Sample-space tables

A sample space lists all possible outcomes. Keep the order of the trials clear.

First toss ↓ / second →	H	T
H	HH	HT
T	TH	TT

Two fair coins are tossed. Find the probability of exactly one head.

Favourable: HT, TH	These are different ordered outcomes.
$P(\text{exactly one head}) = \frac{2}{4}$	Two of the four equally likely outcomes fit.

$\frac{1}{2}$

And or or?

A single path represents a sequence of events. When different paths all satisfy the question, add their probabilities.

Without replacement

The second draw is made from a smaller bag.

A bag has 3 red and 2 blue counters. Two are drawn without replacement. Find the probability both are red.

$P(R_1) = \frac{3}{5}$	At the first draw, 3 of 5 counters are red.
$P(R_2 \mid R_1) = \frac{2}{4}$	After drawing a red, 2 of 4 remaining counters are red.
$P(\text{both red}) = \frac{3}{5} \times \frac{2}{4} = \frac{6}{20}$	Multiply the probabilities along that path.

$\frac{3}{10}$

With replacement

Put the first counter back before the next draw: the composition resets, so here the probability of red on each draw would be 3 out of 5.

More than one possible path

Multiply along each path. Add the probabilities of the separate paths that fit.

From the same bag, draw twice without replacement. Find the probability of one red and one blue.

$P(RB)=\frac{3}{5}\times\frac{2}{4}=\frac{6}{20}$	First red, then blue.
$P(BR)=\frac{2}{5}\times\frac{3}{4}=\frac{6}{20}$	First blue, then red.
$P(RB \text{ or } BR)=\frac{6}{20}+\frac{6}{20}$	These two paths cannot happen on the same pair of draws.

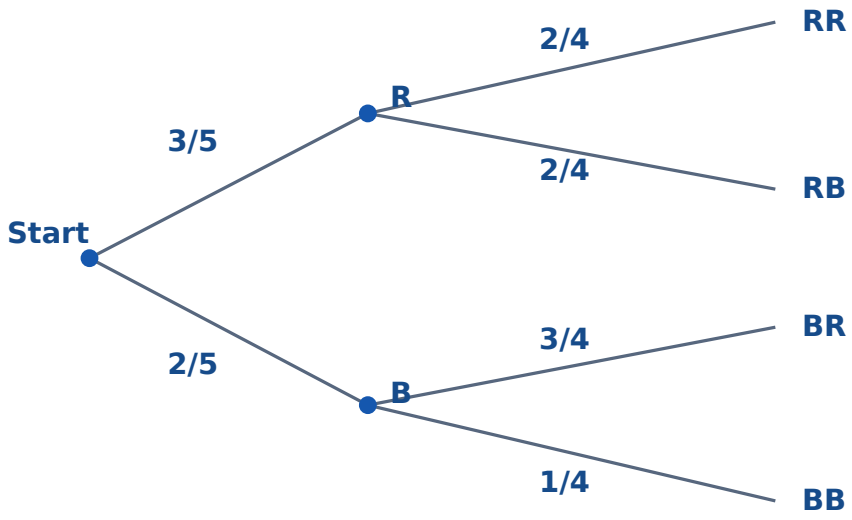
$\frac{3}{5}$

At least one

For “at least one”, it can be quicker to subtract the probability of none from 1.

Reading a probability tree

Each final outcome follows one complete path from the start.



The branches show two draws without replacement from 3 red and 2 blue counters.

Find the probability of at least one red.

$P(BB)=\frac{2}{5}\times\frac{1}{4}=\frac{1}{10}$	The only path with no red is blue, then blue.
$1-\frac{1}{10}=\frac{9}{10}$	Subtract the probability of no red from 1.

$\frac{9}{10}$

Combined Events

Try the questions before opening the separate answer key. Show one step per line.

27.1 · Fluency

List the sample space for two fair coin tosses.

27.2 · Variation

A bag holds three red and two blue counters. Find P(two red) without replacement.

27.3 · Choose a method

With the same bag, find the chance of one counter of each colour without replacement.

Check your work: are the sign, units and level of accuracy appropriate?