FUNCTIONS AND GRAPHS

What is a function?

A function gives exactly one output for every allowed input.



READ F(X) CORRECTLY

f(x) means the output of function f when the input is x. It does not mean f multiplied by x.

Worked example · f(x) = 2x + 3 — ONE CHANGE PER LINE

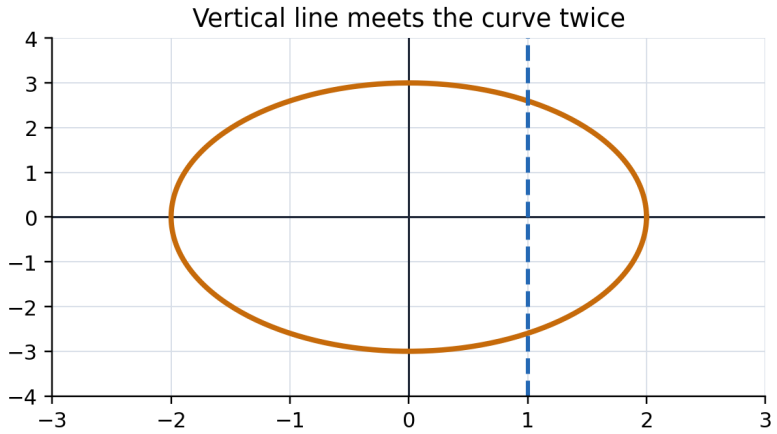
$f(4) = 2(4) + 3$	Replace x by 4.
$f(4) = 8 + 3$	Multiply before adding.
FINAL ANSWER $f(4) = 11$	Input 4 produces output 11.

FUNCTION

Each input has exactly one output.

NOT A FUNCTION

At least one input has more than one output.



VERTICAL-LINE TEST

If a vertical line can meet a graph more than once, the graph is not a function.

FUNCTIONS AND GRAPHS

Finding outputs and inputs

First decide which quantity is known: the input x or the output f(x).

FIND AN OUTPUT
A value of x is given. Substitute it into the rule.

FIND AN INPUT
A value of f(x) is given. Form an equation and solve for x.

A · Find f(-3) when f(x) = x² - 2x + 1 — ONE CHANGE PER LINE

$f(-3)=(-3)^2-2(-3)+1$	Use brackets around the negative input.
$f(-3)=9+6+1$	Square first, then multiply.
FINAL ANSWER $f(-3)=16$	The output is 16.

B · For what x is f(x) = 16? — ONE CHANGE PER LINE

$x^2-2x+1=16$	Set the rule equal to the given output.
$x^2-2x-15=0$	Bring all terms to one side.
$(x-5)(x+3)=0$	Factorise.
FINAL ANSWER $x=5 \quad \text{or} \quad x=-3$	Two different inputs can have the same output.

IMPORTANT
A function may give the same output for different inputs. It is still a function because each individual input has only one output.

FUNCTIONS AND GRAPHS

Domain and range

The domain contains allowed inputs. The range contains outputs actually produced.

DOMAIN
The set of possible x-values.

RANGE
The set of resulting f(x)-values.

INPUT x	RULE $f(x)=x^2$	OUTPUT f(x)
-2	$(-2)^2$	4
-1	$(-1)^2$	1
0	0^2	0
1	1^2	1
2	2^2	4

State the domain and range — ONE CHANGE PER LINE

$\text{Domain} = \{-2, -1, 0, 1, 2\}$	List every allowed input once.
$\text{Range} = \{0, 1, 4\}$	List each output once; do not repeat 1 or 4.

RESTRICTIONS
A denominator cannot be zero. A square root cannot contain a negative number when working with real numbers.

Example · g(x) = 1/(x - 3) — ONE CHANGE PER LINE

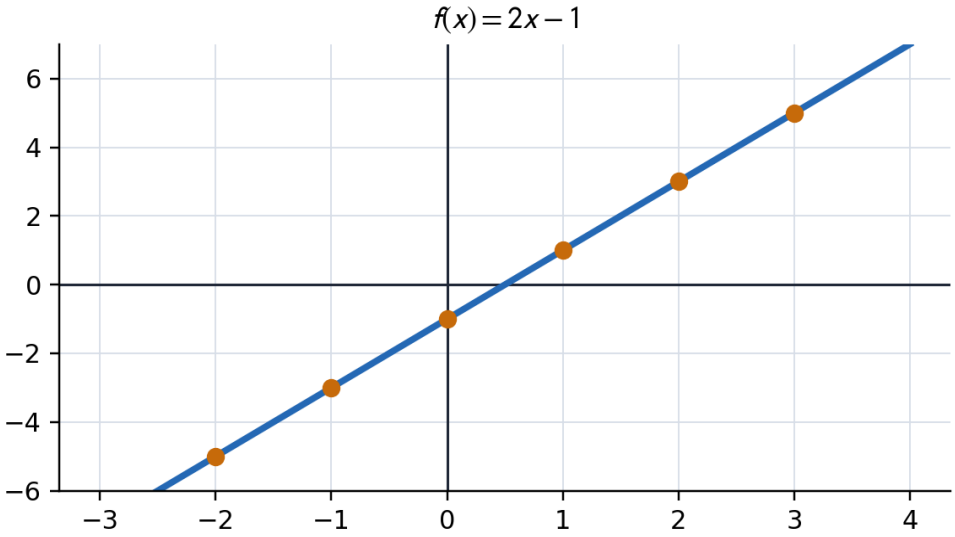
$x - 3 \neq 0$	The denominator cannot equal zero.
$x \neq 3$	Solve the restriction.
FINAL ANSWER $\text{Domain}: \ x \in \mathbb{R}, \ x \neq 3$	Every real input except 3 is allowed.

FUNCTIONS AND GRAPHS

Drawing a graph from a function

Choose x-values, calculate y=f(x), plot the coordinates and draw the graph.

x	-2	-1	0	1	2	3
f(x)=2x-1	-5	-3	-1	1	3	5
(x,y)	(-2,-5)	(-1,-3)	(0,-1)	(1,1)	(2,3)	(3,5)



GRAPHING CHECKLIST

Label both axes. Use equal, sensible scales. Plot each point accurately. A straight-line function may be joined with a ruler. State the equation beside the graph.

Read a point	ONE CHANGE PER LINE
$(2, 3)$	The horizontal coordinate is the input x=2.
$f(2) = 3$	The vertical coordinate is the output.
FINAL ANSWER $2(2) - 1 = 3$	The point satisfies the rule.

FUNCTIONS AND GRAPHS

Reading intercepts and roots

Intercepts occur where a graph meets an axis.

Y-INTERCEPT
Set x=0. The point lies on the vertical axis.

ROOT OR X-INTERCEPT
Set f(x)=0. The point lies on the horizontal axis.

For f(x)=2x-6 — ONE CHANGE PER LINE

$f(0)=2(0)-6=-6$	Set x=0 for the y-intercept.
$2x-6=0$	Set f(x)=0 for the root.
$2x=6$	Add 6 to both sides.
FINAL ANSWER $y\text{–intercept}=(0,\ -6),\quad \text{root}=3$	The x-intercept is (3,0).

FROM A GRAPH
Read a root from the x-axis and a y-intercept from the y-axis. Always give a coordinate if the question asks for a point.

Sign of a function

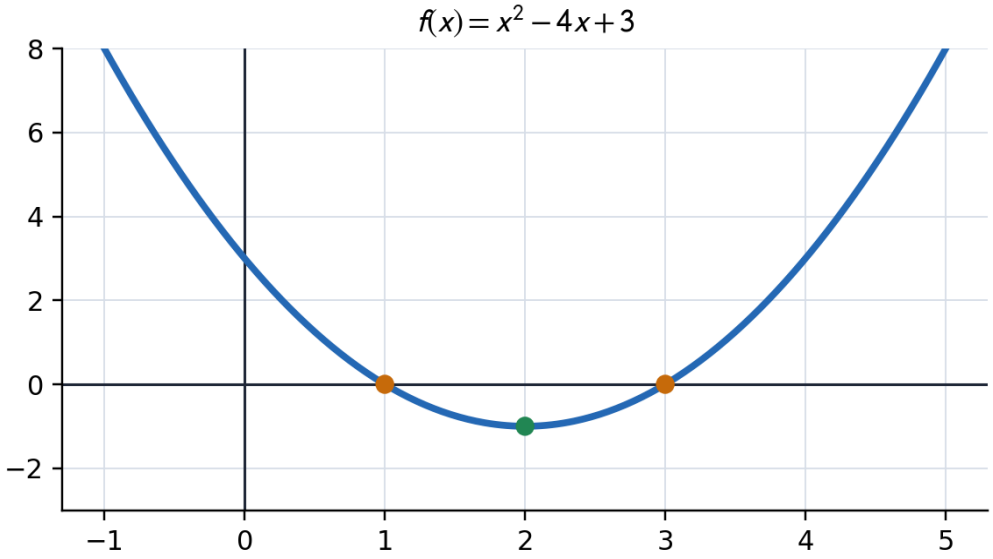
ABOVE THE X-AXIS
f(x)>0.

BELOW THE X-AXIS
f(x)<0.

AT THE X-AXIS
f(x)=0 exactly at a root.

Quadratic graphs

A quadratic graph is a parabola with roots, a turning point and an axis of symmetry.



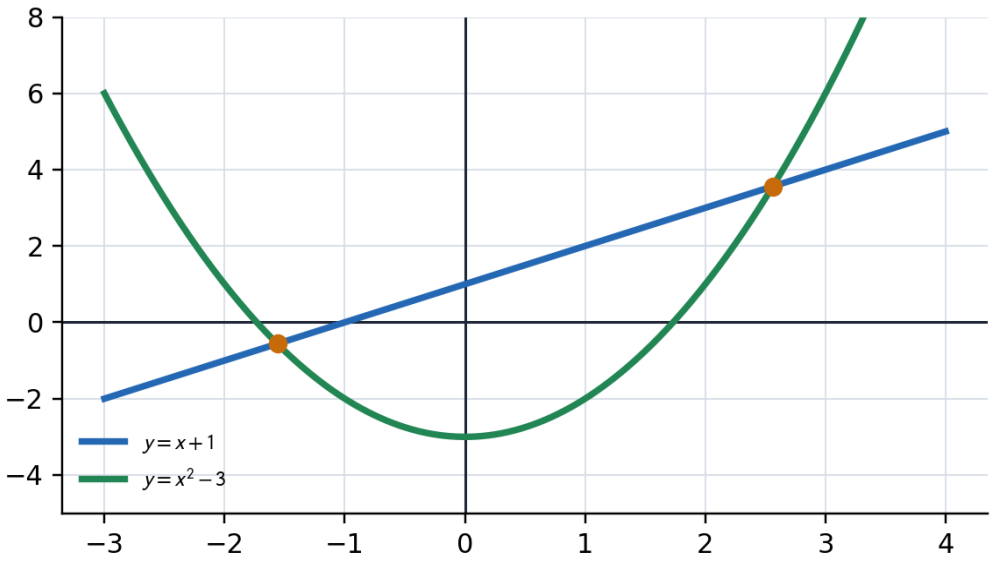
FEATURE	HOW TO READ IT	FOR THIS GRAPH
Roots	Where f(x)=0	x=1 and x=3
y-intercept	Where x=0	(0,3)
Minimum	Lowest point	(2,-1)
Axis of symmetry	Vertical line through turning point	x=2

SHAPE

If the coefficient of x^2 is positive, the graph opens upward and has a minimum. If it is negative, it opens downward and has a maximum.

Where two graphs intersect

At an intersection, both functions have the same x-value and the same y-value.



GRAPHICAL SOLUTION

The x-coordinates of the crossing points solve f(x)=g(x). Read every crossing, not just the first.

Solve x+1 = x²-3 algebraically — ONE CHANGE PER LINE

$x+1=x^2-3$	Set the two rules equal.
$x^2-x-4=0$	Bring all terms to one side.
$x=\frac{1\pm\sqrt{17}}{2}$	Use the quadratic formula.

FINAL ANSWER

$x\approx -1.56$ or $x\approx 2.56$

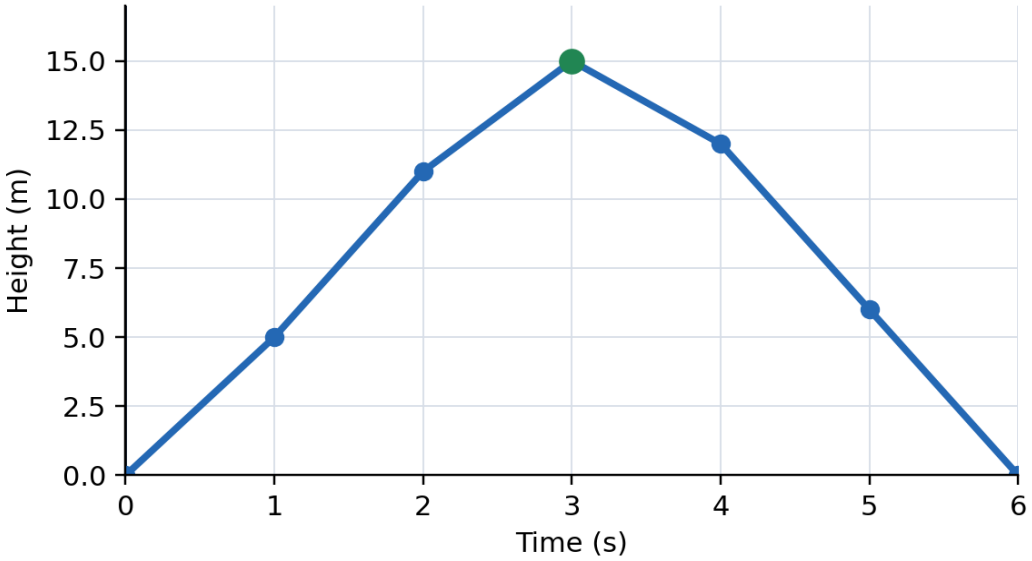
These match the two crossings on the graph.

APPROXIMATE ANSWERS

When reading a graph, use the scale carefully and state the requested accuracy, for example x is approximately 2.6 to one decimal place.

Functions in context

Interpret both coordinates and include units.



Reading the height-time graph	ONE CHANGE PER LINE
$(3, 15)$	At 3 seconds, the height is 15 metres.
maximum height $= 15$ m	Read the vertical coordinate of the highest point.
$h(0) = 0, \quad h(6) = 0$	The object is at ground level at 0 s and 6 s.
FINAL ANSWER maximum at $t = 3$ s	The turning point gives both the time and the greatest height.

QUESTION	LOOK FOR
Find f(4)	Substitute x=4.
Solve f(x)=7	Set the rule equal to 7 and solve.
Find a root	Set f(x)=0 or read the x-axis.
Find the maximum	Read the highest turning point.
Solve f(x)=g(x)	Find every intersection.

JUNIOR CYCLE HIGHER LEVEL · CHAPTER PRACTICE

Functions and graphs

Try the questions before opening the separate answer key. Show one step per line.

14.1 · Fluency

For f(x) = 3x − 6, find f(4).

14.2 · Variation

Find the two intercepts of y = 2x − 8.

14.3 · Choose a method

Sketch f(x) = x + 3 and g(x) = 2x. Use the sketch to identify where f(x) is above g(x).

Check your work: are the sign, units and level of accuracy appropriate?