Straight lines and their equations

Junior Cycle · Higher Level

Understand the idea

How to start

In $y = mx + c$, m tells you the slope and c tells you where the line meets the y-axis.

1. Identify the slope and a known point, or read them from the graph.
2. Put the point coordinates into the line equation to find a missing constant.
3. Check that the completed equation gives the correct y for the known x.

Key terms & definitions

Y-intercept
The y-value when x is zero.

Point-slope form
An equation using one known point and the slope.

General form
A line written as $ax + by + c = 0$ with a and b not both zero.

Key ideas

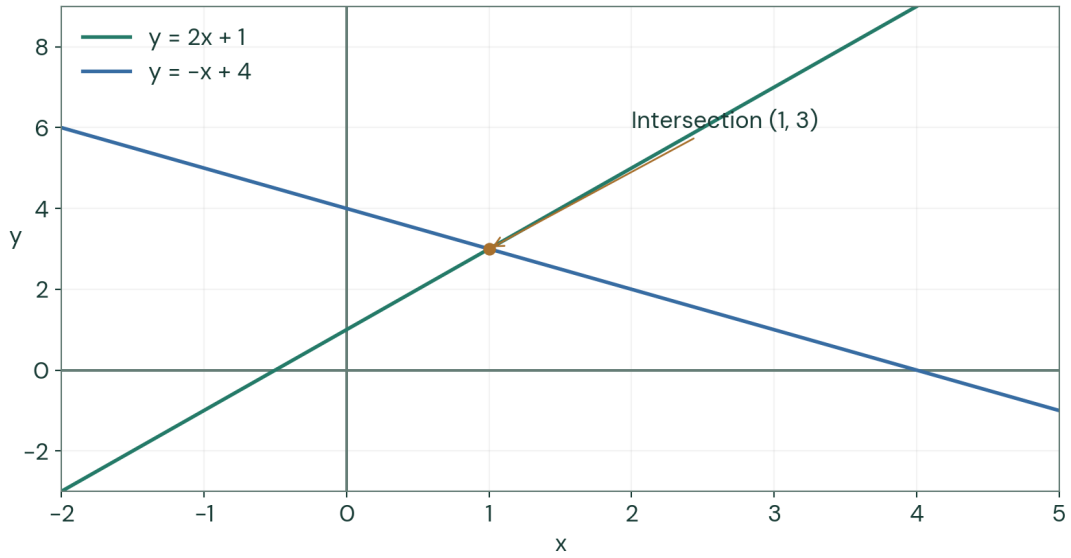
• In $y = mx + c$, m is the slope and c is the y-intercept.

• Given a point (x_1, y_1) and slope m, use $y - y_1 = m(x - x_1)$.

• Parallel non-vertical lines have equal slopes. Perpendicular lines with defined non-zero slopes satisfy $m_1 m_2 = -1$.

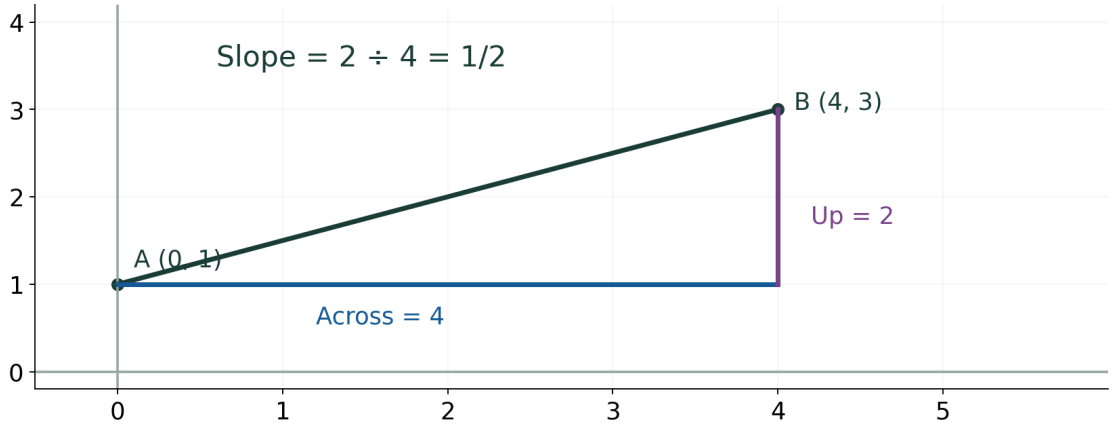
• For $ax + by + c = 0$ with $b \neq 0$, the slope is $-\frac{a}{b}$. If b is zero, the line is vertical.

• Find an intersection by solving both equations together. Interpret the point in the context.



The two lines meet at $(1, 3)$, which satisfies both equations.

Slope: vertical change divided by horizontal change



From $A(0, 1)$ to $B(4, 3)$, move 4 across and 2 up. The slope is vertical change over horizontal change : $\frac{2}{4} = \frac{1}{2}$.

Quick reminder

• $y = mx + c$: m is slope; c is y-intercept.

• Parallel lines have equal slopes; check vertical-line exceptions.

• At an intersection, set the two expressions for y equal.

Worked examples

Example 01 · A parallel line through a point

A line passes through $P(-2, -3)$ and is parallel to $y = -3x + 2$. What is the y-intercept of this line?

Why this method?

Identify the slope and any known point. Use these to form an equation, and check it using the original information.

$y = -3x + c$	**Step 1** The given line has slope -3. A parallel line has the same slope, but may have a different intercept c.
$-3 = -3(-2) + c$	**Step 2** The new line passes through $(-2, -3)$, so put $x = -2$ and $y = -3$ into its equation.
$-3 = 6 + c$	**Step 3** Calculate $-3 \times (-2) = +6$.
$-3 - 6 = c$	**Step 4** Subtract 6 from both sides to leave c on its own.
$c = -9$	**Step 5** The y-intercept value is -9, at the point $(0, -9)$.
$-3 = -3(-2) - 9$	**Step 6** Check the known point: the right side is $6 - 9 = -3$.

Think about it: Before moving on: why does a parallel line have the same slope as the line it is parallel to?

Example 02 · Where two lines meet

The lines $y = 1x + 6$ and $y = 2x + 3$ meet at one point. What is the x-coordinate of that point?

Why this method?

Identify the slope and any known point. Use these to form an equation, and check it using the original information.

$x + 6 = 2x + 3$	**Step 1** At the intersection both expressions have the same y-value. Set them equal.
$6 = x + 3$	**Step 2** Subtract x from both sides to leave one x on the right.
$x = 3$	**Step 3** Subtract 3 from both sides.
$y = 3 + 6 = 9$	**Step 4** The question asks only for x. To see the complete point, substitute $x = 3$ into either line.
$y = 2(3) + 3 = 9$. The intersection is $(3, 9)$.	**Step 5** Check the other line; it must give the same y.

Think about it: Before moving on: why do the two lines have the same y-value at the point where they meet?

Example 03 · Rate of change with a starting amount

A tank holds 55 litres and loses 6 litres each minute. How many litres are in the tank after 4 minutes?

Why this method?

Identify the slope and any known point. Use these to form an equation, and check it using the original information.

$6 \times 4 = 24$	**Step 1** Find the total lost : 6 litres each minute for 4 minutes is 24 litres.
$55 - 24 = 31$	**Step 2** Subtract the loss from the starting 55 litres.

There are 31 litres left.	**Step 3** The result is smaller than 55 litres, as expected for a leak.

Think about it: Before moving on: why is the starting amount needed as well as the rate?

Example 04 · The equation of a line from its slope and a point

A straight line has slope -2 and passes through the point $(2, -5)$. Find its equation in the form $y = mx + c$.

Why this method?

Identify the slope and any known point. Use these to form an equation, and check it using the original information.

$c = -5 - (-2 \times 2) = -1$	**Step 1** Substitute the slope m and the point (x_1, y_1) into $c = y_1 - m \times x_1$
$y = -2x - 1$	**Step 2** Write the line with that intercept

Think about it: Before moving on: why does the intercept c come from subtracting $m \times x_1$ from y_1?

Example 05 · Slope of a perpendicular line

A line has slope 3. Which is the slope of a line perpendicular to it?

Why this method?

Identify the slope and any known point. Use these to form an equation, and check it using the original information.

$3 \times \text{slope} = -1$	**Step 1** Perpendicular slopes multiply to -1
$-\frac{1}{3}$	**Step 2** So the perpendicular slope is the negative reciprocal

Think about it: Before moving on: why does the sign change as well as the fraction turning over?

Example 06 · Slope of a line in the general form

The line $7x + 2y + 6 = 0$ has a slope. What is its slope?

Why this method?

Identify the slope and any known point. Use these to form an equation, and check it using the original information.

$y = -\frac{7}{2x} + \ -\frac{6}{2}$	**Step 1** Rearrange to $y =$ mx $+k$: take ax and c to the other side, then divide by b
$m = -\frac{7}{2}$	**Step 2** The slope is the coefficient of x

Think about it: Before moving on: why is the slope not simply the coefficient of x, but $-\frac{a}{b}$?

Common mistake

Using a negative slope to mean “perpendicular.” A line of slope 2 is perpendicular to one of slope $-\frac{1}{2}$, not -2. For two finite slopes, the product must be -1. A horizontal line is perpendicular to a vertical line, whose slope is undefined.

Exam technique

Substitute the given point into your final equation to check it.

Try it yourself

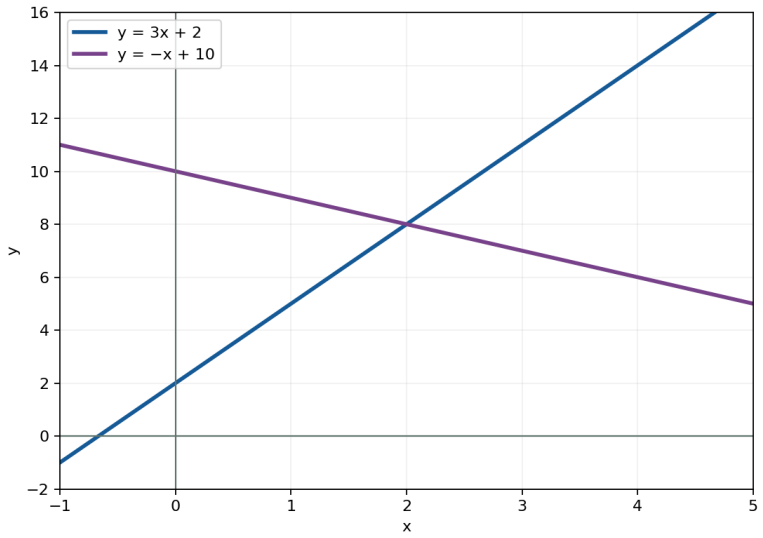
Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

A line passes through $P(4, 9)$ and is parallel to $y = 2x - 3$. What is the y-intercept of this line?

Question 2

The lines $y = 3x + 2$ and $y = -1x + 10$ meet at one point. What is the x-coordinate of that point?



Two lines $y = 3x + 2$ and $y = -x + 10$ on numbered axes. Use their intersection to read the x-coordinate.

Question 3

A tank holds 90 litres and loses 7 litres each minute. How many litres are in the tank after 5 minutes?

Question 4

A straight line has slope 2 and passes through the point $(3, 1)$. Find its equation in the form $y = mx + c$.

A. $y = 2x + 1$

B. $y = 2x + 7$

C. $y = 2x$

D. $y = 2x - 5$

Question 5

A line has slope 14. Which is the slope of a line perpendicular to it?

A. $-\frac{1}{14}$

B. -14

C. 14

D. $\frac{1}{14}$

Question 6

The line $5x + 2y + 7 = 0$ has a slope. What is its slope?

A. $-\frac{5}{2}$

B. $\frac{5}{2}$

C. $-\frac{2}{5}$

Watch an explanation

Cognito — What is 'y = mx + c'

Cognito — Finding the Equation of a Line from 2 Coordinates

Cognito — Perpendicular Lines

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

1

Question 2

2

Question 3

55

Question 4

$y = 2x - 5$

Question 5

$-\frac{1}{14}$

Question 6

$-\frac{5}{2}$

Open the full chapter online