Simultaneous equations

Line up matching terms and add vertically when their coefficients are opposites.

Row	x-term	y-term	Right side
A	x	+y	7
+ B	x	−y	3
A + B	2x	0	10

Solve x + y = 7 and x − y = 3.

$(x+x)+(y-y)=7+3$	Add the x, y and right-hand columns separately.
$2x=10$	The y-terms cancel.
$x=5$	Divide both sides by 2.
$5+y=7$	Substitute x = 5 into equation A.

$x=5,\quad y=2$

Remember

If coefficients do not cancel, label the originals and write each row operation explicitly, e.g. A × (−2) then add B.

Quadratics by factorising

Rearrange to make one side zero before using the zero-product rule.

Solve x² − x − 12 = 0.

$(x-4)(x+3)=0$	The numbers −4 and 3 multiply to −12 and add to −1.
$x-4=0 \quad \text{or} \quad x+3=0$	At least one factor must be zero.
$x=4 \quad \text{or} \quad x=-3$	

Remember

Working backwards, roots 4 and −3 give the monic equation (x − 4)(x + 3) = 0.

The quadratic formula

Write the equation as $ax^2 + bx + c = 0$ and keep the signs with the coefficients.

Solve $2x^2 + x - 4 = 0$.

$a=2,\quad b=1,\quad c=-4$	Read the three coefficients.
$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$	Use the quadratic formula.
$x=\dfrac{-1\pm\sqrt{1-4(2)(-4)}}{4}$	Substitute, with brackets around negative values.
$x=\dfrac{-1\pm\sqrt{33}}{4}$	

Remember

The discriminant $b^2 - 4ac$ determines the real roots: positive gives two distinct roots, zero gives one repeated root, and negative gives no real roots.

Dividing algebraic expressions

Divide the leading terms, multiply back, subtract and bring down.

Divide x³ + 2x² − 5x − 6 by x + 1.

$x^3 \div x = x^2$	Subtract x²(x + 1). The remaining expression is x² − 5x − 6.
$x^2 \div x = x$	Subtract x(x + 1). The remaining expression is −6x − 6.
$(-6x) \div x = -6$	Subtract −6(x + 1). The remainder is zero.

$x^2 + x - 6$

Remember

Check by multiplying the quotient by x + 1. The same method works for a quadratic dividend.

Doubling and tripling patterns

A constant multiplier is different from a constant difference.

Term number	1	2	3	4	5
Term value	4	12	36	108	324

The sequence starts 4, 12, 36, 108. Find the next term.

$12 \div 4 = 3, \quad 36 \div 12 = 3$	Each term is three times the previous term.
$108 \times 3 = 324$	Apply the same rule once more.

324

Remember

Linear: constant first difference. Quadratic: constant non-zero second difference. Doubling and tripling: multiply by 2 or 3 each step.

Build a quadratic from its roots

Each root r gives a factor x − r. Use both factors, then expand if asked.

Write a monic quadratic equation with roots −2 and 5.

$x-(-2)=x+2$	The negative root gives a plus sign.
$(x+2)(x-5)=0$	Both factors belong in the equation.
$x^2-5x+2x-10=0$	Expand the brackets.
$x^2-3x-10=0$	

Remember

Monic means the coefficient of x² is 1. Check by substituting each root. Non-zero multiples of this equation have the same roots.

JUNIOR CYCLE HIGHER LEVEL · CHAPTER PRACTICE

Equations and Extensions

Try the questions before opening the separate answer key. Show one step per line.

12.1 · Fluency

Write a monic quadratic equation with roots 3 and −4.

12.2 · Variation

Solve $2x + y = 7$ and $x - y = 2$.

12.3 · Choose a method

Does $3x^2 - 2x + 2 = 0$ have real solutions? Justify without finding roots.

Check your work: are the sign, units and level of accuracy appropriate?