PATTERNS

Reading a pattern

Separate the position of a term from its value before doing any calculation.

THE ESSENTIAL DISTINCTION

n is the position number. T_n is the value at that position. In the sequence 5, 8, 11, 14, ... the number 3 is a position, while 11 is the value at that position.

Position n	1	2	3	4	5
Term value	5	8	11	14	17
1st difference		3	3	3	3

POSITION QUESTION

"Find the 20th term" gives n = 20. You must find T_{20}.

VALUE QUESTION

"Which term has value 62?" gives $T_n = 62$. You must find n.

BEFORE SOLVING

Write: I know _____. I need to find _____. This prevents a term value from being substituted as a position number.

How patterns are classified

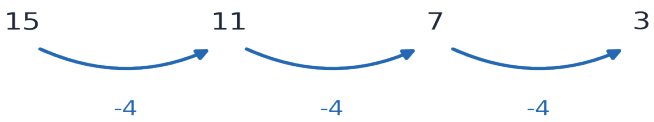
LINEAR	QUADRATIC	MULTIPLICATIVE
Constant first difference. Example: 5, 8, 11, 14, ...	Constant second difference. Example: 3, 8, 15, 24, ...	Constant ratio. Example: 3, 6, 12, 24, ...

PATTERNS

Finding the general term of a linear sequence

Use the common difference and $T_n = a + (n - 1)d$.

Worked example · 15, 11, 7, 3, ...



Common difference d = -4

STEP 1 · IDENTIFY A
The first term is a = 15.

STEP 2 · IDENTIFY D
Each term decreases by 4, so d = -4.

LINEAR-PATTERN FORMULA	MEANING OF THE LETTERS
$T_n = a + (n - 1)d$	a = first term d = common difference n = position number T_n = value at position n

Step 3 · Substitute and simplify — ONE CHANGE PER LINE

$T_n = a + (n - 1)d$	Start with the formula.
$T_n = 15 + (n - 1)(-4)$	Substitute a = 15 and d = -4.
$T_n = 15 - 4n + 4$	Expand the bracket carefully.
FINAL ANSWER $T_n = -4n + 19$	This is the general term.

WHY N - 1?
The first term is already present. Reaching term n requires n - 1 changes.

STEP 4 · CHECK
$T_1 = -4(1) + 19 = 15$ and $T_4 = -4(4) + 19 = 3$.

PATTERNS

Three different question types

Classify the task before substituting into the formula.

QUESTION TYPE	WHAT IS GIVEN?	WHAT DO YOU DO?
Find the rule	The sequence	Keep n. Simplify T_n.
Find a term value	A position n	Substitute the position for n.
Find the position	A value T_n	Set the rule equal to the value and solve for n.

A · Find the 20th term of 5, 8, 11, ... — ONE CHANGE PER LINE

I know: n = 20 I need: T_{20}	The given number is a position.
$T_n = 3n + 2$	Use the rule for this sequence.
$T_{20} = 3(20) + 2$	Substitute 20 for n.
FINAL ANSWER $T_{20} = 62$	The answer is a term value.

B · Which term has value 62? — ONE CHANGE PER LINE

I know: $T_n = 62$ I need: n	The given number is a value.
$62 = 3n + 2$	Set the rule equal to 62.
$60 = 3n$	Subtract 2 from both sides.
FINAL ANSWER $n = 20$	The answer is a position: 62 is the 20th term.

DO NOT DO THIS

T_{62} means the term at position 62. It does not mean "the term whose value is 62".

PATTERNS

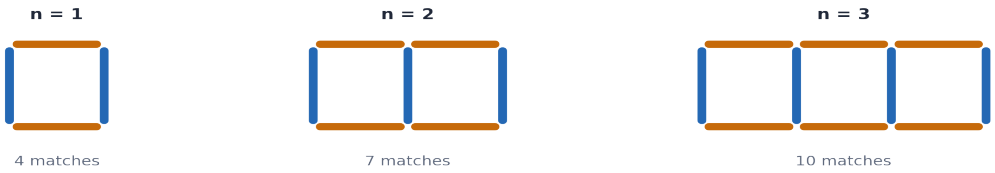
Patterns shown with diagrams or tables

Count each diagram, make a value row, and then use the same linear method.

RELIABLE METHOD

1. Label the diagrams n = 1, 2, 3, ... 2. Count the required objects. 3. Find the difference. 4. Build and check the rule.

Worked example · Matchsticks



Position n	1	2	3	4
Term value	4	7	10	13
1st difference		3	3	3

Build the rule — ONE CHANGE PER LINE

$a=4 \quad d=3$	Identify the first term and common difference.
$T_n=4+(n-1)3$	Use the linear-pattern formula.
$T_n=4+3n-3$	Expand.

FINAL ANSWER

$T_n=3n+1$

The 3n counts three matches for each repeated section; the extra 1 completes the first end. For 10 squares, $T_{10} = 31$.

When the table starts at n = 0

Position n	0	1	2	3
Term value	2	6	10	14
1st difference		4	4	4

Use the supplied positions — ONE CHANGE PER LINE

$T_n=4n+2$	At n = 0, the 4n part is zero, leaving 2.

FINAL ANSWER

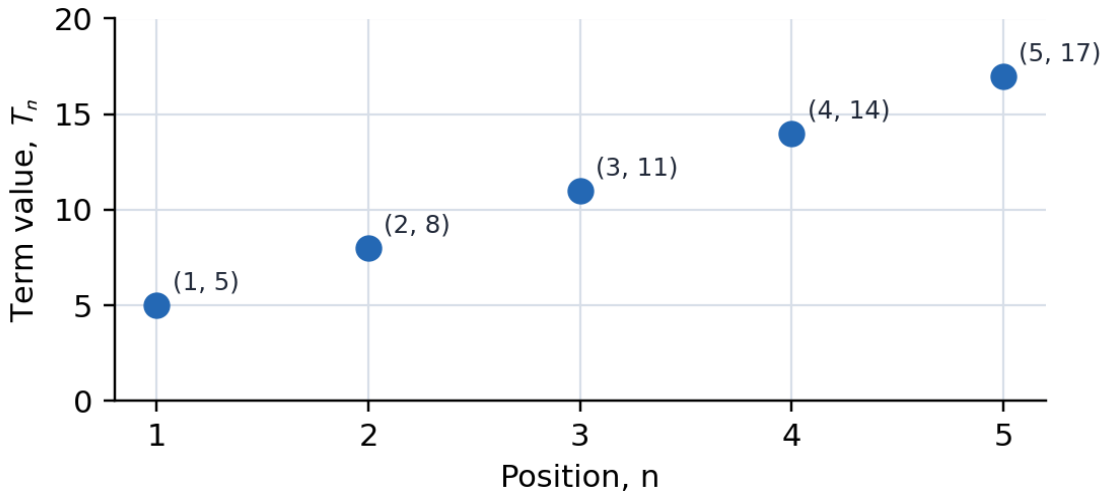
$T_n=4n+2$

Always use the position labels supplied in the question.

PATTERNS

Graphing a sequence

A sequence is discrete: plot separate points for whole-number positions.



HORIZONTAL AXIS
Position n. For an ordinary sequence, n = 1, 2, 3, ...

VERTICAL AXIS
Term value T_n. The point (3, 11) means the third term has value 11.

DO NOT JOIN THE POINTS
Values between positions are not terms of the sequence. A line may show the linear trend, but the sequence itself consists of the plotted points.

Reading a graph	ONE CHANGE PER LINE
$(n, T_n) = (4, 14)$	The x-coordinate is the position.
$T_4 = 14$	The y-coordinate is the term value.
$T_n = 3n + 2$	The plotted points follow this rule.
FINAL ANSWER $T_5 = 17$	At n = 5, the next plotted point is (5, 17).

PATTERNS

Recognising quadratic patterns

A quadratic pattern has changing first differences but constant second differences.

Position n	1	2	3	4
Term value	13	17	29	49
1st difference		4	12	20
2nd difference			8	8

FIRST DIFFERENCES
4, 12, 20 are not constant, so the pattern is not linear.

SECOND DIFFERENCES
8, 8 are constant, so a quadratic rule is appropriate.

KEY FACT
For $T_n = an^2 + bn + c$, the constant second difference is 2a. Therefore a is half the second difference.

Find the coefficient of n^2 — ONE CHANGE PER LINE

$\text{second difference} = 8$	Read the constant second difference.
$2a = 8$	The second difference equals 2a.
FINAL ANSWER $a = 4$	Now use two terms to find b and c.

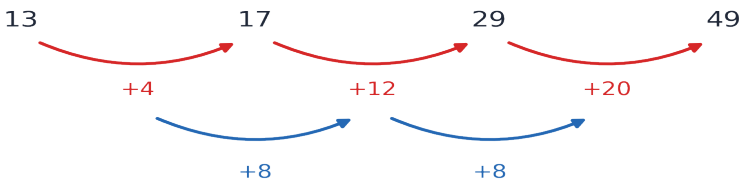
COMMON MISTAKE
Do not use the whole second difference as the coefficient of n^2. Divide it by 2 first.

PATTERNS

Finding a quadratic nth-term rule

Use $T_n = an^2 + bn + c$. Find a first, then solve two equations for b and c.

Step 1 · Find the second difference



Second difference = 8

Step 2 · Find a

Use second difference = 2a	ONE CHANGE PER LINE
$2a = 8$	The second difference is 8.
FINAL ANSWER $a = 4$	Substitute a = 4 into $T_n = an^2 + bn + c$.

Step 3 · Form two equations

USE THE FIRST TERM	USE THE SECOND TERM
$T_1 = 4(1)^2 + b(1) + c = 13$	$T_2 = 4(2)^2 + b(2) + c = 17$
$4 + b + c = 13$	$16 + 2b + c = 17$
$b + c = 9$	$2b + c = 1$

Step 4 · Solve the simultaneous equations

Eliminate c, then substitute	ONE CHANGE PER LINE
$(b + c) - (2b + c) = 9 - 1$	Subtract the second equation from the first.
$-b = 8$	The c terms cancel.
$b = -8$	Multiply both sides by -1.
$-8 + c = 9$	Substitute b = -8 into b + c = 9.
$c = 17$	Add 8 to both sides.
FINAL ANSWER $T_n = 4n^2 - 8n + 17$	Check: $T_3 = 4(3^2) - 8(3) + 17 = 29$.

PATTERNS

Choosing the correct method

Use the structure of the pattern and the wording of the question.

WHAT YOU SEE	WHAT IT MEANS	NEXT STEP
Constant first difference	Linear pattern	Use $T_n = a + (n - 1)d$.
Constant second difference	Quadratic pattern	Use 2a, then form two equations for b and c.
Constant ratio	Multiplicative pattern	Continue by multiplying by the ratio.
"Find the 30th term"	A position is given	Substitute n = 30.
"Which term is 143?"	A value is given	Set $T_n = 143$ and solve for n.

Final paired example · $T_n = 4n - 5$ — ONE CHANGE PER LINE

$T_{12} = 4(12) - 5$	"Find the 12th term" gives n = 12.
$T_{12} = 43$	The answer is a value.
$143 = 4n - 5$	"Which term has value 143?" gives $T_n = 143$.
$148 = 4n$	Add 5 to both sides.

FINAL ANSWER

$n = 37$

143 is the 37th term, not the 143rd term.

FINAL CHECK

1. Did I distinguish position from value? 2. Did I use the correct difference? 3. Does my answer have the right form: rule, value or position? 4. Does it reproduce the original terms?

Patterns

Try the questions before opening the separate answer key. Show one step per line.

13.1 · Fluency

Find the general term of 12, 17, 22, 27, …

13.2 · Variation

Which term of that sequence has value 72?

13.3 · Choose a method

Classify 3, 9, 27, 81, … and 2, 5, 10, 17, …, giving a reason for each.