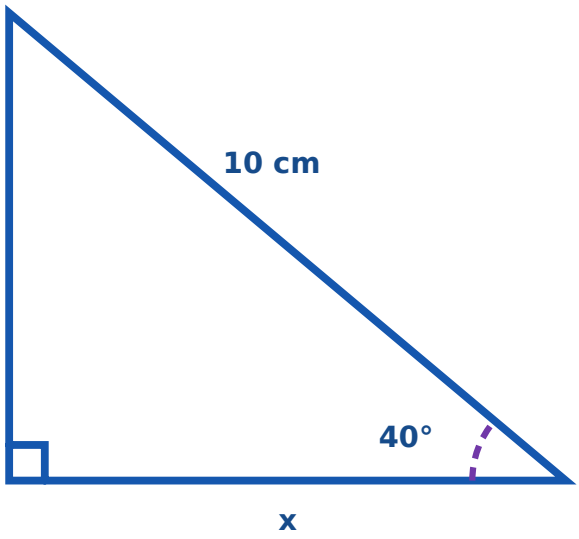
About $17.13\,\text{m}$

Remember

Depression is measured downward from a horizontal. Parallel horizontals give equal alternate angles.

Using cosine

The adjacent side touches the marked angle and is not the hypotenuse.



Here x touches the 40° angle and is not the sloping side.

The hypotenuse is 10 cm and the angle at the right-hand tip is 40°. Find the adjacent leg.

$\cos 40^\circ = \frac{x}{10}$	Cosine uses adjacent over hypotenuse.
$x = 10\cos 40^\circ$	Multiply by 10.
$x \approx 7.66\,\text{cm}$	

Remember

Common mistake: choosing the ratio by the look of the triangle. Label opposite, adjacent and hypotenuse from the marked angle first.

Trigonometry

Try the questions before opening the separate answer key. Show one step per line.

17.1 · Fluency

A right triangle has hypotenuse 14 cm and angle 36°. Find the adjacent leg to two decimal places.

17.2 · Variation

The opposite and adjacent legs are 9 cm and 12 cm. Find the angle to one decimal place.

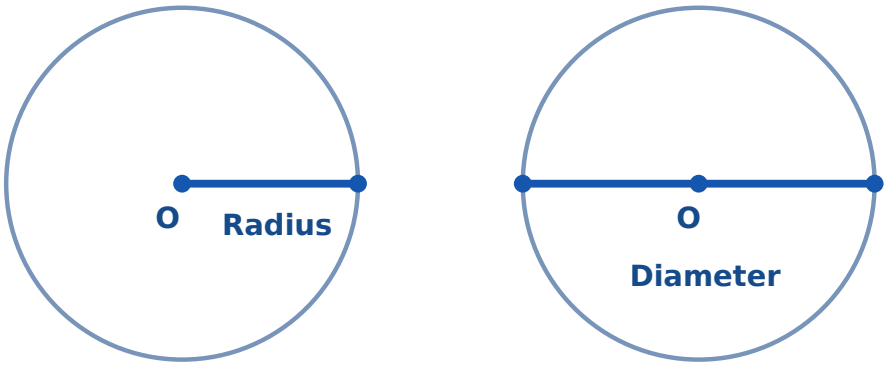
17.3 · Choose a method

A ladder reaches 4.2 m up a wall and is 1.8 m out from the foot. Find its angle with the ground.

Check your work: are the sign, units and level of accuracy appropriate?

Circumference and area

Circumference is the distance around. Area is the space inside.



Radius: centre to circle. Diameter: circle to circle through the centre.

Find the circumference and area when the diameter is 10 cm.

$r = \frac{10}{2} = 5$	Halve the diameter.
$C = 2\pi r = 2\pi(5)$	Circumference has length units.
$A = \pi r^2 = \pi(5)^2$	Area has square units.

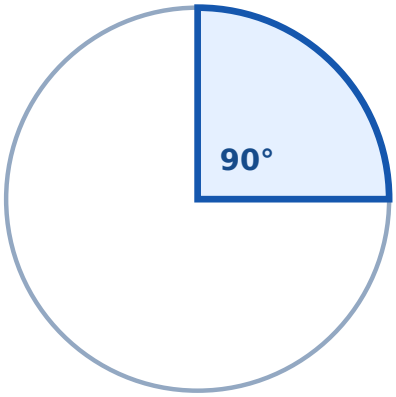
$C = 10\pi \text{ cm}, \quad A = 25\pi \text{ cm}^2$

Circle vocabulary

A chord joins two points on the circle. An arc is a curved part of the circumference. A diameter is a chord through the centre.

Arcs and sectors

Both the arc and the sector are fractions of the full circle.



A 90° sector takes up one quarter of the circle.

A 90° sector has radius 8 cm. Find its arc length and area.

$\frac{90}{360}=\frac{1}{4}$	Find the fraction of a full turn.
$L=\frac{1}{4}(2\pi\cdot 8)=4\pi$	Use the fraction of the circumference.
$A=\frac{1}{4}(\pi\cdot 8^2)=16\pi$	Use the same fraction of the circle's area.

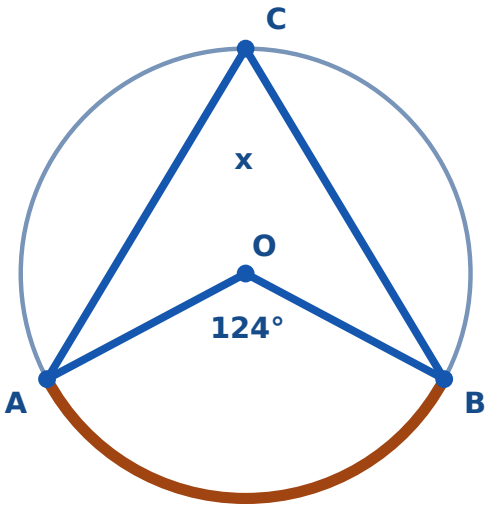
$L=4\pi$ cm, $A=16\pi$ cm^2

Exam application

For the perimeter of a sector, add the arc length and both radii. In the example this is 4π + 16 cm. A semicircle also has a straight edge: its diameter.

Centre and circumference

Theorem 19: the angle at the centre is twice the angle at the circumference standing on the same arc.



Both angles look towards the orange arc AB. The vertices are O and C.

The central angle AOB is 124°. Find angle ACB.

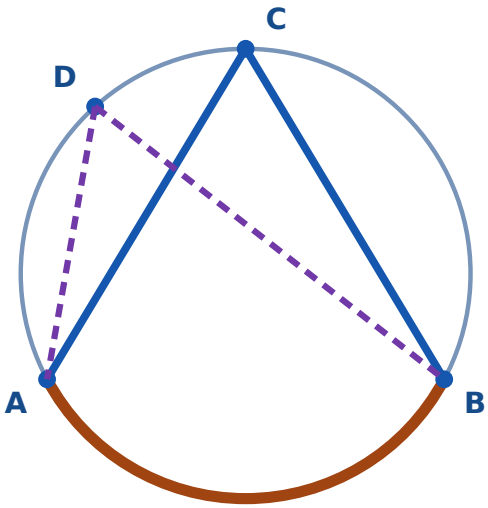
$2x = 124^\circ$	Both angles stand on the highlighted minor arc.
$x = \frac{124^\circ}{2}$	Halve the central angle.
$x = 62^\circ$	

Minor or major arc?

124° is the smaller central angle here. A central angle of 240° is reflex and stands on a major arc, not a minor arc.

Equal angles on the same arc

Corollary 2: angles at the circumference standing on the same arc are equal.



C and D are on the same side of chord AB. Both angles stand on orange arc AB.

Angle ACB is 62°. Find angle ADB.

$\angle ADB = \angle ACB$	Same endpoints and same arc. Each is half the same central angle.

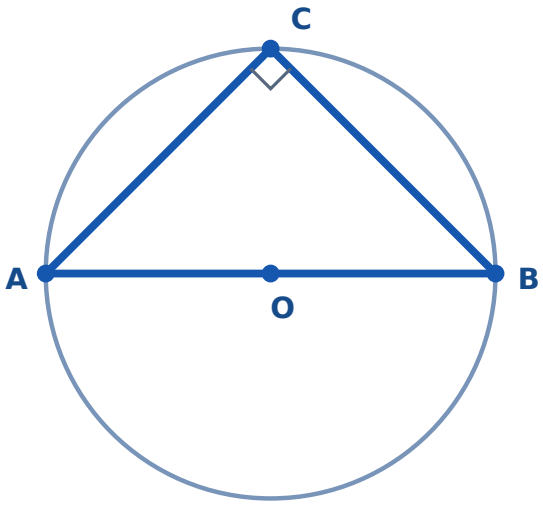
$\angle ADB = 62^\circ$

Converse

If C and D are on the same side of AB and angle ACB equals angle ADB, then A, B, C and D lie on one circle. A shared chord alone is not enough.

The angle in a semicircle

Corollaries 3 and 4: a diameter gives a right angle, and a right angle standing on a chord identifies a diameter.



AB passes through O. The right-angle mark is at C, on the circumference.

AB is a diameter. Find angle ACB.

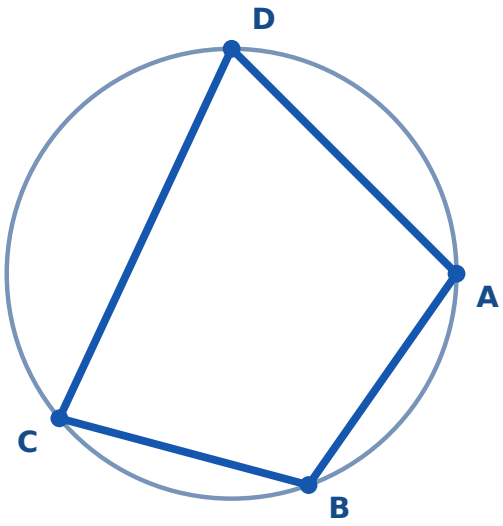
$\angle AOB = 180^\circ$	The diameter gives a straight angle at the centre.
$\angle ACB = \frac{180^\circ}{2}$	Use the centre and circumference rule.
$\angle ACB = 90^\circ$	

Use it backwards

If C lies on the circle and angle ACB is 90°, then AB is a diameter.

Cyclic quadrilaterals

Corollary 5: when all four vertices lie on one circle, opposite angles add to 180°.



A and C are opposite vertices. B and D are the other opposite pair.

ABCD is cyclic. Angle A is 100°. Find angle C.

$100^\circ + C = 180^\circ$	A and C are opposite. The two central angles for their arcs make 360°; halve that to get 180°.
$C = 180^\circ - 100^\circ$	Subtract the known angle.

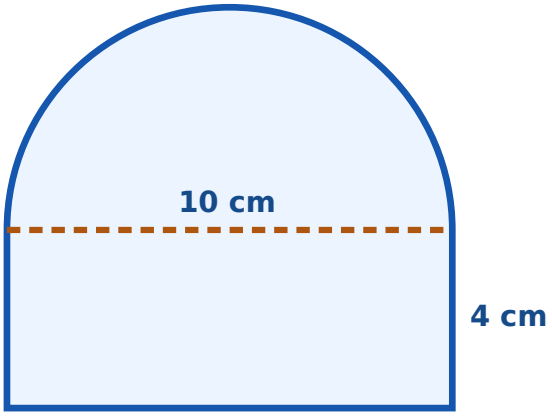
$C = 80^\circ$

Converse

If opposite angles of a convex quadrilateral add to 180°, it is cyclic.

Composite shapes with circles

Area measures the region. Perimeter measures only the outside boundary.



A semicircle sits on top of a 10 cm by 4 cm rectangle. The dashed join is inside the shape.

Find the exact area and perimeter.

$r = 10 \div 2 = 5$	Halve the diameter.
$A = 10(4) + \frac{1}{2}\pi(5)^2$	Add the rectangle and semicircle areas.
$P = 10 + 4 + 4 + \frac{1}{2}(2\pi \cdot 5)$	Add the three outside straight sides and the curved edge. Do not count the dashed join.

$A = 40 + 12.5\pi \,\text{cm}^2, \quad P = 18 + 5\pi \,\text{cm}$

Circles

Try the questions before opening the separate answer key. Show one step per line.

18.1 · Fluency

A circle has diameter 12 cm. Find its circumference and area in terms of π.

18.2 · Variation

An arc subtends 146° at the centre. Find the angle on the same arc at the circumference.

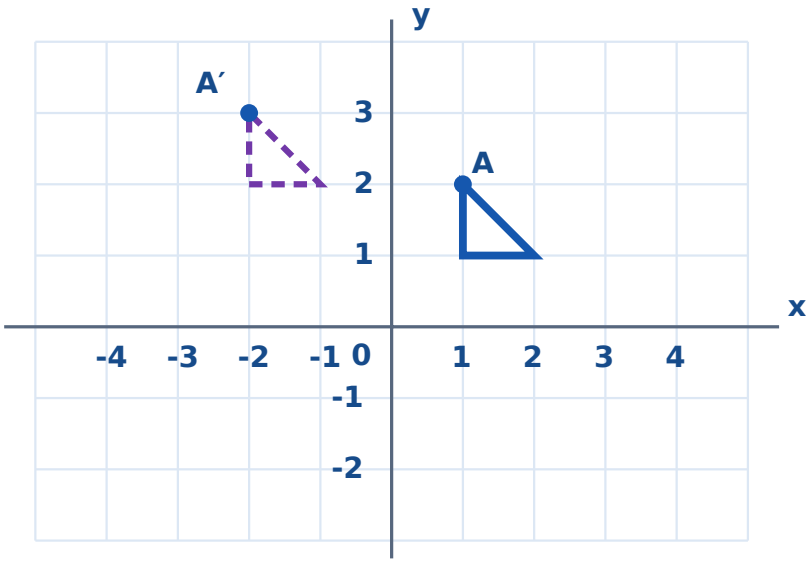
18.3 · Choose a method

A cyclic quadrilateral has one angle of 112°. Find the angle opposite and state the fact used.

Check your work: are the sign, units and level of accuracy appropriate?

Translations

A translation slides every point the same distance in the same direction.



A(1, 2) moves 3 left and 1 up to A′(−2, 3). The whole triangle makes the same move.

Translate A(1, 2) by 3 left and 1 up.

$x = 1 - 3 = -2$	Left means subtract from x.
$y = 2 + 1 = 3$	Up means add to y.

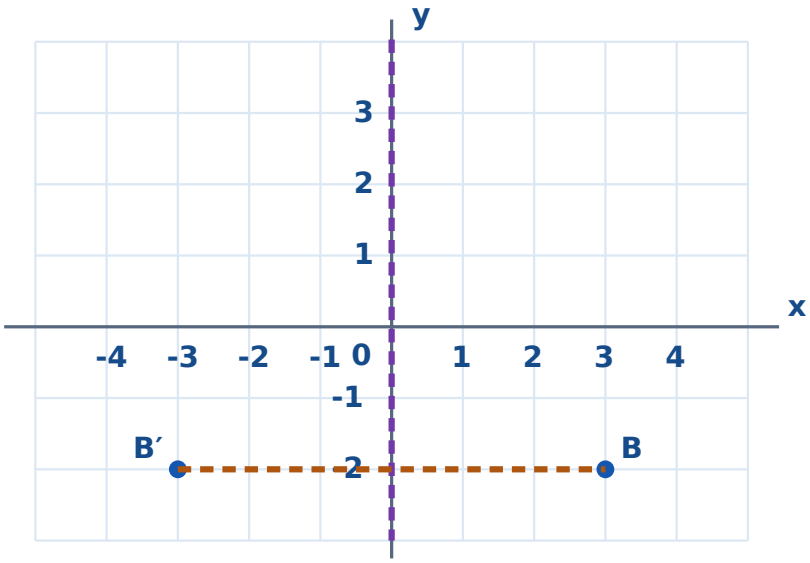
$A' = (-2, 3)$

Check

The image has the same size and shape as the original. The prime mark names the image point.

Reflections

A reflection flips the shape across a mirror line.



B and B′ are the same perpendicular distance from the y-axis.

Reflect B(3, −2) in the y-axis.

$x = -3$	Change the sign of the x-coordinate.
$y = -2$	The y-coordinate stays the same.

$B' = (-3, -2)$

Other mirror lines

In the x-axis, keep x and change the sign of y. In y = x, swap x and y.

Rotations

Always identify the centre before you move any point.

Rotate C(2, 1) through 90° anticlockwise about (0, 0).

$(x, y) \longrightarrow (-y, x)$	For this quarter-turn about the origin, swap coordinates and negate the new x.
$(2, 1) \longrightarrow (-1, 2)$	Use that rule with the original coordinates.

$C' = (-1, 2)$

Check

A rotation moves every point through the same angle about the same centre, without changing the shape or size.

Describing a transformation

Give enough information for someone else to reproduce the movement.

Transformation	What to state
Translation	Horizontal and vertical movement
Reflection	The equation or name of the mirror line
Central symmetry	Centre of symmetry (equivalent to a half-turn)
Rotation	Centre, angle and direction

Find the image of (4, 3) under central symmetry about (2, 1).

$(4-2,\ 3-1)=(2,2)$	The point is 2 right and 2 up from the centre.
$(2-2,\ 1-2)=(0,\ -1)$	Go the same distance on the opposite side of the centre.

$(0,\ -1)$

Check

The centre (2, 1) is the midpoint between (4, 3) and its image (0, −1).

Transformations

Try the questions before opening the separate answer key. Show one step per line.

19.1 · Fluency

Reflect (−3, 4) in the y-axis.

19.2 · Variation

Rotate (2, −5) through 90° anticlockwise about the origin.

19.3 · Choose a method

Draw a non-square rectangle and show all its axes of symmetry. Does it have central symmetry?

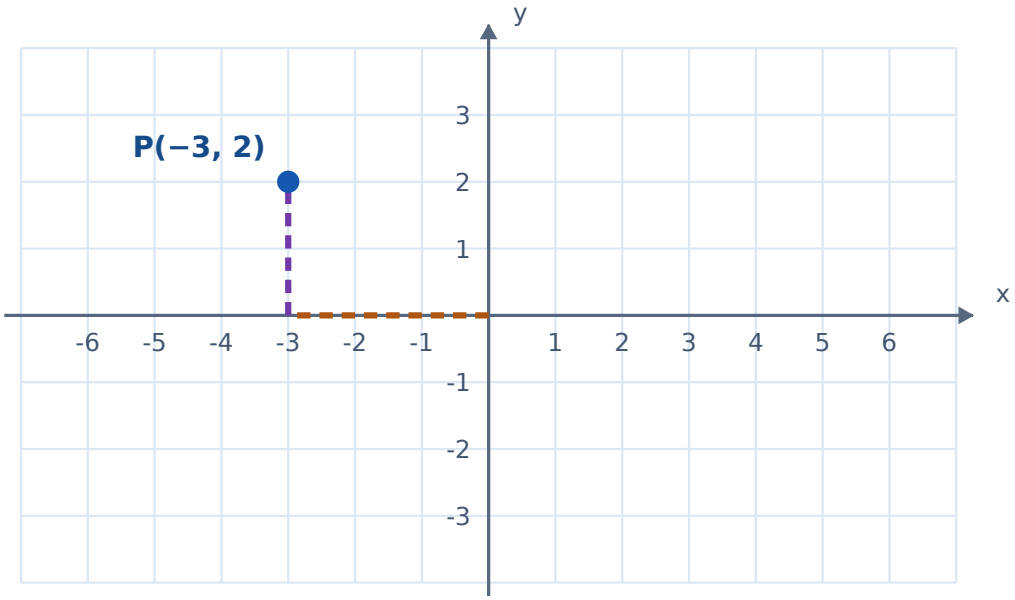
Check your work: are the sign, units and level of accuracy appropriate?

Coordinates and quadrants

Down the hall, up the stairs: read x first, then y.

The ordered pair (x, y)

x tells you the horizontal movement (over and back). y tells you the vertical movement (up and down). A negative x means left; a negative y means down.



From the origin, move 3 units left, then 2 units up. P is in Quadrant II.

II · top left

x negative, y positive

I · top right

x positive, y positive

III · bottom left

x negative, y negative

IV · bottom right

x positive, y negative

On an axis?

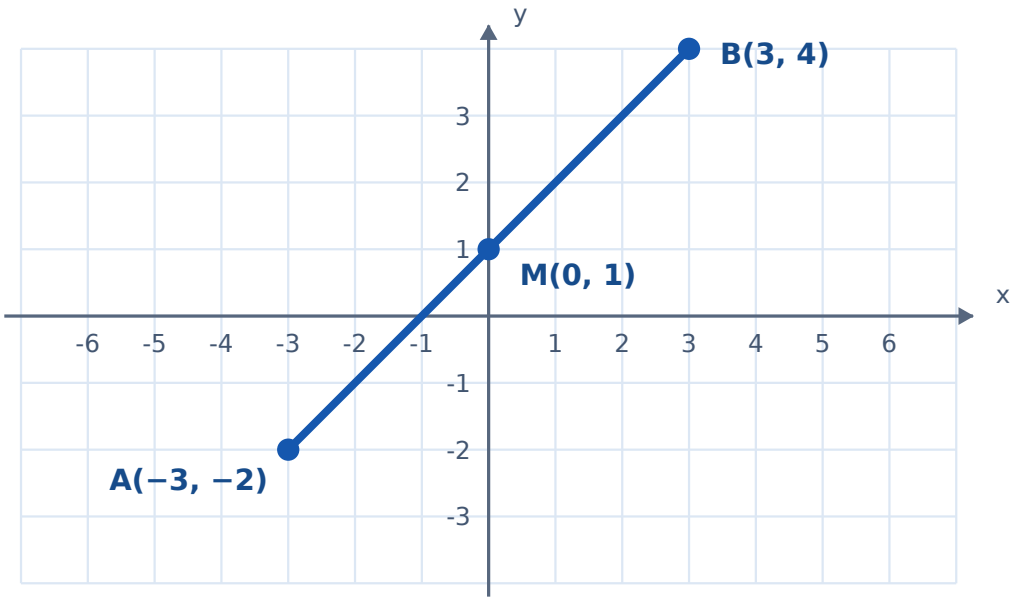
If x = 0 or y = 0, the point lies on an axis, not in a quadrant. The origin is (0, 0).

Midpoint

Average the x-coordinates and the y-coordinates separately.

Formulae and Tables · printed page 18

Find the formula under Co-ordinate geometry → Line.



M lies halfway along the segment from A to B.

Find the midpoint of A(−3, −2) and B(3, 4).

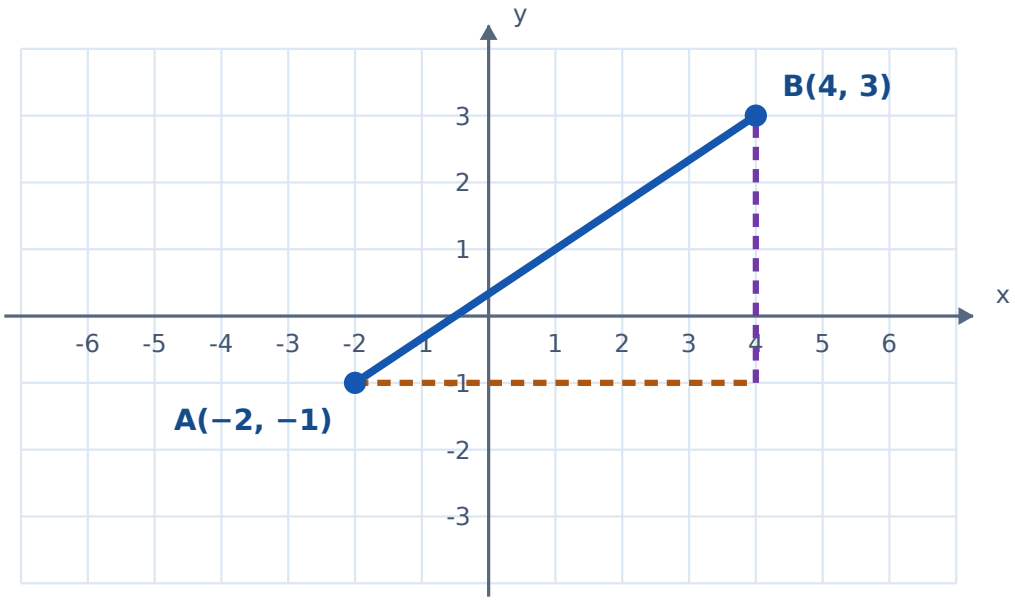
$M=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$	Label A as point 1 and B as point 2.
$M=\left(\frac{-3+3}{2},\frac{-2+4}{2}\right)$	Substitute the x-values together and the y-values together.
$M=\left(\frac{0}{2},\frac{2}{2}\right)$	Add each pair before dividing.
$M=(0,1)$	

Distance between two points

The distance formula gives the straight-line length from A to B.

Formulae and Tables · printed page 18

Find the formula under Co-ordinate geometry → Line.



The dashed sides show 6 units across and 4 units up. The blue segment is the distance.

Find the distance from A(−2, −1) to B(4, 3).

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$	Write the distance formula. A is point 1; B is point 2.
$d=\sqrt{(4-(-2))^2+(3-(-1))^2}$	Subtract the coordinates in the same order. Keep brackets around negative values.
$d=\sqrt{6^2+4^2}=\sqrt{52}$	Square each difference, then add.

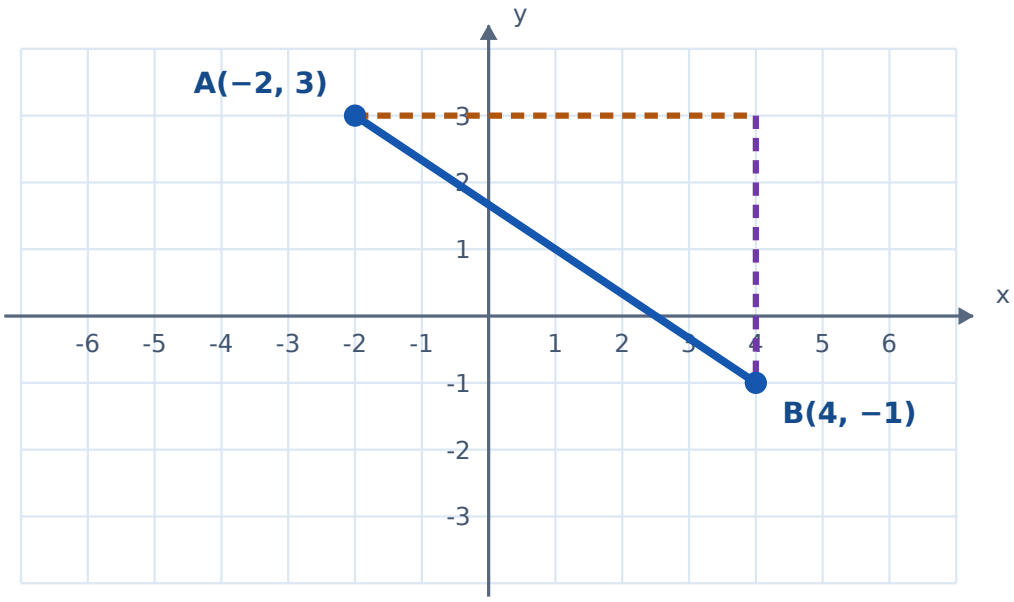
$d=2\sqrt{13}\approx 7.21$ units

Slope of a line

Slope compares the up/down change with the over/back change.

Formulae and Tables · printed page 18

Find the formula under Co-ordinate geometry → Line.



From A to B: move 6 units right and 4 units down. The line falls from left to right.

Find the slope through A(−2, 3) and B(4, −1).

$m = \frac{y_2 - y_1}{x_2 - x_1}$	Write the slope formula. The y-values go on top.
$m = \frac{-1 - 3}{4 - (-2)}$	Subtract A from B both above and below the bar.
$m = \frac{-4}{6}$	Simplify the numerator and denominator.
$m = -\frac{2}{3}$	

Reading slope at a glance

Read the line from left to right to decide whether the slope is positive, negative, zero or undefined.

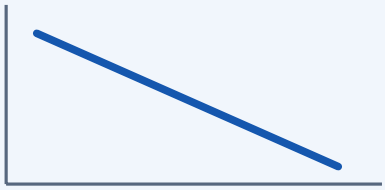
Positive

The line rises.



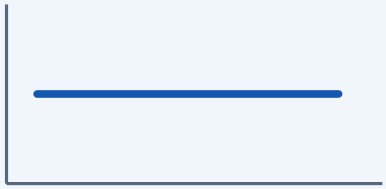
Negative

The line falls.



Zero

The line is horizontal.



Undefined

The line is vertical.



Why is vertical slope undefined?

$x_2 - x_1 = 0$

On a vertical line, the x-coordinates are the same. You cannot divide by zero.

Quick check

Midpoint: average. Distance: square, add and square-root. Slope: y-change over x-change. For subtraction, keep the same point order.

Coordinate Geometry 1

Try the questions before opening the separate answer key. Show one step per line.

20.1 · Fluency

Find the midpoint of $(-4, 1)$ and $(2, 7)$.

20.2 · Variation

Find the slope through $(1, -2)$ and $(5, 4)$.

20.3 · Choose a method

Find the distance between $(-2, 3)$ and $(4, -5)$.

Check your work: are the sign, units and level of accuracy appropriate?

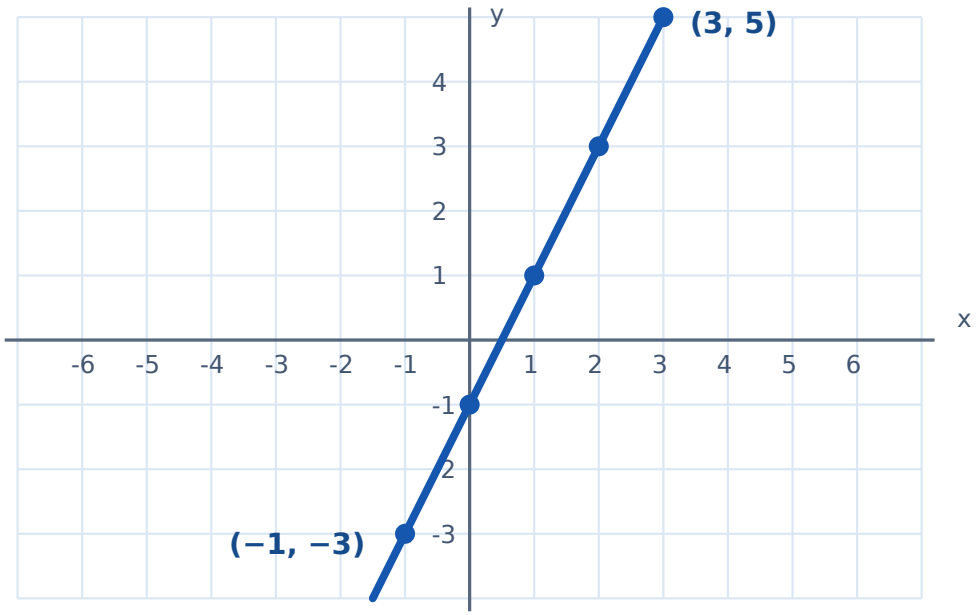
Drawing a line from a table

Every non-vertical straight line can be written as y = mx + c.

What do m and c tell you?

m is the slope (gradient). c is the y-value where the line crosses the y-axis, at (0, c).

x	−1	0	1	2	3
y	−3	−1	1	3	5



Plot each (x, y) pair and draw a straight line through the points.

Work out the first point on y = 2x − 1.

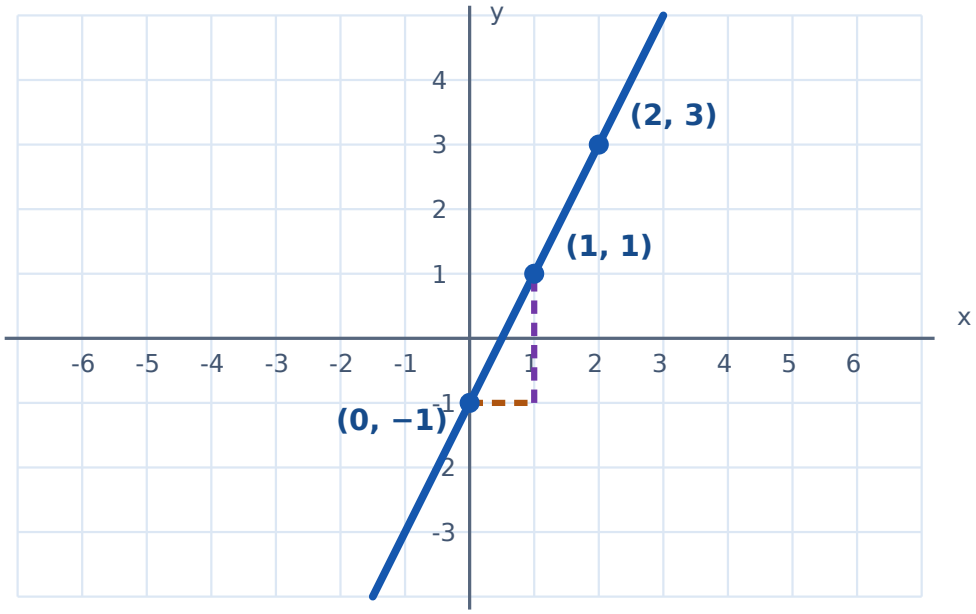
$y = 2(-1) - 1$	Put −1 in place of x.
$y = -2 - 1$	Multiply first, then subtract 1.
$(-1, -3)$	

Drawing with slope and intercept

Draw the same line, y = 2x − 1, without making a table.

Formulae and Tables · printed page 18

See Co-ordinate geometry → Line for the slope and straight-line equations.



Start at (0, −1). The dashed steps show 1 right, then 2 up.

Use c to start and m to find a second point.

$c = -1$	Plot (0, −1) on the y-axis.
$m = \frac{2}{1}$	The numerator says up 2. The denominator says right 1.
$(0, -1) \longrightarrow (1, 1)$	Plot the next point and draw the line. Use (2, 3) as a check.

$y = 2x - 1$

Intercepts and special lines

The intercept is where the line meets an axis.

Which coordinate is zero?

On the y-axis, x = 0. On the x-axis, y = 0.

Find both intercepts of y = 3x − 6.

$y = 3(0) - 6 = -6$	Put x = 0 to find the y-intercept.
$(0, -6)$	Give the intercept as a point.
$0 = 3x - 6$	Put y = 0 to find the x-intercept.
$6 = 3x$	Add 6 to both sides.
$x = 2$	Divide both sides by 3.

$(0, -6)$ and $(2, 0)$

Horizontal and vertical lines

A horizontal line through (5, −2) has equation y = −2. A vertical line through the same point has equation x = 5. A vertical line has undefined slope.

Finding an equation from two points

Find the slope first. Use one point to find c, then check with the other.

Formulae and Tables · printed page 18

See Co-ordinate geometry → Line for the slope and straight-line equations.

Find the equation through (1, 5) and (4, 11).

$m=\frac{y_2-y_1}{x_2-x_1}$	Write the slope formula.
$m=\frac{11-5}{4-1}$	Substitute both points in the same order.
$m=\frac{6}{3}=2$	Simplify to get the slope.
$y=2x+c$	Insert the slope. If the slope is given, start here with that value.
$5=2(1)+c$	Use the point (1, 5).
$5=2+c$	Multiply first.
$c=5-2=3$	Subtract 2 from both sides.
$y=2x+3$	

Check the other point

When x = 4, y = 2(4) + 3 = 11. So (4, 11) fits. From a graph, you can use any two clearly marked points in the same way.

Parallel and perpendicular lines

Parallel lines go in the same direction. Perpendicular lines meet at a right angle.

Parallel lines

Distinct non-vertical lines with the same slope are parallel. If they also have the same intercept, they are the same line. Two distinct vertical lines are also parallel.

Find the parallel to y = 2x + 5 through (4, 1).

$y = 2x + c$	Keep the slope 2.
$1 = 2(4) + c$	Substitute the point.
$c = 1 - 8 = -7$	Subtract 8 from both sides.
$y = 2x - 7$	

Find the perpendicular to y = 2x + 5 through (4, 1).

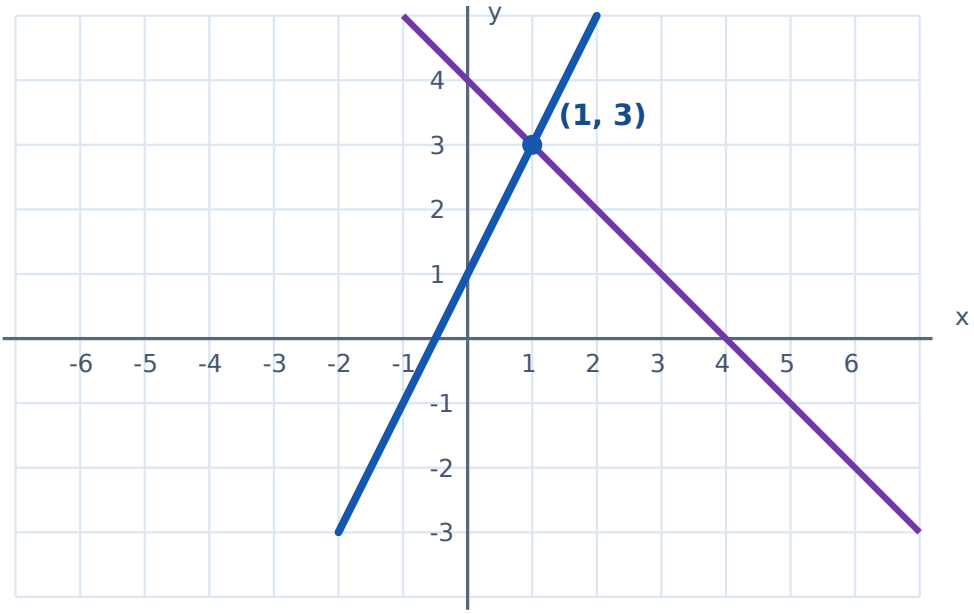
$m_2 = -\frac{1}{2}$	Flip the slope 2 over 1 and change the sign.
$1 = -\frac{1}{2}(4) + c$	Use the new slope and substitute the point.
$1 = -2 + c$	Multiply first.
$c = 1 + 2 = 3$	Add 2 to both sides.
$y = -\frac{1}{2}x + 3$	

Special case

A horizontal line is perpendicular to a vertical line. Do not try to take the reciprocal of slope zero.

Finding where two lines meet

At the intersection, the same x and y satisfy both equations.



The blue line is y = 2x + 1. The purple line is y = −x + 4.

Find the intersection of y = 2x + 1 and y = −x + 4.

$2x+1=-x+4$	Both expressions equal y, so set them equal.
$3x=3$	Add x to both sides, then subtract 1.
$x=1$	Divide both sides by 3.
$y=2(1)+1=3$	Substitute x into either original equation.

$(1,3)$

Check both lines

The second equation gives y = −1 + 4 = 3 too. The point fits both lines.

Coordinate Geometry 2

Try the questions before opening the separate answer key. Show one step per line.

21.1 · Fluency

Write the line through (2, 7) with slope 3 in y = mx + c form.

21.2 · Variation

Find the equation of the line perpendicular to y = 4x + 1 through (0, 3).

21.3 · Choose a method

Find the intersection of y = 3x − 2 and y = −x + 6.

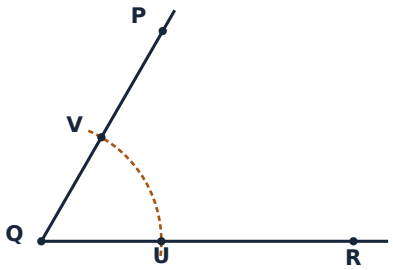
Check your work: are the sign, units and level of accuracy appropriate?

1. Bisect an angle

Given angle PQR with vertex Q.

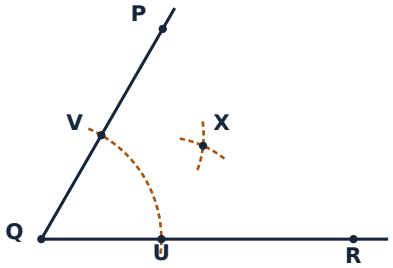
STEP 1

From Q, draw one arc cutting both arms at U and V.



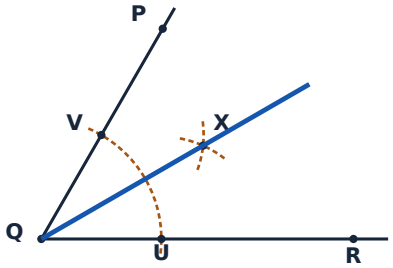
STEP 2

With one compass width, draw arcs from U and V that cross inside the angle at X.



STEP 3

Draw the ray from Q through X.



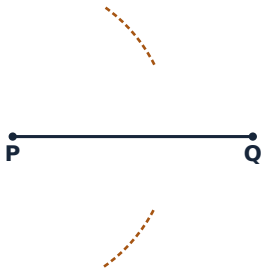
Check: The two angles on either side of QX are equal.

2. Perpendicular bisector

Given segment PQ.

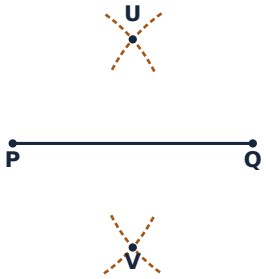
STEP 1

Open the compass wider than half of PQ. From P, draw arcs above and below PQ.



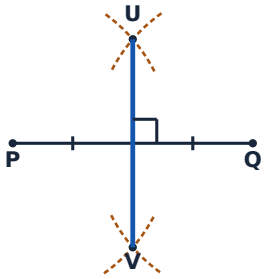
STEP 2

Keep the same width. Repeat from Q, crossing the first arcs at U and V.



STEP 3

Draw UV through the two intersections.



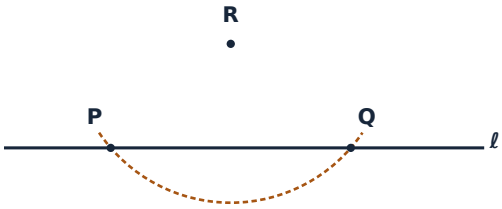
Check: The new line meets PQ at 90° and bisects it.

3. Perpendicular through an external point

HIGHER LEVEL ONLY · Given a point R outside line ℓ.

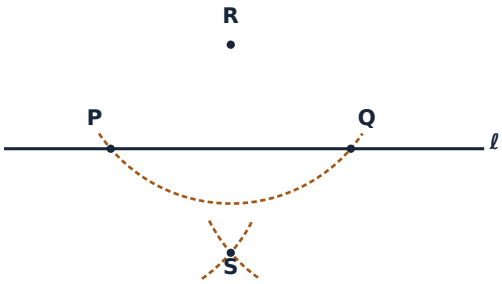
STEP 1

From R, draw an arc wide enough to cut ℓ twice. Label the cuts P and Q.



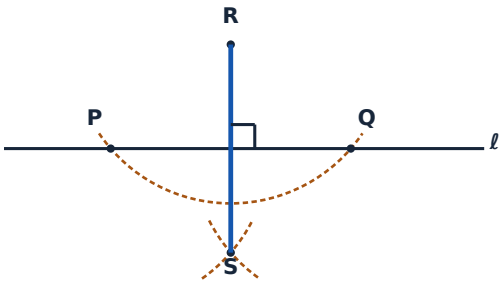
STEP 2

Keep the same radius. From P and Q draw arcs meeting on the far side of ℓ at S.



STEP 3

Draw RS and extend it to the line.



Check: R and S are both equidistant from P and Q, so RS is their perpendicular bisector.

4. Perpendicular at a point on a line

Given a point K on line ℓ.

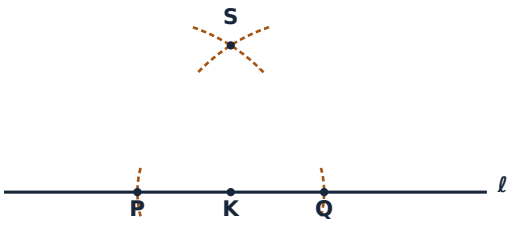
STEP 1

Use one radius from K to mark P and Q on the line, the same distance from K.



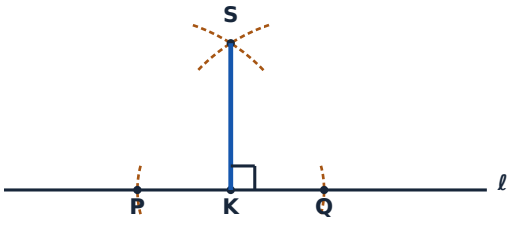
STEP 2

Increase the width. From P and Q draw equal-radius arcs crossing at S.



STEP 3

Draw KS.



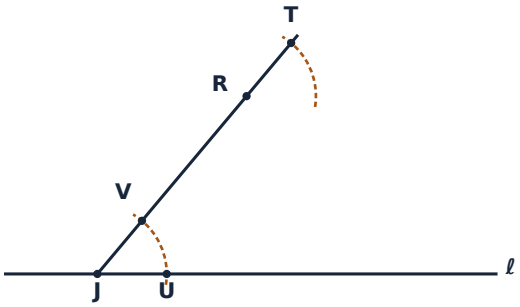
Check: KS is perpendicular to ℓ at K.

5. Parallel through a point

Given line ℓ and point R outside it.

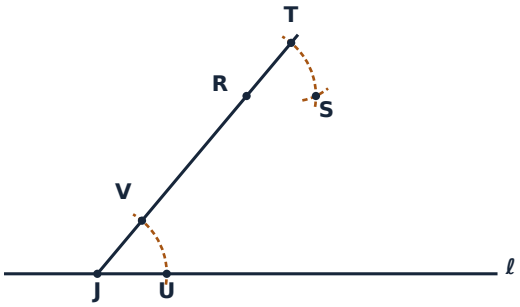
STEP 1

Draw a transversal through R meeting ℓ at J. Use the same radius at J and R to draw the two arcs shown.



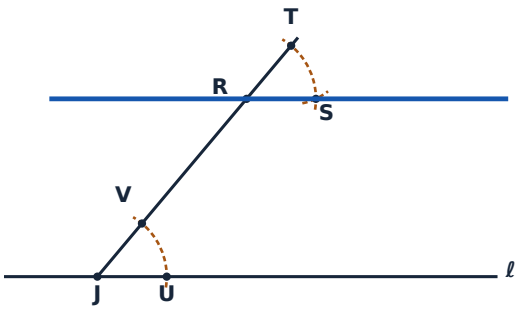
STEP 2

Set the compass to chord UV. With centre T, mark S on the upper arc on the same side of the transversal as U.



STEP 3

Draw the line through R and S.



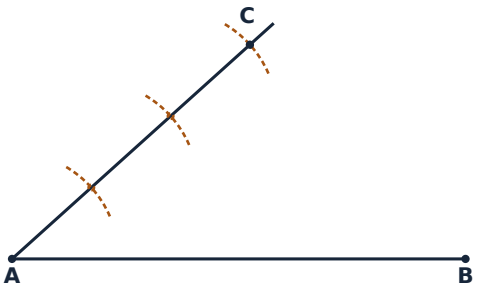
Check: The copied corresponding angles are equal, so RS is parallel to ℓ.

6. Divide a segment into two or three

Given segment AB. The example shows three parts.

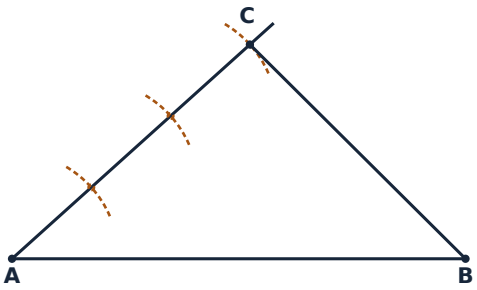
STEP 1

Draw a slanted ray from A. Step one compass width along it 3 times. Call the last mark C.



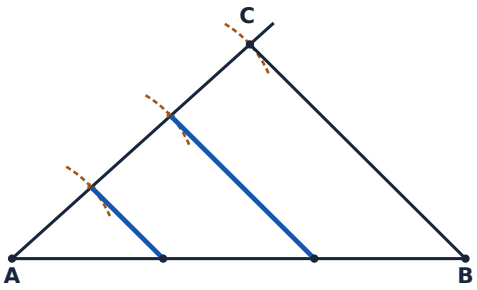
STEP 2

Join C to B. This is the direction for all the parallel lines in the next step.



STEP 3

Use Construction 5 or slide a set square to draw a parallel to CB through each earlier mark. Their intersections divide AB equally.



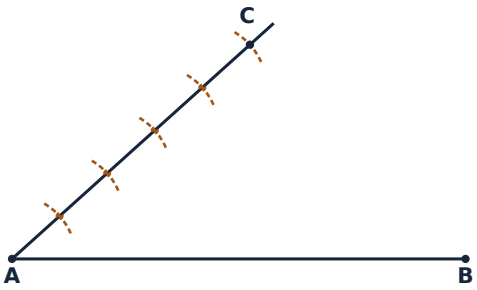
Check: For two parts use two steps; for any n use n steps. The intercept theorem gives equal parts on AB.

7. Divide a segment into n equal parts

HIGHER LEVEL ONLY · Given segment AB. The example shows n = 5.

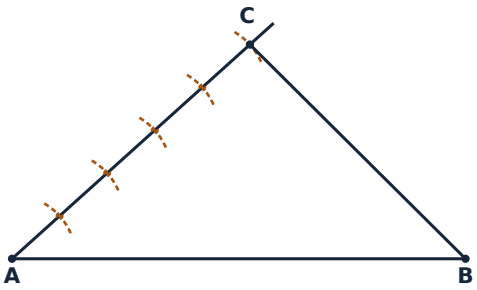
STEP 1

Draw a slanted ray from A. Step one compass width along it 5 times. Call the last mark C.



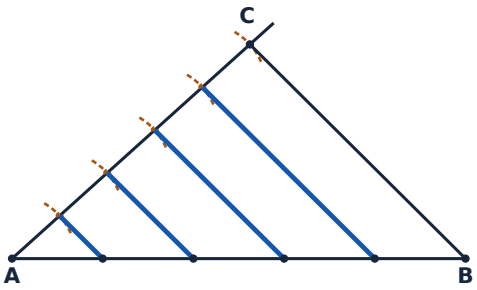
STEP 2

Join C to B. This is the direction for all the parallel lines in the next step.



STEP 3

Use Construction 5 or slide a set square to draw a parallel to CB through each earlier mark. Their intersections divide AB equally.



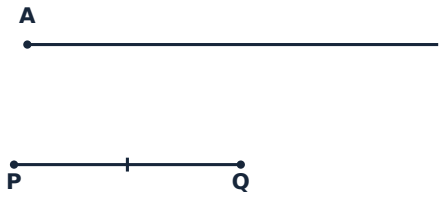
Check: For two parts use two steps; for any n use n steps. The intercept theorem gives equal parts on AB.

8. Copy a length onto a ray

Given segment PQ and a ray starting at A.

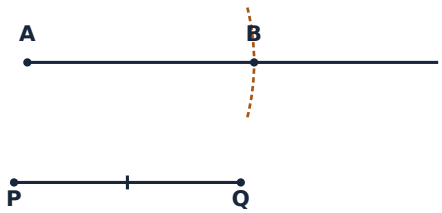
STEP 1

Set the compass width to PQ without measuring it with a ruler.



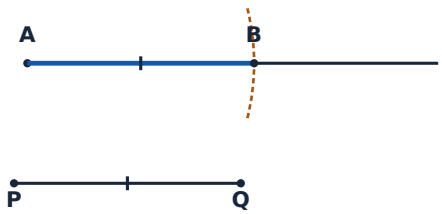
STEP 2

Keeping the width fixed, place the point at A and mark the ray at B.



STEP 3

Identify the copied segment AB.



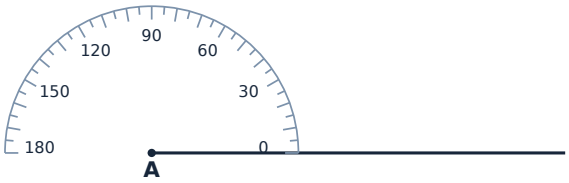
Check: AB and PQ have the same length.

9. Construct an angle of a given size

Example: construct an angle of 55° on the given ray.

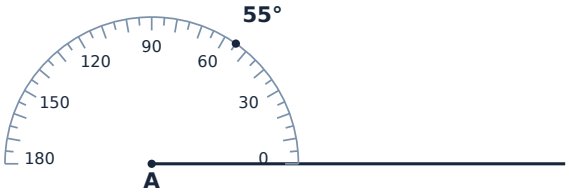
STEP 1

Place the protractor centre on A and align the zero ray with the given ray.



STEP 2

Read from that zero. Mark the 55° direction.



STEP 3

Remove the protractor and draw the second ray through the mark.



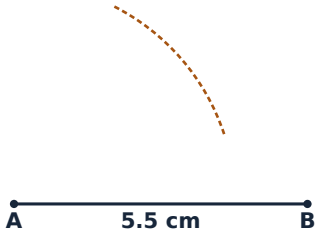
Check: Read the scale that starts at 0° on the given ray.

10. Triangle from three sides (SSS)

Example: AB = 5.5 cm, AC = 4.15 cm and BC = 3.6 cm.

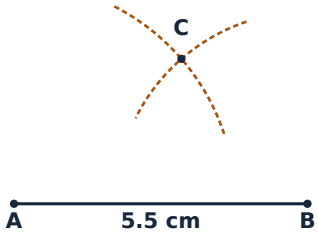
STEP 1

Copy AB. With centre A and compass width AC, draw the first arc.



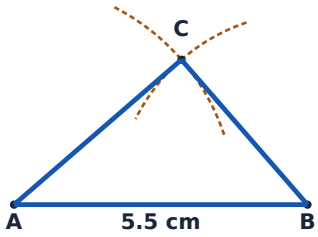
STEP 2

From B draw an arc with radius BC. Label its intersection with the first arc C.



STEP 3

Join A to C and B to C. Leave both arcs visible.



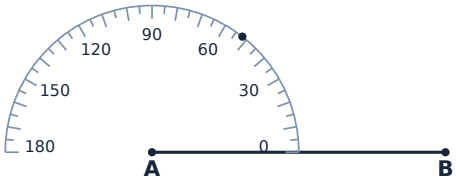
Check: The same three lengths also give a reflected triangle on the other side of AB.

11. Triangle from two sides and included angle (SAS)

Example: AB = 5.5 cm, AC = 4.45 cm and angle A = 52°.

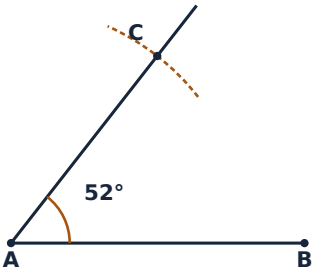
STEP 1

Copy AB. At A, use a protractor to mark a ray 52° from AB.



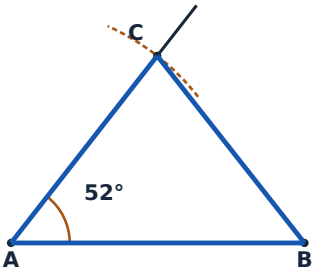
STEP 2

Open the compass to AC. With centre A, cut the new ray at C.



STEP 3

Join C to B to complete the triangle.



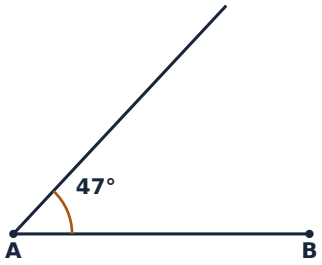
Check: The 52° angle is between the two given sides.

12. Triangle from two angles and included side (ASA)

Example: AB = 5.55 cm, angle A = 47° and angle B = 58°.

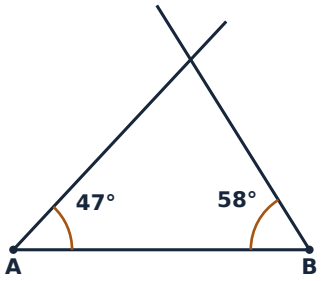
STEP 1

Copy AB. At A construct a 47° ray measured from AB.



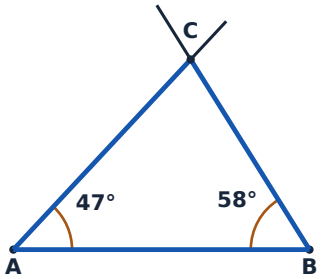
STEP 2

At B construct a 58° ray measured from BA, on the same side of AB.



STEP 3

Label their intersection C. The three points determine triangle ABC.



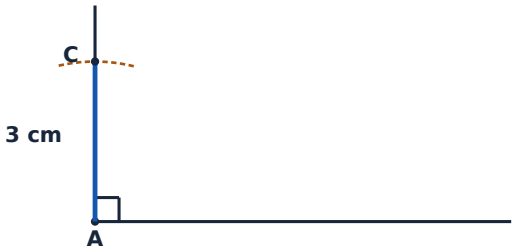
Check: The third angle is 75°. At B start from BA, not from the extension to the right.

13. Right triangle from hypotenuse and a leg (RHS)

Example: right angle at A, AC = 3 cm and BC = 5 cm.

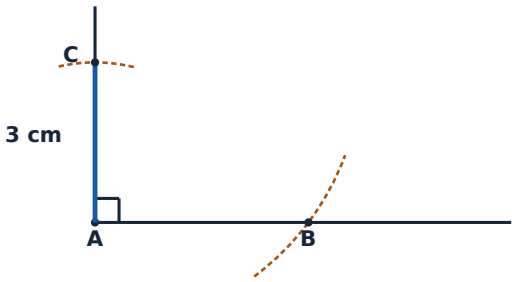
STEP 1

Draw perpendicular rays at A and mark the 3 cm leg AC on one ray.



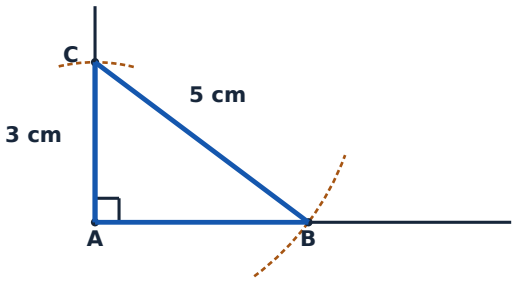
STEP 2

With centre C and radius 5 cm, cut the other ray at B.



STEP 3

Join B to C.



Check: The hypotenuse BC is opposite the right angle.

14a. Right triangle: given the adjacent leg

Example: AB = 6.2 cm and angle B = 25°.

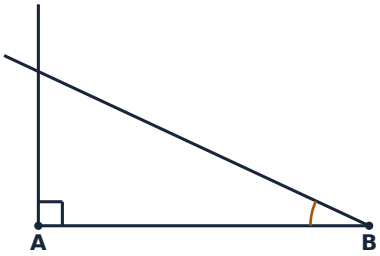
STEP 1

Draw AB and construct a perpendicular ray at A.



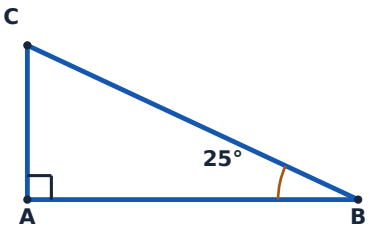
STEP 2

At B, measure 25° from BA and draw the ray on the side of the perpendicular shown.



STEP 3

The two rays meet at C.



Check: The acute angles are B = 25° and C = 65°; the right angle is at A.

14b. Right triangle: given the opposite leg

Example: AC = 3 cm and angle B = 25°.

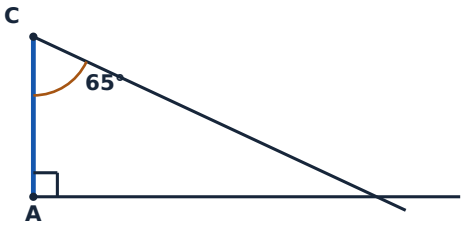
STEP 1

Draw the 3 cm leg AC. At A, construct the perpendicular ray AB.



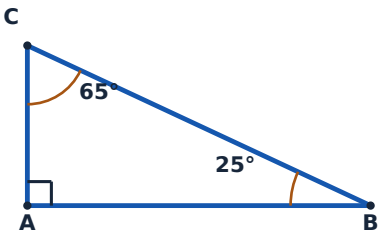
STEP 2

At C, measure 65° from CA and draw the ray towards the perpendicular.



STEP 3

The two rays meet at B. The angle at B is 25°.



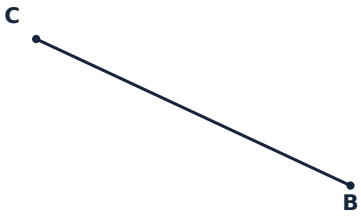
Check: The acute angles are B = 25° and C = 65°; the right angle is at A.

14c. Right triangle: given the hypotenuse

Example: BC = 6.5 cm and angle B = 25°.

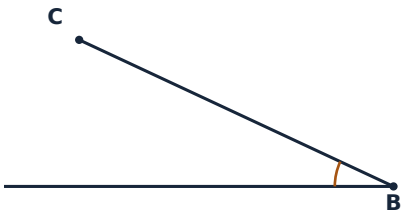
STEP 1

Draw the 6.5 cm hypotenuse BC.



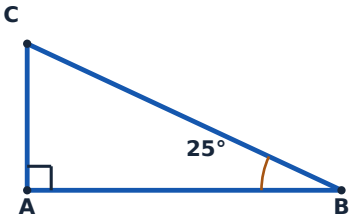
STEP 2

At B, draw ray BA at 25° below BC as shown.



STEP 3

Through C, construct a perpendicular to BA. Its foot is A.



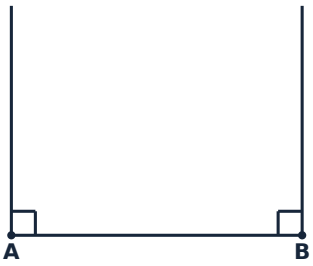
Check: The acute angles are B = 25° and C = 65°; the right angle is at A.

15. Rectangle from its side lengths

Example: length 5.45 cm, width 3.05 cm.

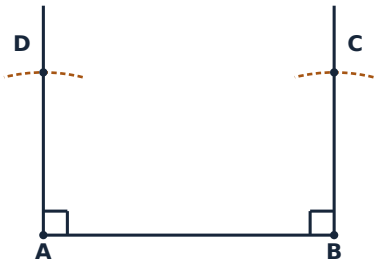
STEP 1

Copy AB to the required length. Construct perpendiculars at A and B on the same side.



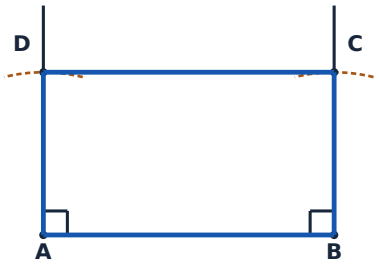
STEP 2

Use one compass width to mark the given width at D and C.



STEP 3

Join D to C to complete the rectangle.



Check: Opposite sides are equal, and all four angles are right angles.

Constructions 1–15

Try the questions before opening the separate answer key. Show one step per line.

22.1 · Fluency

Construct the perpendicular bisector of a 7 cm segment using compass and straight edge only.

22.2 · Variation

Construct triangle ABC with AB = 6 cm, AC = 4 cm and BC = 5 cm.

22.3 · Choose a method

Construct a right triangle with right angle A, AC = 4 cm and angle B = 30°. Leave guide lines visible.

Check your work: are the sign, units and level of accuracy appropriate?

Asking a statistical question

A statistical question expects different answers and can be investigated with data.

Worked example

Instead of “How long does Ava spend travelling to school?”, ask “How many minutes do third-year students spend travelling to school on a typical day?”

Variable	Type	Example
Travel method	Categorical	Bus
Number of buses	Numerical, discrete	2
Journey time	Numerical, continuous	18.5 minutes

What are you measuring?

Name the population, the variable and its units before you collect data. Rounding a measurement to a whole number does not make the underlying variable discrete.

Population, sample and bias

The population is the whole group. The sample is the group you actually collect data from.

Worked example

To estimate travel times for all third-year students, collect responses across the year group. Asking only students at the bicycle rack would over-represent cyclists.

A better sample

Randomly select students from a complete list of third-year students. If you need every class represented, randomly select within each class in proportion to its size.

A fair question

Ask “How many minutes did your journey take today?” rather than “Was your journey too long?” Keep the wording neutral.

From data to a conclusion

Use a number from the data to support your answer, and say who the result applies to.

Journey time	Under 15 minutes	15 minutes or more
Number of students	18	12

What percentage of the 30 respondents travelled for under 15 minutes?

$18 + 12 = 30$	Find the number of responses.
$\frac{18}{30} \times 100 = 60$	Divide the group by the total, then multiply by 100.

60% of respondents

Report the limitation

This was 60% of the students who answered. How well it represents all third-year students depends on how the sample was chosen.

Statistical Investigations

Try the questions before opening the separate answer key. Show one step per line.

23.1 · Fluency

Classify walking time and method of travel as numerical or categorical.

23.2 · Variation

Rewrite “Do students travel far?” as a statistical question with a measurable variable.

23.3 · Choose a method

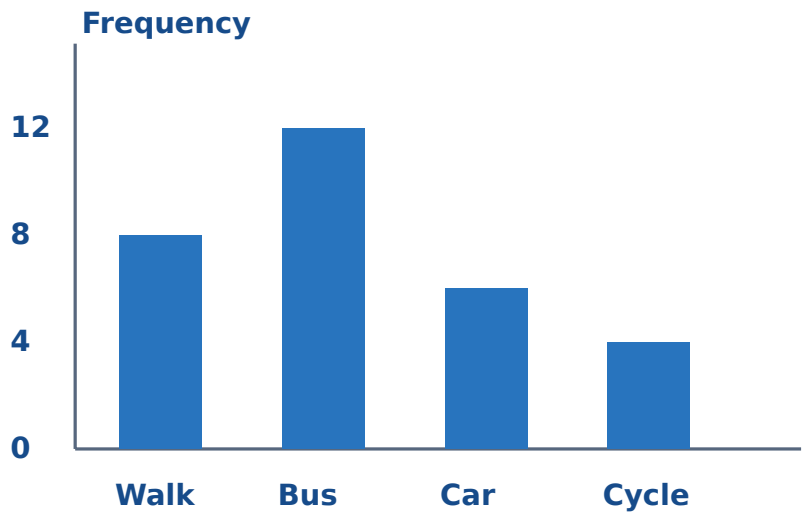
A survey about the library asks only pupils currently in the library. Explain the concern and propose a change.

Check your work: are the sign, units and level of accuracy appropriate?

Frequency tables and bar charts

Frequency means how many times a value or category occurs.

Travel method	Walk	Bus	Car	Cycle
Frequency	8	12	6	4



Equal-width bars have gaps between categories. Bus has the highest frequency.

Check the scale

Read the axis first: one tick interval may represent more than one student. Start a frequency bar chart at zero.

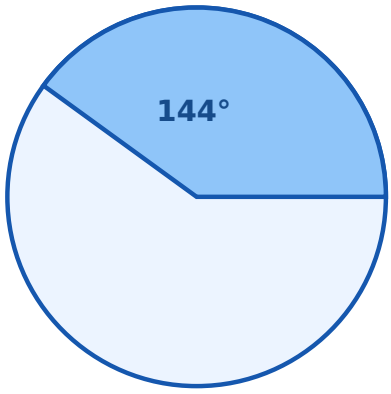
Pie charts

The whole pie represents the total frequency and has an angle of 360°.

12 of 30 students travel by bus. Find the angle for the bus sector.

$\frac{12}{30}=\frac{2}{5}$	Use the bus frequency over the total frequency.
$\frac{2}{5}\times 360^\circ = 144^\circ$	Convert that fraction of a full turn into degrees.

144°



The bus sector represents 40% of all students.

Check

All of the sector angles should add up to 360°, allowing for rounding.

Which display fits the data?

Choose a graph that helps answer the question.

Display	Use it to show
Bar chart	Counts for categories
Pie chart	Shares of the whole
Histogram	Grouped numerical data
Scatter plot	Relationship between two numerical variables

Histogram or bar chart?

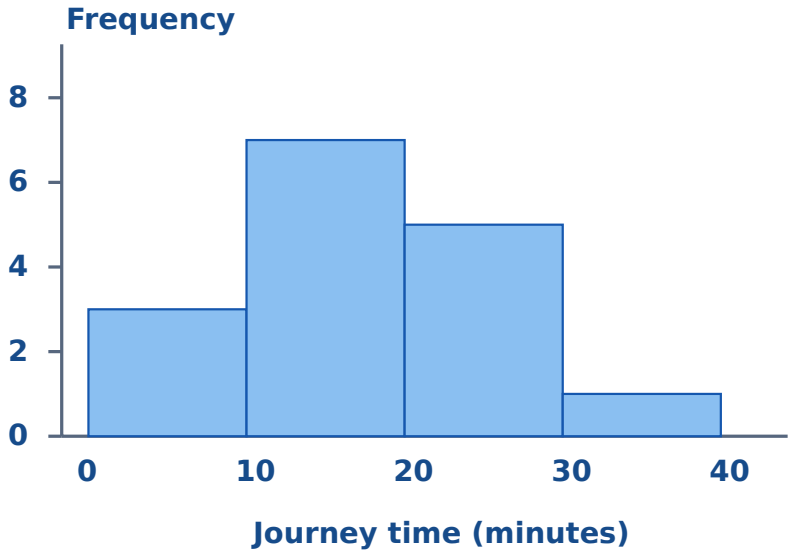
Histogram bars touch for adjoining intervals. With equal interval widths, compare bar heights. With unequal widths, use frequency density so the bar area represents frequency.

Worked interpretation

If a scatter plot of revision time and scores tends to rise from left to right, it shows positive association. It does not by itself prove that extra study caused the difference.

Reading a histogram

For equal-width intervals, the height of a bar gives its frequency when the vertical axis is labelled frequency.



Each bar covers a ten-minute interval. The second bar represents 10 minutes up to, but not including, 20 minutes.

How many students took less than 20 minutes?

$3+7=10$	Add the frequencies of the first two intervals.

10 students

Charts

Try the questions before opening the separate answer key. Show one step per line.

24.1 · Fluency

In a pie chart, 18 out of 60 pupils choose music. Find the sector angle.

24.2 · Variation

Histograms show counts 3, 5 and 8 for intervals [0, 10), [10, 20) and [20, 30). How many observations are below 20?

24.3 · Choose a method

Choose a chart for favourite sports and one for grouped journey times with equal intervals.

Check your work: are the sign, units and level of accuracy appropriate?

Mean, median, mode and range

Each measure answers a different question about the same data.

Find the mean, median, mode and range of 4, 5, 5, 7, 9.

$\text{Mean} = \frac{4+5+5+7+9}{5} = 6$	Add the values, then divide by how many there are.
$\text{Median} = 5$	The ordered list has 5 values, so take the third.
$\text{Mode} = 5$	5 occurs most often.
$\text{Range} = 9 - 4 = 5$	Subtract the smallest value from the largest.

$6, \quad 5, \quad 5, \quad 5$

Find the median of 2, 4, 7, 9.

$\frac{4+7}{2} = \frac{11}{2}$	With an even number of values, average the two middle values.

5.5

Mean from a frequency table

Multiply each value by its frequency before adding.

Value x	1	2	3	Total
Frequency f	2	5	3	10
Product fx	2	10	9	21

Find the mean of the values in the table.

$\sum f = 10$	There are 10 observations, not 3.
$\sum fx = 21$	Add the products in the last row.
$\bar{x} = \frac{\sum fx}{\sum f} = \frac{21}{10}$	Divide the sum of products by the total frequency.

$\bar{x} = 2.1$

Grouped data

For an estimated mean from intervals, use each interval midpoint in place of x. The exact values inside the intervals are unknown.

Estimated mean from grouped data

Use the midpoint of each interval to represent its values.

Time (min)	Frequency	Midpoint	Product
0 to under 10	2	5	10
10 to under 20	3	15	45
20 to under 30	1	25	25

Estimate the mean time.

$\sum f = 2 + 3 + 1 = 6$	Count the observations.
$\sum fx = 10 + 45 + 25 = 80$	Add the midpoint × frequency products.
$\bar{x} \approx \frac{80}{6}$	Divide by the total frequency. It is an estimate because the actual times are unknown.

$\bar{x} \approx 13.3 \text{ min}$

Comparing centre and spread

Use the mean, median or mode for a typical value. Use the range for spread.

Compare A: 4, 5, 6 and B: 1, 5, 9.

$\text{Median of A} = \text{Median of B} = 5$	Both groups have the same middle value.
$\text{Range of A} = 6 - 4 = 2$	Subtract the smallest value from the largest.
$\text{Range of B} = 9 - 1 = 8$	The larger range means the values in B span a wider interval.

A is less spread out by range.

Watch out for extremes

A single unusually large value can raise the mean and the range. The median is usually less affected.

Averages and Spread

Try the questions before opening the separate answer key. Show one step per line.

25.1 · Fluency

Find the median, mode and range of 2, 6, 6, 9, 12.

25.2 · Variation

Values 2, 4 and 7 occur 3, 5 and 2 times respectively. Find the mean.

25.3 · Choose a method

Two classes both have median score 65. Their ranges are 20 and 44. What can you conclude and what can you not conclude?

Check your work: are the sign, units and level of accuracy appropriate?

Theoretical probability

List the outcomes first. Use favourable over total only when the outcomes are equally likely.

A fair six-sided die is rolled. Find the probability of rolling more than 4.

$\{1,2,3,4,5,6\}$	These are the six equally likely outcomes.
$\{5,6\}$	Only 5 and 6 are greater than 4.
$P(X>4)=\frac{2}{6}$	Put the favourable count above the total count.

$P(X>4)=\frac{1}{3}$

Probability scale

0 means impossible, 1 means certain, and all probabilities lie between 0 and 1.

Complementary outcomes

The complement of an event is everything that is not in that event.

A bag has 4 red, 3 blue and 5 green counters. Find the probability of not choosing red.

$P(\text{red}) = \frac{4}{12} = \frac{1}{3}$	There are 12 counters in all.
$P(\text{not red}) = 1 - P(\text{red})$	Red and not red together cover all outcomes.
$1 - \frac{1}{3} = \frac{3}{3} - \frac{1}{3}$	Use a common denominator.
$P(\text{not red}) = \frac{2}{3}$	

Experimental probability

Relative frequency is the fraction of trials in which the event occurred.

A spinner lands on blue 27 times in 60 spins. Estimate the probability of blue.

$P(\text{blue}) \approx \frac{27}{60}$	Use observed successes over trials.
$\frac{27}{60} = \frac{9}{20} = 0.45$	Simplify or convert to a decimal.

$P(\text{blue}) \approx 45\%$

Using that estimate, predict the number of blue results in 200 spins.

$200(0.45) = 90$	Multiply the number of trials by the estimated probability.

90 blue results (estimate)

Probability

Try the questions before opening the separate answer key. Show one step per line.

26.1 · Fluency

A fair die is rolled. Find the probability of a multiple of 3.

26.2 · Variation

A spinner lands on green 27 times out of 90. Estimate the chance of green.

26.3 · Choose a method

The probability of bus being late is 0.18. Find the chance it is not late and predict the number of not-late days in 50 days.

Check your work: are the sign, units and level of accuracy appropriate?

Sample-space tables

A sample space lists all possible outcomes. Keep the order of the trials clear.

First toss ↓ / second →	H	T
H	HH	HT
T	TH	TT

Two fair coins are tossed. Find the probability of exactly one head.

Favourable: $HT,\ TH$ — These are different ordered outcomes.

$P(\text{exactly one head}) = \frac{2}{4}$ — Two of the four equally likely outcomes fit.

$\frac{1}{2}$

And or or?

A single path represents a sequence of events. When different paths all satisfy the question, add their probabilities.

Without replacement

The second draw is made from a smaller bag.

A bag has 3 red and 2 blue counters. Two are drawn without replacement. Find the probability both are red.

$P(R_1) = \frac{3}{5}$	At the first draw, 3 of 5 counters are red.
$P(R_2 \mid R_1) = \frac{2}{4}$	After drawing a red, 2 of 4 remaining counters are red.
$P(\text{both red}) = \frac{3}{5} \times \frac{2}{4} = \frac{6}{20}$	Multiply the probabilities along that path.

$\frac{3}{10}$

With replacement

Put the first counter back before the next draw: the composition resets, so here the probability of red on each draw would be 3 out of 5.

More than one possible path

Multiply along each path. Add the probabilities of the separate paths that fit.

From the same bag, draw twice without replacement. Find the probability of one red and one blue.

$P(RB)=\frac{3}{5}\times\frac{2}{4}=\frac{6}{20}$	First red, then blue.
$P(BR)=\frac{2}{5}\times\frac{3}{4}=\frac{6}{20}$	First blue, then red.
$P(RB \text{ or } BR)=\frac{6}{20}+\frac{6}{20}$	These two paths cannot happen on the same pair of draws.

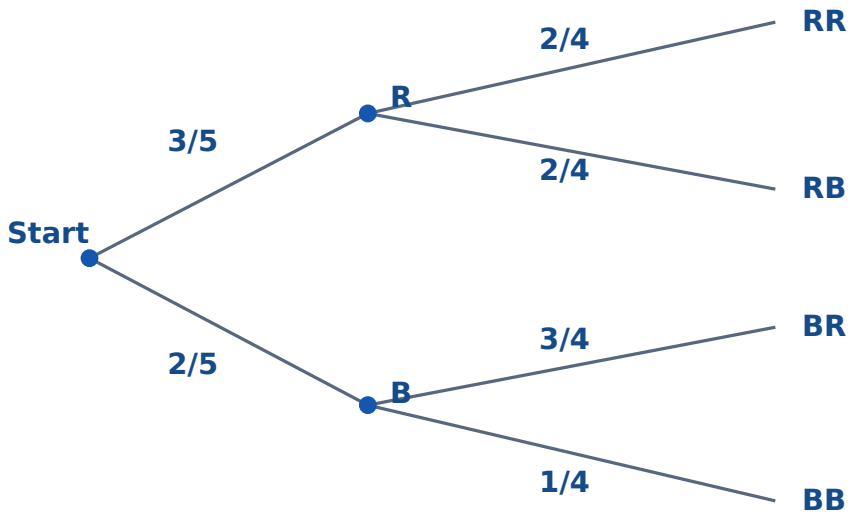
$\frac{3}{5}$

At least one

For “at least one”, it can be quicker to subtract the probability of none from 1.

Reading a probability tree

Each final outcome follows one complete path from the start.



The branches show two draws without replacement from 3 red and 2 blue counters.

Find the probability of at least one red.

$P(BB)=\frac{2}{5}\times\frac{1}{4}=\frac{1}{10}$	The only path with no red is blue, then blue.
$1-\frac{1}{10}=\frac{9}{10}$	Subtract the probability of no red from 1.

$\frac{9}{10}$

Combined Events

Try the questions before opening the separate answer key. Show one step per line.

27.1 · Fluency

List the sample space for two fair coin tosses.

27.2 · Variation

A bag holds three red and two blue counters. Find P(two red) without replacement.

27.3 · Choose a method

With the same bag, find the chance of one counter of each colour without replacement.

Check your work: are the sign, units and level of accuracy appropriate?

Three forms of a line

The same line can be written in point–slope, slope–intercept or general form.

Write the line through (2, 5) with slope 3 in all three forms.

$y-5=3(x-2)$	Substitute into $y - y_1 = m(x - x_1)$.
$y-5=3x-6$	Expand the bracket.
$y=3x-1$	Add 5 to write slope–intercept form.

$3x-y-1=0$

Remember

General form is $ax + by + c = 0$. For the coordinate geometry formulas, see the Formulae and Tables reference in the coordinate chapters.

Ordered stem-and-leaf plots

Put the leaves in order, repeat duplicate values, and include a key.

A leaves	Stem	B leaves
7 4 2	1	3 5 5
4 1	2	2 6

Compare A: 12, 14, 17, 21, 24 and B: 13, 15, 15, 22, 26.

$\text{Median A} = 17$	The third of the five ordered values.
$\text{Median B} = 15$	The repeated 15 must appear twice.
$\text{Range A} = 24 - 12 = 12$	Subtract the smallest value from the largest.

$$\text{Range B} = 26 - 13 = 13$$

Remember

In a back-to-back plot, read the left leaves from the stem outwards. Key: stem 1 and leaf 7 mean 17.

Counting choices and independent events

Multiply the number of choices at each stage when every choice in the first stage can pair with every choice in the next.

A fair coin and a fair six-sided die are used. Find the probability of tails and a die score above 4.

$2\times6=12$	There are 12 equally likely outcomes.
$\{(T,5),(T,6)\}$	These are the two successful outcomes.

$$\frac{2}{12}=\frac{1}{6}$$

Remember

A two-way table can show independent events; use a tree when later probabilities depend on earlier outcomes.

Parallel cuts and areas

A line parallel to one triangle side creates a similar smaller triangle.

D is on AB and E is on AC. DE is parallel to BC. AD = 4, DB = 2 and AE = 6. Find EC.

$\frac{AD}{DB}=\frac{AE}{EC}$	The parallel cut divides the two sides in the same ratio.
$\frac{4}{2}=\frac{6}{EC}$	Substitute the known lengths.
$EC=3$	

Remember

Parallelogram area is base × perpendicular height. A triangle with that same base and height has half the area.

Symmetry and central symmetry

Axial symmetry is a reflection in a line. Central symmetry is a half-turn about a point.

Find the image of A(4, 0) under central symmetry about C(1, 2).

$A - C = (4-1, 0-2) = (3, -2)$	Find the move from the centre to A.
$A' = C - (3, -2) = (-2, 4)$	Go equally far in the opposite direction.

$A' = (-2, 4)$

Remember

The centre is the midpoint of A and its image. A non-square rectangle has two reflection axes and half-turn symmetry.

Comparing changes in context

An absolute comparison uses subtraction; a relative comparison compares to a starting value.

Class A goes from 10 to 15. Class B goes from 20 to 26. Which increases more?

$15-10=5, \quad 26-20=6$	B has the larger absolute increase.
$\frac{5}{10}\times 100=50, \quad \frac{6}{20}\times 100=30$	A has the larger percentage increase.

Absolute: B. Relative: A.

Remember

State which comparison you mean. If two graphs use different scales, compare their values rather than their visual steepness.

Line plots and misleading displays

A line plot shows frequency using marks over a number line. A time-series graph has time on the horizontal axis.

The values are 1, 2, 2, 4, 4, 4. Find the mode and range.

$f(1)=1,\quad f(2)=2,\quad f(4)=3$	Count how many times each value appears.
$4-1=3$	Range is the largest value minus the smallest.

Mode 4, range 3

Remember

An axis starting above zero can exaggerate differences on a bar chart. Check scale, labels and sample size before interpreting a display.

Make a conjecture, then test it

Examples can suggest a rule. An argument explains why it always works. A counterexample can show a rule is false.

Is the sum of two consecutive odd whole numbers always a multiple of 4?

$3+5=8, \quad 5+7=12$	Two examples suggest that it is.
$(2n+1)+(2n+3)=4n+4$	Write two consecutive odd numbers in general form.
$4n+4=4(n+1)$	Factor out 4.

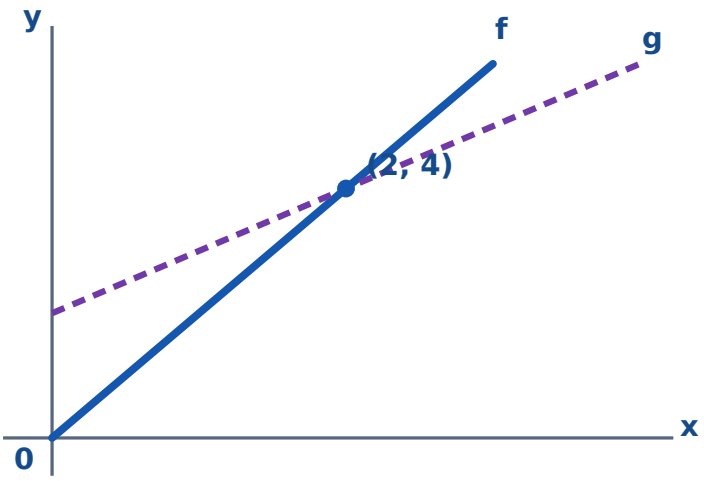
Yes, the sum is a multiple of 4.

Remember

A single counterexample disproves an “always” claim. For instance, 3 + 7 = 10 shows that two odd numbers need not sum to a multiple of 4 if they are not consecutive.

Read an inequality from two graphs

Compare the heights of the two graphs at the same x.



Blue: f(x) = 2x. Purple: g(x) = x + 2. The blue graph is lower to the left of the crossing.

For f(x) = 2x and g(x) = x + 2, use the graph to find where f(x) is below g(x).

$2x = x + 2 \Rightarrow x = 2$	The graphs meet at (2, 4).
$f(0) = 0, \quad g(0) = 2$	On the left of the crossing, the blue graph is lower.

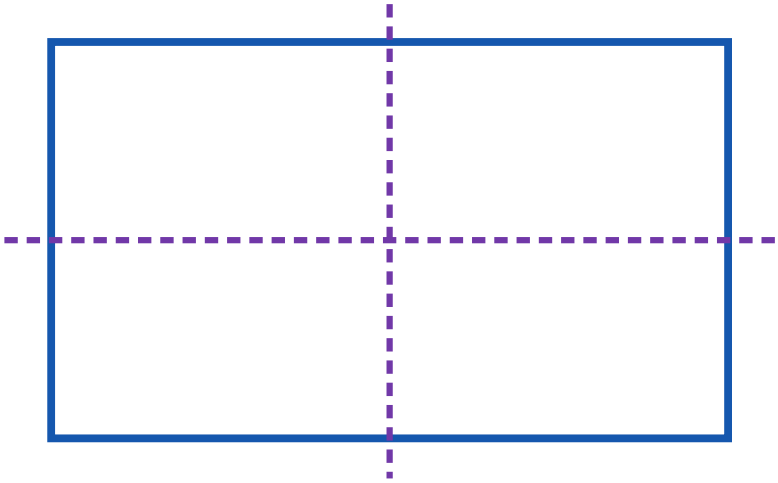
$x < 2$

Remember

Do not include x = 2 for a strict inequality. The answer is a set of x-values, not a shaded region in the coordinate plane.

Draw axes of symmetry

An axis is a line you can fold along so that the two halves match exactly.



The dashed horizontal and vertical lines pass through the centre.

Draw every axis of symmetry of a rectangle that is not a square.

Left half matches right half.	Draw one line vertically through the centre.
Top half matches bottom half.	Draw one line horizontally through the centre.

Two axes of symmetry.

Remember

A diagonal of a non-square rectangle does not fold one half onto the other.

Course Review

Try the questions before opening the separate answer key. Show one step per line.

28.1 · Fluency

Plan A rises from €35 to €42. Plan B rises from €80 to €92. Compare absolute and relative rises.

28.2 · Variation

Prove that two consecutive even integers add to a number two more than a multiple of four.

28.3 · Choose a method

A 1 : 25,000 map shows 9 cm between two places. A walker covers that distance in 45 minutes. Estimate the average speed and state an assumption.

Check your work: are the sign, units and level of accuracy appropriate?