Patterns and nth terms

Junior Cycle · Higher Level

Understand the idea

How to start

The term number tells you the position. The term value tells you the number at that position.

1. Write the position numbers above the terms, starting with 1 unless told otherwise.
2. Find the differences between neighbouring terms.
3. Test a proposed rule using more than one position.

Key terms & definitions

Sequence
An ordered list of terms.

First difference
The change from one term to the next.

Nth term
A rule for the term at position n.

Key ideas

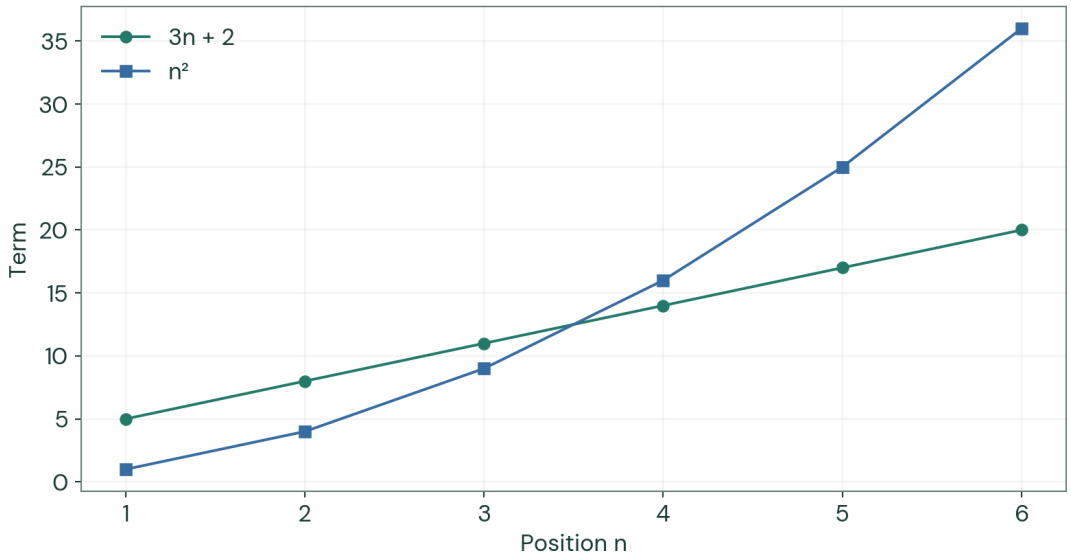
• The position number n starts at 1 unless the question states otherwise.

• A constant first difference suggests a linear sequence. For first term a and difference d, $T_n = a + (n - 1)d$.

• A constant non-zero second difference suggests a quadratic sequence $T_n = an^2 + bn + c$; its second difference is $2a$.

• A constant ratio suggests a geometric pattern, such as repeated doubling or tripling.

• Check a proposed rule against several terms and the context. A short list alone does not prove a unique general rule.



Linear pattern $3n + 2$ has constant first difference; quadratic pattern n^2 has constant second difference.

Quick reminder

• Write the term positions, starting at the stated n-value.

• Constant first difference → linear; constant non-zero second difference → quadratic.

• Check your rule against the original terms.

Worked examples

Example 01 · Continuing a numerical pattern

The pattern starts $3, 8, 13, 18, \ldots$ What is the next term?

Why this method?

Write the position numbers beside the terms. Check differences or ratios before choosing the form of the rule.

$8 - 3 = 5$	**Step 1** Find the change from the first term to the second.
$13 - 8 = 5, \quad 18 - 13 = 5$	**Step 2** Check the same change happens in the next two steps.
$18 + 5 = 23$	**Step 3** Continue the pattern by adding 5 to the last term shown.

Think about it: Before moving on: how can you check the common difference is right for every pair of neighbouring terms?

Example 02 · The term at any position

A pattern starts with 2 and goes up by 4 each time. What is term 10?

Position	1	2	3	4	5
Term	2	6	10	14	18

Why this method?

Write the position numbers beside the terms. Check differences or ratios before choosing the form of the rule.

$T_1 = 2$	**Step 1** The first term is 2. No increases have happened yet.
$10 - 1 = 9$	**Step 2** From position 1 to position 10 there are nine steps.
$9 \times 4 = 36$	**Step 3** Each step adds 4, so the total increase is 36.
$T_{10} = 2 + 36 = 38$	**Step 4** Add the increase to the first term.

Think about it: Before moving on: how many differences are added to reach term 10?

Example 03 · Finding a term of a linear pattern

The pattern starts $3, 9, 15, 21, \ldots$ What is term number 12?

Why this method?

Write the position numbers beside the terms. Check differences or ratios before choosing the form of the rule.

$9 - 3 = 6$	**Step 1** The pattern adds 6 each time.
$12 - 1 = 11$	**Step 2** Term 12 is eleven steps after term 1.
$11 \times 6 = 66$	**Step 3** Find the total increase across eleven steps.

$T_{12} = 3 + 66 = 69$	**Step 4** Start at the first term, 3, then add the increase.

Think about it: Before moving on: why is there one fewer difference than the term number?

Example 04 · Classifying quadratic, doubling and tripling patterns

What kind of pattern is $1, 2, 4, 8, 16, \ldots$?

Why this method?

Write the position numbers beside the terms. Check differences or ratios before choosing the form of the rule.

ratios $: \frac{2}{1}, \frac{4}{2}, \frac{8}{4}, \frac{16}{8},$ all equal to 2, so exponential	**Step 1** Check the differences, the second differences, or the ratios
Exponential (doubling)	**Step 2** Name the kind of pattern

Think about it: Before moving on: how can you tell a quadratic pattern from an exponential one, using only the terms?

Example 05 · Classifying quadratic, doubling and tripling patterns — another example

What kind of pattern is $1, 4, 9, 16, 25, \ldots$?

Why this method?

Write the position numbers beside the terms. Check differences or ratios before choosing the form of the rule.

differences : $3, 5, 7, 9$; second differences : $2, 2, 2$, all equal, so quadratic	**Step 1** Check the differences, the second differences, or the ratios
Quadratic	**Step 2** Name the kind of pattern

Example 06 · Quadratic nth-term rules

The first five terms, starting at $n = 1$, are $4, 15, 30, 49, 72$. Find a rule for the nth term.

Why this method?

Write the position numbers beside the terms. Check differences or ratios before choosing the form of the rule.

$15 - 4 = 11, 30 - 15 = 15, 49 - 30 = 19,$ $72 - 49 = 23.$	**Step 1** Subtract each term from the next to find the first differences.
$15 - 11 = 4, 19 - 15 = 4, 23 - 19 = 4.$	**Step 2** Subtract neighbouring first differences. The second difference is constant, so try a quadratic rule.
$2a = 4,$ so $a = 2.$	**Step 3** For $an^2 + bn + c$, the second difference is $2a$. Divide 4 by 2 to find a.
$2,\ 8,\ 18,\ 32,\ 50$	**Step 4** Work out $2n^2$ for positions 1 to 5.
$2,\ 7,\ 12,\ 17,\ 22$	**Step 5** Subtract these quadratic values from the original terms. The leftover values follow a linear rule.
$5(1) + c = 2,$ so $c = -3.$	**Step 6** The leftover difference is 5, so try $5n + c$. At $n = 1$, the leftover is 2, giving $c = -3$.
$T_n = 2n^2 + 5n - 3$	**Step 7** Combine the quadratic part and the leftover linear part.
$T_3 = 2(3)^2 + 5(3) - 3 = 18 + 15 - 3 = 30$	**Step 8** Check a term from the original list.

Think about it: Why does using the whole second difference give the wrong coefficient of n^2?

Example 07 · Quadratic nth-term rules — another example

The first five terms, starting at $n = 1$, are $1, 0, 1, 4, 9$. Find a rule for the nth term.

Why this method?

Write the position numbers beside the terms. Check differences or ratios before choosing the form of the rule.

First differences : $-1, 1, 3, 5$. Second differences : $2, 2, 2$.	**Step 1** Find the first and second differences
$2 \div 2 = 1$	**Step 2** Half the second difference gives the coefficient of n^2
The remaining rule is $-4n + 4$. Hence the nth term is $n^2 - 4n + 4$.	**Step 3** Subtract the quadratic part and find the remaining linear rule

Common mistake

Starting at $n = 0$ when the question numbers the first term as 1. For $T_n = 3n + 2$, the first term is $3(1) + 2 = 5$, not 2.

Exam technique

Write a position row and a value row before finding a rule.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

The pattern starts $6, 11, 16, 21, \ldots$ What is the next term?

Question 2

A pattern starts with 8 and goes up by 3 each time. What is term 15?

Position	1	2	3	4
Term	8	11	14	17

Question 3

The pattern starts $6, 13, 20, 27, \ldots$ What is term number 15?

Question 4

What kind of pattern is $5, 15, 45, 135, 405, \ldots$?

A. Linear

B. Quadratic

C. Exponential (doubling)

D. Exponential (tripling)

Question 5

The first five terms, starting at $n = 1$, are $12, 29, 54, 87, 128$. Find a rule for the nth term.

A. $8n^2 + 5n + 3$

B. $4n^2 + 5n + 3$

C. $4n^2 + 3$

D. $4n^2 - 3n + 2$

Watch an explanation

Cognito — Finding the nth Term (Arithmetic Sequences)

Cognito — Finding the nth Term (Quadratic Sequences)

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

26

Question 2

50

Question 3

104

Question 4

Exponential (tripling)

Question 5

$4n^2 + 5n + 3$

Open the full chapter online