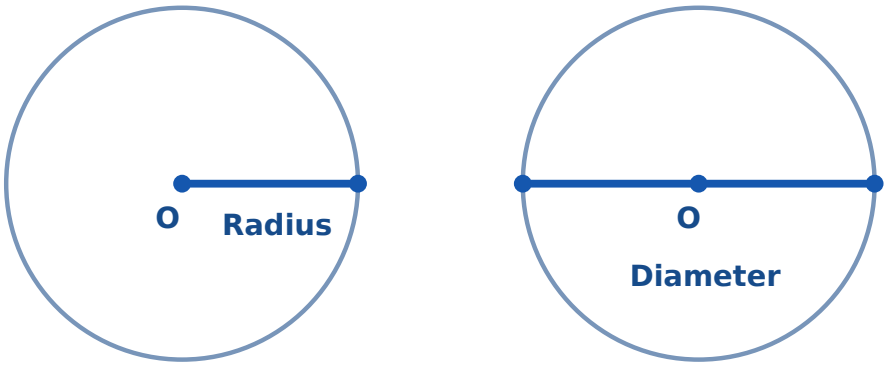
Circumference and area

Circumference is the distance around. Area is the space inside.



Radius: centre to circle. Diameter: circle to circle through the centre.

Find the circumference and area when the diameter is 10 cm.

$r = \frac{10}{2} = 5$	Halve the diameter.
$C = 2\pi r = 2\pi(5)$	Circumference has length units.
$A = \pi r^2 = \pi(5)^2$	Area has square units.

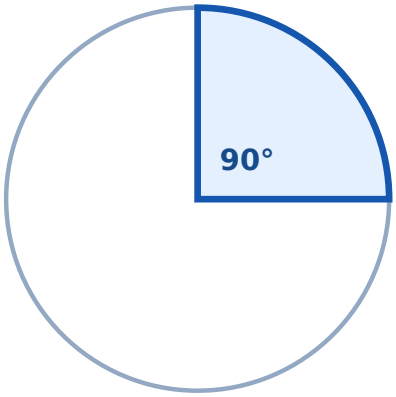
$C = 10\pi \text{ cm}, \quad A = 25\pi \text{ cm}^2$

Circle vocabulary

A chord joins two points on the circle. An arc is a curved part of the circumference. A diameter is a chord through the centre.

Arcs and sectors

Both the arc and the sector are fractions of the full circle.



A 90° sector takes up one quarter of the circle.

A 90° sector has radius 8 cm. Find its arc length and area.

$\frac{90}{360}=\frac{1}{4}$	Find the fraction of a full turn.
$L=\frac{1}{4}(2\pi\cdot 8)=4\pi$	Use the fraction of the circumference.
$A=\frac{1}{4}(\pi\cdot 8^2)=16\pi$	Use the same fraction of the circle's area.

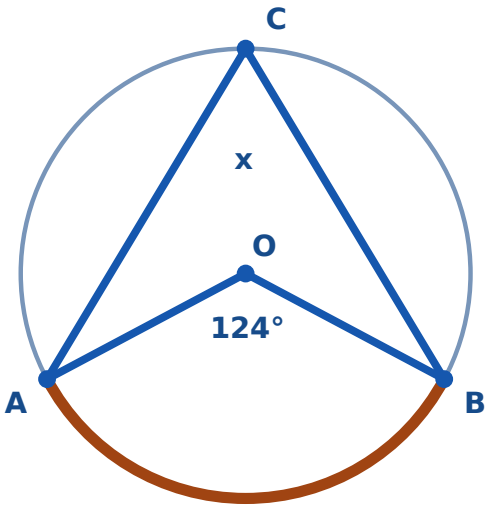
$L=4\pi$ cm, $A=16\pi$ cm^2

Exam application

For the perimeter of a sector, add the arc length and both radii. In the example this is 4π + 16 cm. A semicircle also has a straight edge: its diameter.

Centre and circumference

Theorem 19: the angle at the centre is twice the angle at the circumference standing on the same arc.



Both angles look towards the orange arc AB. The vertices are O and C.

The central angle AOB is 124°. Find angle ACB.

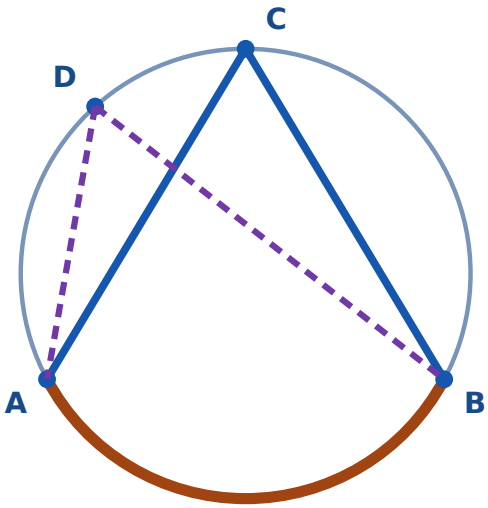
$2x = 124^\circ$	Both angles stand on the highlighted minor arc.
$x = \frac{124^\circ}{2}$	Halve the central angle.
$x = 62^\circ$	

Minor or major arc?

124° is the smaller central angle here. A central angle of 240° is reflex and stands on a major arc, not a minor arc.

Equal angles on the same arc

Corollary 2: angles at the circumference standing on the same arc are equal.



C and D are on the same side of chord AB. Both angles stand on orange arc AB.

Angle ACB is 62°. Find angle ADB.

$\angle ADB = \angle ACB$	Same endpoints and same arc. Each is half the same central angle.

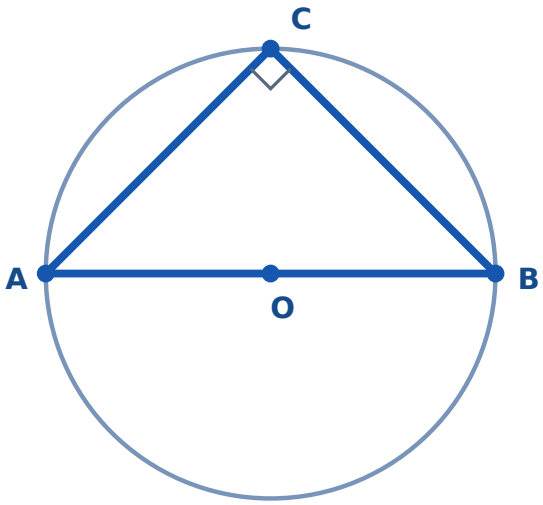
$\angle ADB = 62^\circ$

Converse

If C and D are on the same side of AB and angle ACB equals angle ADB, then A, B, C and D lie on one circle. A shared chord alone is not enough.

The angle in a semicircle

Corollaries 3 and 4: a diameter gives a right angle, and a right angle standing on a chord identifies a diameter.



AB passes through O. The right-angle mark is at C, on the circumference.

AB is a diameter. Find angle ACB.

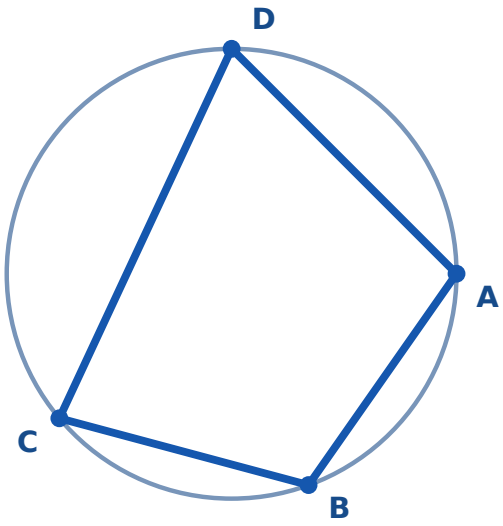
$\angle AOB = 180^\circ$	The diameter gives a straight angle at the centre.
$\angle ACB = \frac{180^\circ}{2}$	Use the centre and circumference rule.
$\angle ACB = 90^\circ$	

Use it backwards

If C lies on the circle and angle ACB is 90°, then AB is a diameter.

Cyclic quadrilaterals

Corollary 5: when all four vertices lie on one circle, opposite angles add to 180°.



A and C are opposite vertices. B and D are the other opposite pair.

ABCD is cyclic. Angle A is 100°. Find angle C.

$100^\circ + C = 180^\circ$	A and C are opposite. The two central angles for their arcs make 360°; halve that to get 180°.
$C = 180^\circ - 100^\circ$	Subtract the known angle.

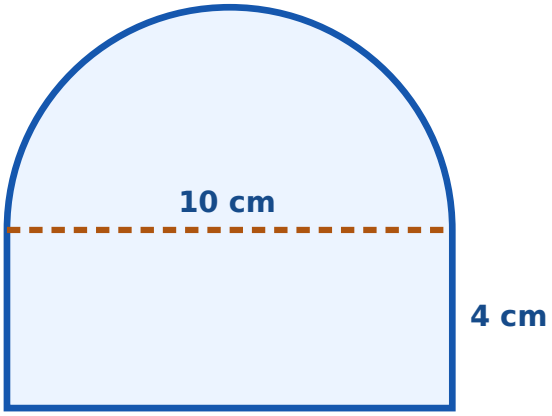
$C = 80^\circ$

Converse

If opposite angles of a convex quadrilateral add to 180°, it is cyclic.

Composite shapes with circles

Area measures the region. Perimeter measures only the outside boundary.



A semicircle sits on top of a 10 cm by 4 cm rectangle. The dashed join is inside the shape.

Find the exact area and perimeter.

$r = 10 \div 2 = 5$	Halve the diameter.
$A = 10(4) + \frac{1}{2}\pi(5)^2$	Add the rectangle and semicircle areas.
$P = 10 + 4 + 4 + \frac{1}{2}(2\pi \cdot 5)$	Add the three outside straight sides and the curved edge. Do not count the dashed join.

$A = 40 + 12.5\pi \,\text{cm}^2, \quad P = 18 + 5\pi \,\text{cm}$

Circles

Try the questions before opening the separate answer key. Show one step per line.

18.1 · Fluency

A circle has diameter 12 cm. Find its circumference and area in terms of π.

18.2 · Variation

An arc subtends 146° at the centre. Find the angle on the same arc at the circumference.

18.3 · Choose a method

A cyclic quadrilateral has one angle of 112°. Find the angle opposite and state the fact used.

Check your work: are the sign, units and level of accuracy appropriate?