Solving linear equations

Junior Cycle · Higher Level

Understand the idea

How to start

Solve means find a value that makes the equation true. The equals sign means both sides have the same value.

1. Keep the equation balanced: do the same operation to both sides.
2. Undo the operation on the variable, writing the complete new equation each time.
3. Put your answer into the original equation to check both sides agree.

Key terms & definitions

Solution
A value that makes the original equation true.

Inverse operation
An operation that undoes another, such as subtracting to undo addition.

Identity
An equation true for every permitted value.

Key ideas

• An equation is a balance. Do the same operation to both sides to preserve equality.

• Undo addition or subtraction, then multiplication or division. Choose an order that simplifies the equation.

• Expand brackets and collect like terms before isolating the variable.

• For fractions, multiply every term on both sides by a common denominator.

• Check the solution by substituting it into the original equation. A fractional solution can be fully correct.

Quick reminder

• Goal: leave the variable on its own.

• Do the same operation to both sides.

• Check in the original equation.

Worked examples

Example 01 · Equality as a relationship

$14 + 9 = 18 + \square$

Why this method?

Our goal is to leave the variable on its own. Use inverse operations on both sides so that the equation stays balanced.

$14 + 9 = 18 + \square$	**Step 1** Write the complete equation
$23 = 18 + \square$	**Step 2** Evaluate the left side
$\square = 5$	**Step 3** Find the missing value
$14 + 9 = 23$ and $18 + 5 = 23$	**Step 4** Check both sides

Example 02 · One-step additive equations

$x + 8 = 21$

Purple marks the same subtraction on both sides of the equation.

Why this method?

Our goal is to leave the variable on its own. Use inverse operations on both sides so that the equation stays balanced.

$x + 8 = 21$	**Step 1** The x has 8 added to it. Undo this by subtracting 8 from both sides.

$x + 8 - 8 = 21 - 8$	**Step 2** Both sides must change by the same amount.
$x = 13$	**Step 3** The $+8$ and -8 cancel. Calculate $21 - 8$.
$13 + 8 = 21$	**Step 4** Check the result in the original equation.

Example 03 · One-step multiplicative equations

$6x = 42$

Purple marks division on both sides. Each whole side is divided by 6.

Why this method?

Our goal is to leave the variable on its own. Use inverse operations on both sides so that the equation stays balanced.

$6x = 42$	**Step 1** Six x means 6 multiplied by x.
$\frac{6x}{6} = \frac{42}{6}$	**Step 2** Undo multiplication by 6 by dividing both sides by 6.
$x = 7$	**Step 3** Six divided by six is one, leaving one x.
$6(7) = 42$	**Step 4** Check the answer in the original equation.

Example 04 · Two-step equations

$3x + 5 = 20$

Purple marks the matching operation on both sides: first subtract 5, then divide by 3.

Why this method?

Our goal is to leave the variable on its own. Use inverse operations on both sides so that the equation stays balanced.

$3x + 5 = 20$	**Step 1** The goal is x on its own.

$3x + 5 - 5 = 20 - 5$	**Step 2** Undo the added 5 by subtracting 5 from both sides.
$3x = 15$	**Step 3** Simplify both sides.
$\frac{3x}{3} = \frac{15}{3}$	**Step 4** Undo multiplication by 3 by dividing both sides by 3.
$x = 5$	**Step 5** Calculate the divisions.
$3(5) + 5 = 20$	**Step 6** Check: the original left-hand side is $15 + 5 = 20$.

Example 05 · Equations involving brackets

$3(x + 4) = 27$

Why this method?

Our goal is to leave the variable on its own. Use inverse operations on both sides so that the equation stays balanced.

$3(x + 4) = 27$	**Step 1** Write the original equation
$3x + 12 = 27$	**Step 2** Expand: multiply each term in the bracket by 3
$3x = 15$	**Step 3** Subtract 12 from both sides to remove that term while keeping the equation balanced.
$x = 5$	**Step 4** Divide every term on both sides by 3 so that the coefficient of the variable becomes 1.
$3(5 + 4) = 3 \times 9 = 27$ □	**Step 5** Put the answer back into the original information and check that all the stated conditions are satisfied.

Example 06 · Variables on both sides

$5x + 3 = 2x + 18$

Why this method?

Our goal is to leave the variable on its own. Use inverse operations on both sides so that the equation stays balanced.

$5x + 3 = 2x + 18$	**Step 1** Write the original equation
$3x + 3 = 18$	**Step 2** Subtract $2x$ from both sides to remove that term while keeping the equation balanced.
$3x = 15$	**Step 3** Subtract 3 from both sides to remove that term while keeping the equation balanced.
$x = 5$	**Step 4** Divide every term on both sides by 3 so that the coefficient of the variable becomes 1.
$5(5) + 3 = 28$ and $2(5) + 18 = 28$ □	**Step 5** Put the answer back into the original information and check that all the stated conditions are satisfied.

Example 07 · Linear equations with rational coefficients on both sides

Solve $\frac{((-5)x + (12))}{2} = \frac{((-1)x + (9))}{5}$. Give x as a reduced fraction.

Amber marks the multiplier 10 on both sides. It multiplies each whole fraction.

Why this method?

Our goal is to leave the variable on its own. Use inverse operations on both sides so that the equation stays balanced.

$10 \times \frac{-5x + 12}{2} = 10 \times \frac{-x + 9}{5}$	**Step 1** The denominators are 2 and 5. Their lowest common multiple is 10, so multiply every term on both sides by 10.

$5(-5x+12)=2(-x+9)$	**Step 2** Divide out the denominators : $10 \div 2 = 5$ and $10 \div 5 = 2$.
$-25x+60=-2x+18$	**Step 3** Expand both brackets, including the negative signs.
$60=23x+18$	**Step 4** Add $25x$ to both sides so the x terms collect on the right.
$42=23x$	**Step 5** Subtract 18 from both sides.
$x=\frac{42}{23}$	**Step 6** Divide both sides by 23 and write the exact solution.

Think about it: Why must clearing denominators multiply every term, including terms inside brackets?

Example 08 · Linear equations with rational coefficients on both sides — another example

Solve $\frac{((-4)x+(-12))}{2}=\frac{((-5)x+(6))}{7}$. Give x as a reduced fraction.

Why this method?

Our goal is to leave the variable on its own. Use inverse operations on both sides so that the equation stays balanced.

$(-28)x+(-84)=(-10)x+(12).$	**Step 1** Multiply both sides by the product of the denominators
$(-18)x=96.$	**Step 2** Collect x terms and constants
$x=-\frac{16}{3}$	**Step 3** Divide and reduce the fraction

Example 09 · Linear equations with fractional solutions

Solve for x : $5x + 8 = 15$. Give the answer as a fraction in its simplest form.

Why this method?

Our goal is to leave the variable on its own. Use inverse operations on both sides so that the equation stays balanced.

$5x = 7$	**Step 1** Subtract the constant from both sides to remove that term while keeping the equation balanced.
$x = \frac{7}{5}$	**Step 2** Divide every term on both sides by the number in front of x so that the coefficient of the variable becomes 1.

Think about it: Before moving on: why must the answer be divided by the number in front of x, and not turned upside down?

Common mistake

Dividing only one term when dividing an equation. From $2x + 6 = 10$, dividing both sides by 2 gives $x + 3 = 5$, not $x + 6 = 5$. Every term must be divided.

Exam technique

Keep an equals sign on each working line; show both sides changing together.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

$9 \times 3 = 20 +$ □. Find □.

Question 2

$23 = k + 9$. Find k.

Question 3

$8p = 100$. Find p.

Question 4

$7k - 9 = 33$. Find k.

Question 5

$5(k - 3) = 35$. Find k.

Question 6

$10k - 7 = 6k + 9$. Find k.

Question 7

Solve
$\frac{((3)x + (9))}{6} = \frac{((-2)x + (-1))}{7}$
. Give x as a reduced fraction.

A. $x = -\frac{23}{11}$

B. $x = \frac{19}{11}$

C. $x = -\frac{23}{7}$

D. $x = -2$

Question 8

Solve for x : $5x - 2 = 1$. Give the answer as a fraction in its simplest form.

A. $x = \frac{3}{5}$

B. $x = -\frac{1}{5}$

C. $x = \frac{5}{3}$

D. $x = 3$

Watch an explanation

Cognito — Algebraic Equations (2 Operations & 1 Subject)

Cognito — Algebraic Equations (2 Subjects)

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

7

Question 2

14

Question 3

12.5

Question 4

6

Question 5

10

Question 6

4

Question 7

$x = -\frac{23}{11}$

Question 8

$x = \frac{3}{5}$

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