Ratio, proportion and rates

Junior Cycle · Higher Level

Understand the idea

How to start

The order matters: red : blue compares red counters first and blue counters second.

1. Put quantities in the same units before comparing them.
2. To share a total, add the ratio parts and find the value of one part.
3. To find a cost or rate, find one unit first when possible.

Key terms & definitions

Ratio
A comparison of quantities by division.

Unit rate
The amount for one unit, such as euro per kilogram.

Constant of proportionality
The fixed multiplier in a directly proportional relationship.

Key ideas

• A ratio compares quantities in the same units. Simplify it by dividing every part by a common factor.

• To share an amount in a ratio, add the ratio parts, find one part, then scale each share.

• Direct proportion means one quantity is a constant multiple of another: $y = kx$. Its graph passes through the origin.

• A rate compares quantities with different units. Average speed is $\frac{\text{totaldistance}}{\text{totaltime}}$.

• Use the stated exchange rate in the correct direction. A fixed starting charge means a relationship is not direct proportion.



Sharing €84 in the ratio 2 : 5 means seven equal parts of €12.

Quick reminder

• Keep the quantities in the stated order.

• Sharing: add the parts, find one part, then scale.

• Rates: put all measurements into compatible units.

Worked examples

Example 01 · What a ratio means

There are 2 red and 5 blue counters. Write red : blue.

Why this method?

Keep the quantities in the stated order. Find the value of one equal part or one unit, then scale to the amount requested.

red : blue	**Step 1** Read the order: red first, then blue
2 : 5	**Step 2** Put in the two numbers for those colours
2 : 5 is not 2 : 7	**Step 3** Do not add the counters together, because that compares red with all counters

Think about it: Before moving on: why is 5 : 2 not the same ratio as 2 : 5?

Example 02 · Equivalent ratios

$2 : 3 = \ ? \ : 12$

Why this method?

Keep the quantities in the stated order. Find the value of one equal part or one unit, then scale to the amount requested.

$\times 4$	**Step 1** Find the scale factor : $3 \times 4 = 12$
$2 \times 4 = 8$	**Step 2** Do the same to the other part : $2 \times 4 = 8$
$8 : 12$	**Step 3** So $2 : 3 = 8 : 12$

Think about it: Before moving on: what would have happened if you added 4 to both parts instead?

Example 03 · Find one, then scale

6 tickets cost €24. How much do 9 tickets cost?

Why this method?

Keep the quantities in the stated order. Find the value of one equal part or one unit, then scale to the amount requested.

$24 \div 6 = 4$	**Step 1** Divide the total price by 6 to find the cost of one ticket.
One ticket costs €4.	**Step 2** This is the unit price. We assume each ticket has the same price.
$9 \times 4 = 36$	**Step 3** Nine tickets cost nine times the unit price.
Nine tickets cost €36.	**Step 4** State what the calculated number means.

Think about it: Before moving on: why is the price of one ticket found first?

Example 04 · Share in a ratio

Share €84 in the ratio 2 : 5.

Why this method?

Keep the quantities in the stated order. Find the value of one equal part or one unit, then scale to the amount requested.

$2 + 5 = 7$	**Step 1** Count ratio parts
$84 \div 7 = 12$	**Step 2** Find one part
$2 \times 12 = 24$ and $5 \times 12 = 60$.	**Step 3** Scale each share
€24 + €60 = €84 and 24 : 60 simplifies to 2 : 5.	**Step 4** Put the answer back into the original information and check that all the stated conditions are satisfied.

Example 05 · Proportional or not

Pens cost €2 each. Is the total cost directly proportional to the number of pens?

Why this method?

Keep the quantities in the stated order. Find the value of one equal part or one unit, then scale to the amount requested.

0 pens → €0	**Step 1** Check zero : 0 pens cost €0
€2 ÷ 1 = €2, €4 ÷ 2 = €2, €6 ÷ 3 = €2	**Step 2** Check the cost ÷ number of pens for several amounts
directly proportional	**Step 3** The ratio is always the same, so yes

Think about it: Before moving on: what would change if the shop charged a €1 bag fee on every order?

Example 06 · Currency conversion

A task gives the rate €1 = 1.20*X*. Convert €50 to X.

Why this method?

Keep the quantities in the stated order. Find the value of one equal part or one unit, then scale to the amount requested.

$1\text{euro} = 1.20X$	**Step 1** The stated rate gives the amount of X for each euro.
$50 \times 1.20 = 60$	**Step 2** There are 50 euros, so multiply the amount per euro by 50.
€50 = 60*X*.	**Step 3** The result is in X, the currency requested.

Think about it: Why do we multiply by 1.20 in this direction?

Example 07 · Average speed, distance and time

A car travels 30 km in 45 minutes. Find the average speed in km/*h*.

Why this method?

Keep the quantities in the stated order. Find the value of one equal part or one unit, then scale to the amount requested.

$45\text{ min} = \frac{45}{60}h = 0.75h$	**Step 1** Convert 45 minutes to hours
$30 \div 0.75 = 40$	**Step 2** Use average speed = total distance ÷ total time
$\text{average speed} = 40\text{ km}/h$	**Step 3** Include the unit

Think about it: Why must 45 minutes be converted to hours before the calculation?

Common mistake

Treating the first number in a ratio of $2:3$ as two-thirds of the total. There are $2+3=5$ parts, so the first share is $\frac{2}{5}$ of the total.

Exam technique

Convert minutes to hours before calculating speed in kilometres per hour.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Pears : apples $= 9 : 3$. How many pears for each apple?

Question 2

$2 : 9 = 6 :$?. Find the missing number.

Question 3

3 kg of rice costs €8.40. How much do 5 kg cost?

Question 4

A bus fare is €1.50 per stop, with no start or set-up fee. Is the total fare directly proportional to the number of stops?

A. Cannot tell without calculating several fares

B. No, because the fare changes when the number of stops changes

C. No, because €1.50 is not a whole number

D. Yes

Question 5

Rate: $€1 = 1.25X$. Convert €80 to X.

Question 6

A runner covers 12 km in 1.5 hours. What is the average speed in km/h?

Watch an explanation

Cognito — Ratios - Introduction & how to Simplify

Cognito — Proportional Division

Corbettmaths — Speed, distance and time

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

3

Question 2

27

Question 3

14

Question 4

Yes

Question 5

100

Question 6

8

Open the full chapter online