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# Choosing a factorising method

Recognise the structure first. Then choose the matching method.

**FIRST CHECK**

Is there a common factor in every term? If yes, take out the HCF before doing anything else.

| 2 TERMS | 3 TERMS | 4 TERMS |
|---|---|---|
| Difference of two squares? | In the form $ax^2 + bx + c$? | Can the terms be paired? |
| Perfect square - perfect square. | Use the guide number ac. | Factorise each pair. |
| Use (a - b)(a + b). | Split the middle term. | Take out the repeated bracket. |

## Highest common factor

**Worked example** — ONE CHANGE PER LINE

| | |
|---|---|
| $18x^3y^2 - 12x^2y$ | Start with the full expression. |
| $\text{HCF} = 6x^2y$ | 6 is the largest number dividing both coefficients; $x^2y$ uses the smallest shared powers. |
| $18x^3y^2 \div 6x^2y = 3xy$ | Divide the first term by the HCF. |
| $-12x^2y \div 6x^2y = -2$ | Divide the second term by the HCF. |

**FINAL ANSWER**

$6x^2y(3xy - 2)$

**CHECK** $6x^2y(3xy - 2) = 18x^3y^2 - 12x^2y$

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# Factorising special structures

Use a pattern only when every required feature is present.

## Difference of two squares

**PATTERN**

Two perfect squares separated by a minus sign: $a^2 - b^2 = (a - b)(a + b)$.

**Worked example** — ONE CHANGE PER LINE

| | |
|---|---|
| $25x^2 - 49$ | Both terms are perfect squares and there is a minus sign. |
| $(5x)^2 - 7^2$ | Write each term as a square. |

**FINAL ANSWER**

$(5x - 7)(5x + 7)$

**CHECK** $(5x - 7)(5x + 7) = 25x^2 - 49$

**WATCH OUT** $25x^2 + 49$ is a sum of squares. This identity does not apply.

**SECOND EXAMPLE**

$9a^2 - 16 = (3a)^2 - 4^2 = (3a - 4)(3a + 4)$.

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# Factorising by grouping

Pair four terms, factorise each pair, and take out the bracket that repeats.

**METHOD**

Pair the first two terms and the last two terms. Factorise each pair. Then take out the repeated bracket.

**Worked example** — ONE CHANGE PER LINE

| | |
|---|---|
| $2xy+6x+5y+15$ | Start: there are four terms. |
| $(2xy+6x)+(5y+15)$ | Pair the first two terms and the last two terms. |
| $2x(y+3)+5(y+3)$ | Take 2x from the first pair and 5 from the second. |

**FINAL ANSWER**

$(2x+5)(y+3)$

**CHECK** $(2x+5)(y+3)=2xy+6x+5y+15$

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# Quadratic factorising

Write a, b and c first. Then use the guide number ac.

**GUIDE-NUMBER METHOD**

For $ax^2 + bx + c$: calculate ac, find two numbers that multiply to ac and add to b, then split the middle term.

| | | | |
|---|---|---|---|
| $6x^2+11x+3$ | $a=6$ | $b=11$ | $c=3$ |

**Guide-number example** — ONE CHANGE PER LINE

| | |
|---|---|
| $ac=6\times3=18$ | Multiply a by c. This is the guide number. |
| $9\times2=18$ and $9+2=11$ | Find two numbers that multiply to 18 and add to b = 11. |
| $6x^2+9x+2x+3$ | Replace 11x with 9x + 2x. |
| $(6x^2+9x)+(2x+3)$ | Pair the four terms. |
| $3x(2x+3)+1(2x+3)$ | Factorise each pair. |

**FINAL ANSWER**

$(3x+1)(2x+3)$

**CHECK** $(3x+1)(2x+3)=6x^2+11x+3$

**EASIER CASE WHEN A = 1**

For $x^2 + 5x + 6$, find two numbers that multiply to 6 and add to 5: 2 and 3. Therefore $x^2 + 5x + 6 = (x + 2)(x + 3)$.

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# Simplifying algebraic fractions

Factorise completely before cancelling. Restrictions come from the original denominator.

**RULE**

You may cancel complete factors joined by multiplication. You may not cancel separate terms joined by + or -.

## Worked example

ONE CHANGE PER LINE

| | |
|---|---|
| $\frac{x^2-16}{x^2+7x+12}$ | Start with the original fraction. |
| $x^2-16=(x-4)(x+4)$ | Factorise the numerator using difference of two squares. |
| $x^2+7x+12=(x+3)(x+4)$ | Factorise the denominator. |
| $\frac{(x-4)(x+4)}{(x+3)(x+4)}$ | Now the complete factor (x + 4) can be cancelled. |
| **FINAL ANSWER** $\frac{x-4}{x+3}$ | Write the simplified fraction. |

**RESTRICTIONS**

From the original denominator: x ≠ -3 and x ≠ -4. The restriction x ≠ -4 remains even though (x + 4) cancels.

| YOU MAY CANCEL FACTORS | YOU MAY NOT CANCEL TERMS |
|---|---|
| $\frac{3x(x+2)}{x(x-1)}=\frac{3(x+2)}{x-1}$ | $\frac{x+3}{x}\neq 3$ |

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# Adding and subtracting algebraic fractions

Use the lowest common denominator. Multiply the numerator and denominator by the same missing factor.

## Addition

ONE CHANGE PER LINE

| | |
|---|---|
| $\frac{2}{x}+\frac{3}{x+1}$ | The denominators are different. |
| $\text{LCD}=x(x+1)$ | The lowest common denominator contains both factors. |
| $\frac{2(x+1)}{x(x+1)}+\frac{3x}{x(x+1)}$ | Give both fractions the common denominator. |
| $\frac{2x+2+3x}{x(x+1)}$ | Combine the numerators only. |

**FINAL ANSWER**

$$\frac{5x+2}{x(x+1)}$$

**RESTRICTIONS** $x \neq 0$ and $x \neq -1$. The denominators must be identical before the numerators are combined.

## Subtraction

ONE CHANGE PER LINE

| | |
|---|---|
| $\frac{4}{x}-\frac{1}{x+2}$ | The denominators are different. |
| $\frac{4(x+2)}{x(x+2)}-\frac{x}{x(x+2)}$ | Use the common denominator $x(x+2)$. |
| $\frac{4x+8-x}{x(x+2)}$ | Expand the numerator. Keep the subtraction sign, then collect like terms. |

**FINAL ANSWER**

$$\frac{3x+8}{x(x+2)}$$

**RESTRICTIONS** $x \neq 0$ and $x \neq -2$.

**SIGN CHECK** Use brackets so the minus applies to every term: $-(x+2) = -x-2$, not $-x+2$.

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# Multiplying and dividing algebraic fractions

Factorise first. Cancel complete factors before multiplying or dividing.

## Multiplication

ONE CHANGE PER LINE

| | |
|---|---|
| $\frac{x^2-9}{x} \times \frac{2x}{x+3}$ | Factorise $x^2$ - 9 before cancelling. |
| $\frac{(x-3)(x+3)}{x} \times \frac{2x}{x+3}$ | Write $x^2$ - 9 as (x - 3)(x + 3). |
| $(x-3) \times 2$ | Cancel the complete factors x and (x + 3). |

**FINAL ANSWER**

$2(x-3)$

**DIVIDING FRACTIONS**

Keep the first fraction, change ÷ to ×, and flip the second fraction. Here x ≠ 0 and x ≠ -3; the fraction being divided by must also not equal zero.

## Division

ONE CHANGE PER LINE

| | |
|---|---|
| $\frac{x^2-9}{x^2} \div \frac{x+3}{2x}$ | Start with the division. |
| $\frac{x^2-9}{x^2} \times \frac{2x}{x+3}$ | Change ÷ to × and flip the second fraction. |
| $\frac{(x-3)(x+3)}{x^2} \times \frac{2x}{x+3}$ | Factorise $x^2$ - 9. |
| $\frac{2(x-3)}{x}$ | Cancel x and the complete factor (x + 3). |

**FINAL ANSWER**

$\frac{2(x-3)}{x}$

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# Changing the subject of a formula

Undo operations in reverse order and perform the same operation on both sides.

**METHOD ORDER**

Undo addition or subtraction, then multiplication or division, then powers or roots.

## Example 1 · Make r the subject

ONE CHANGE PER LINE

| | |
|---|---|
| $A = \pi r^2$ | Start with the area formula. |
| $\frac{A}{\pi} = r^2$ | Divide both sides by π. |
| **FINAL ANSWER** $r = \sqrt{\frac{A}{\pi}}$ | Take the positive square root because a radius is positive. |

## Example 2 · Make h the subject

ONE CHANGE PER LINE

| | |
|---|---|
| $V = \frac{\pi r^2 h}{3}$ | Start with the volume formula. |
| $3V = \pi r^2 h$ | Multiply both sides by 3. |
| **FINAL ANSWER** $h = \frac{3V}{\pi r^2}$ | Divide both sides by $\pi r^2$. |

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# When the variable appears twice

Collect the terms containing the required variable, then factorise that variable.

**Example 3 · Make x the subject** — ONE CHANGE PER LINE

| | |
|---|---|
| $y=\frac{2x-1}{x+3}$ | Start with the original formula. |
| $y(x+3)=2x-1$ | Multiply both sides by (x + 3). |
| $xy+3y=2x-1$ | Expand the left-hand side. |
| $xy-2x=-1-3y$ | Take 2x from both sides and take 3y from both sides. |
| $x(y-2)=-1-3y$ | Factorise x on the left-hand side. |
| $x=\frac{-1-3y}{y-2}$ | Divide both sides by (y - 2). |
| **FINAL ANSWER** $x=\frac{1+3y}{2-y}$ | Multiply numerator and denominator by -1 to simplify the signs. |

**FINAL CHECK**

Test a simple value: if y = 0, the rearranged formula gives x = 1/2, and (2 × 1/2 - 1)/(1/2 + 3) = 0. Also check that each operation was applied to the entire side.

JUNIOR CYCLE HIGHER LEVEL · CHAPTER PRACTICE

# Algebra 2: factorising, fractions and formulae

Try the questions before opening the separate answer key. Show one step per line.

### 11.1 · Fluency

Factorise $15x^2 - 10x$ completely.

### 11.2 · Variation

Simplify $2/(x + 1) - 1/(x - 1)$, stating restrictions.

### 11.3 · Choose a method

Make r the subject of $V = \pi r^2 h$, assuming $h > 0$ and r is a radius.

Check your work: are the sign, units and level of accuracy appropriate?