Sets and Venn diagrams

Junior Cycle · Higher Level

Understand the idea

How to start

A set is a collection of items. An overlap contains items that belong to both sets.

1. Name the whole group, called the universal set.
2. Put the shared items in the overlap first.
3. Fill the remaining regions and check that every item is counted once.

Key terms & definitions

Universal set
All elements under consideration, usually labelled U.

Complement
Elements in U that are not in the named set.

Cardinality
The number of elements in a finite set.

Empty set
A set with no elements.

Key ideas

• A set is a collection of distinct elements. State the universal set before finding a complement.

• $A \cup B$ means in A or B or both. $A \cap B$ means in both. $A \setminus B$ means in A but not B.

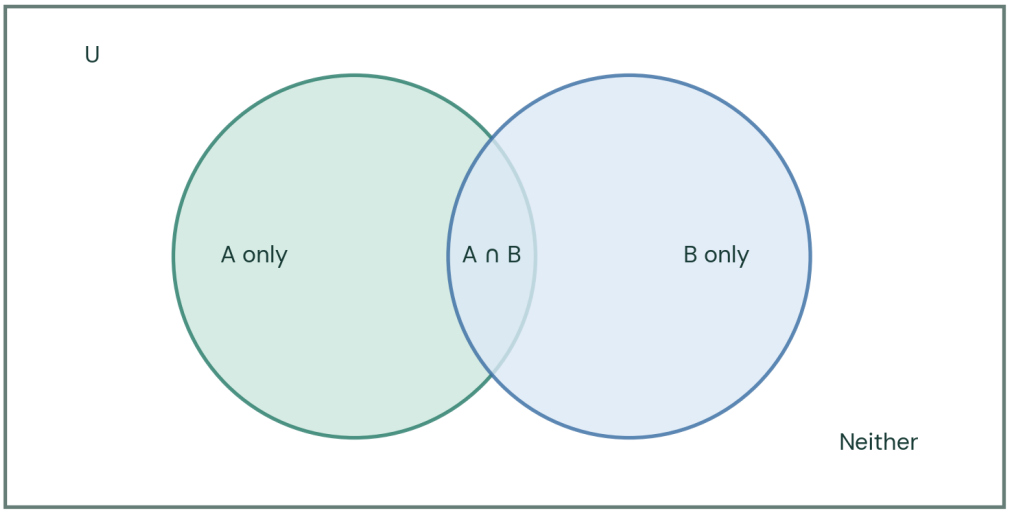
• Fill a Venn diagram from the most specific overlap outwards: the triple intersection first, then pair-only regions, then one-set-only regions.

• For two sets, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. Subtract the union from the universal total to find neither.

• Union and intersection are commutative and associative. These laws regroup sets; they do not change which elements belong.

• A subset contains only elements of another set: $A \subseteq B$. Distinguish an element belonging to a set, $x \in A$, from a set being a subset.

• For three sets,
$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$.



Four separate regions: A only, both A and B, B only, and neither. Together they make the universal set.

Quick reminder

• Union: in either set or both. Intersection: in both.

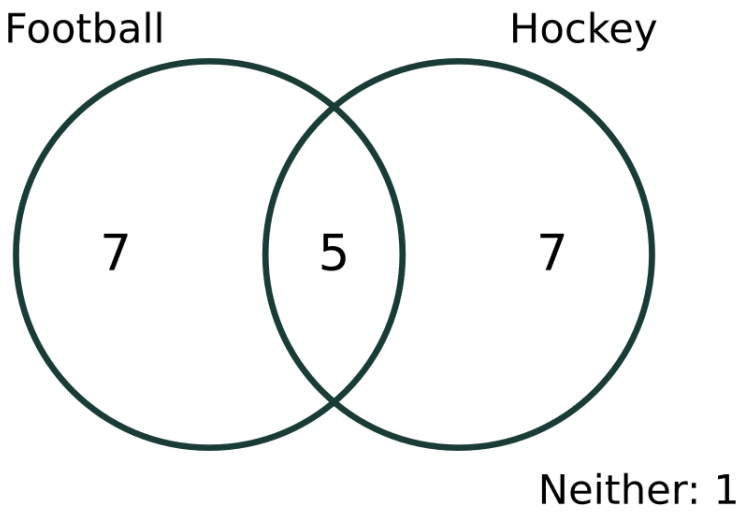
• In Venn diagrams, fill the deepest overlap first.

• Neither = universal total minus the union.

Worked examples

Example 01 · Two-set Venn diagrams

In a group of 20 students, 12 play football, 12 play hockey, and 5 play both. How many play football only?



Two-set Venn diagram for 20 students. Football only : 7. Both : 5. Hockey only : 7. Neither : 1.

Why this method?

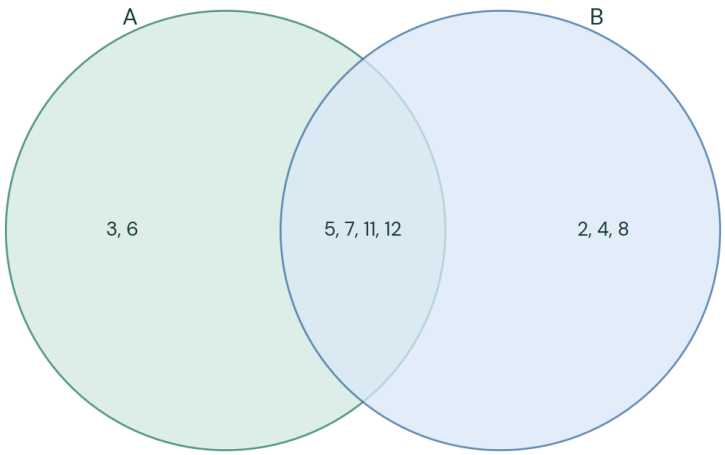
Identify which region or operation the question asks for. Count each element or person only in the regions where they belong.

$\text{Football total} = \text{football only} + \text{both}.$	**Step 1** The 12 football players already include the 5 who play both sports.
$12 = \text{footballonly} + 5$	**Step 2** Put the given numbers into that relationship.
$\text{footballonly} = 12 - 5 = 7$	**Step 3** Subtract the overlap once to leave only the football-only region.

Think about it: Before moving on: why must the overlap be taken away from each circle total?

Example 02 · Set notation: union, intersection and difference

$A = \{3, 5, 6, 7, 11, 12\}$ and $B = \{2, 4, 5, 7, 8, 11, 12\}$. How many elements are in $A \cup B$?



Venn diagram matching the sets in this question. A only : $[3, 6]$. Both : $[5, 7, 11, 12]$. B only : $[2, 4, 8]$.

Why this method?

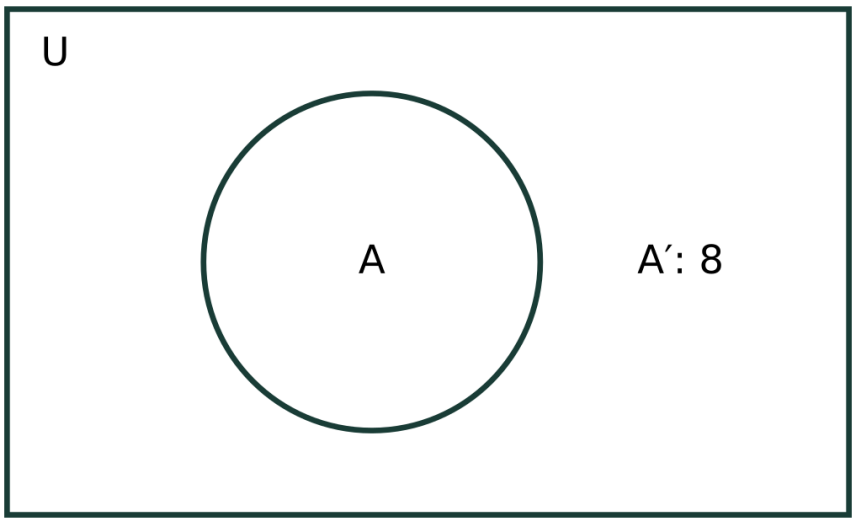
Identify which region or operation the question asks for. Count each element or person only in the regions where they belong.

$A \cup B = \{2, 3, 4, 5, 6, 7, 8, 11, 12\}$	**Step 1** $A \cup B$ contains every element in A or in B, each listed once
9 elements	**Step 2** Count the elements

Think about it: Before moving on: why is an element that is in both sets listed only once in $A \cup B$?

Example 03 · Complement and universal set

The universal set is $U = \{1, 2\}$ …, 12}. $A = \{2, 6, 8, 12\}$. How many elements are in A□, the complement of A?



Universal set $U = \{1\}$ …, 12} drawn as a rectangle, with $A = \{2, 6, 8, 12\}$ as a circle inside it. The complement A□ is the part of U outside the circle, with 8 elements.

Why this method?

Identify which region or operation the question asks for. Count each element or person only in the regions where they belong.

$A□ = \{1, 3, 4, 5, 7, 9, 10, 11\}$	**Step 1** The complement is the elements of U that are not in A
$12 - 4 = 8$	**Step 2** Count the elements of A□

Think about it: Before moving on: why is A□ found by taking A away from U rather than by adding A to U?

Example 04 · Cardinal number and the null set

Hannah writes the set $A = \{2, 2, 9, 12\}$. What is n(A), the number of elements in A?

Why this method?

Identify which region or operation the question asks for. Count each element or person only in the regions where they belong.

Different elements : $\{2, 9, 12\}$	**Step 1** Each element of a set is counted once, even if it is written more than once

$n(A) = 3$	**Step 2** Count the different elements

Think about it: Before moving on: why is the null set not the same as the set $\{0\}$?

Example 05 · Union of three sets

In a school, 14 students take Art, 17 take Music and 13 take Drama. 4 take Art and Music, 7 take Music and Drama, and 3 take Art and Drama. 1 student takes all three. How many students take at least one of the three subjects? The pairwise counts include those who are in all three sets.

Why this method?

Identify which region or operation the question asks for. Count each element or person only in the regions where they belong.

$14 + 17 + 13 = 44$	**Step 1** Add the three set totals
$44 - (4 + 7 + 3) = 30$	**Step 2** Take off each pairwise overlap once
$30 + 1 = 31$	**Step 3** Add back the triple overlap, which was taken off three times and must be counted once

Think about it: Before moving on: why is the triple overlap added back, and why is it added only once?

Example 06 · None of three sets

In a survey of students, the universal set contains 71 people. Set A has 21, set B has 30, and set C has 38. There are 3 in A and B, 9 in A and C, and 9 in B and C. Each pairwise count includes everyone in all three sets. There are 3 in all three sets. How many people are in none of A, B or C?

Why this method?

Identify which region or operation the question asks for. Count each element or person only in the regions where they belong.

$21 + 30 + 38 - (3 + 9 + 9) + 3 = 71$	**Step 1** Count the union: add set totals, subtract inclusive pair counts, add back the triple
$71 - 71 = 0$	**Step 2** Take the complement in the stated universal set

A only 12; B only 21; C only 23; A and B only 0; A and C only 6; B and C only 6; all three 3; none 0. Total 71.	**Step 3** Check all eight disjoint regions total the universal set

Think about it: Why must the universal-set total be known, and why is the triple intersection added back before finding the complement?

Example 07 · None of three sets — another example

In a survey of students, the universal set contains 69 people. Set A has 26, set B has 26, and set C has 30. There are 4 in A and B, 8 in A and C, and 7 in B and C. Each pairwise count includes everyone in all three sets. There are 3 in all three sets. How many people are in none of A, B or C?

Why this method?

Identify which region or operation the question asks for. Count each element or person only in the regions where they belong.

$26 + 26 + 30 - (4 + 8 + 7) + 3 = 66$	**Step 1** Count the union: add set totals, subtract inclusive pair counts, add back the triple
$69 - 66 = 3$	**Step 2** Take the complement in the stated universal set
A only 17; B only 18; C only 18; A and B only 1; A and C only 5; B and C only 4; all three 3; none 3. Total 69.	**Step 3** Check all eight disjoint regions total the universal set

Example 08 · Commutative set operations

$A = \{a, b, g\}, B = \{a, d, f, g\}$. Is $A \cup B$ the same as $B \cup A$?

Why this method?

Identify which region or operation the question asks for. Count each element or person only in the regions where they belong.

$A \cup B = \{a, b, d, f, g\}$	**Step 1** Work out $A \cup B$
$B \cup A = \{a, b, d, f, g\}$	**Step 2** Work out $B \cup A$
They are the same, so the operation is commutative for these sets.	**Step 3** Compare the two results

Think about it: Before moving on: which of union, intersection and difference give the same answer when the sets are swapped, and why?

Example 09 · Associative set operations

$A = \{a, e, f, h\}, B = \{c, f, g, h\}, C = \{e, f\}$. Is $(A \cup B) \cup C$ the same as $A \cup (B \cup C)$?

Why this method?

Identify which region or operation the question asks for. Count each element or person only in the regions where they belong.

$(A \cup B) \cup C = \{a, c, e, f, g, h\}$	**Step 1** Work out $A \cup B$ first, then $\cup C$
$A \cup (B \cup C) = \{a, c, e, f, g, h\}$	**Step 2** Work out $B \cup C$ first, then $A \cup$
They are the same, so the operation is associative for these sets.	**Step 3** Compare the two results

Think about it: Before moving on: which operations let you move the brackets without changing the answer, and what does that mean for the order you do the work in?

Common mistake

Counting the overlap twice when finding how many are in A or B. If $n(A) = 12$, $n(B) = 10$ and $n(A \cap B) = 4$, the total is $12 + 10 - 4 = 18$, not 22.

Exam technique

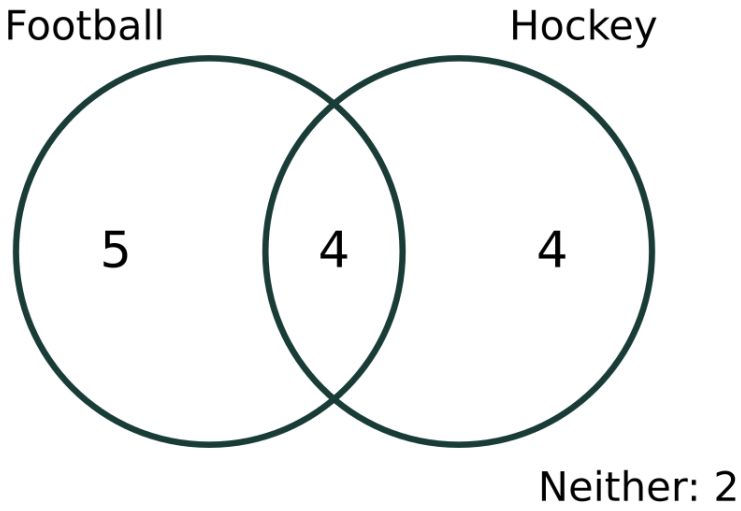
Check that all disjoint regions add to the universal total.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

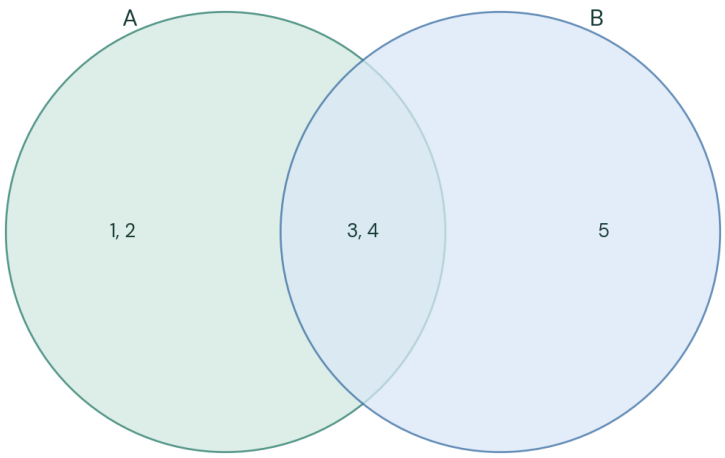
In a group of 15 students, 9 play football, 8 play hockey, and 4 play both. How many play neither sport?



Two-set Venn diagram for 15 students. Football only : 5. Both : 4. Hockey only : 4. Neither : 2.

Question 2

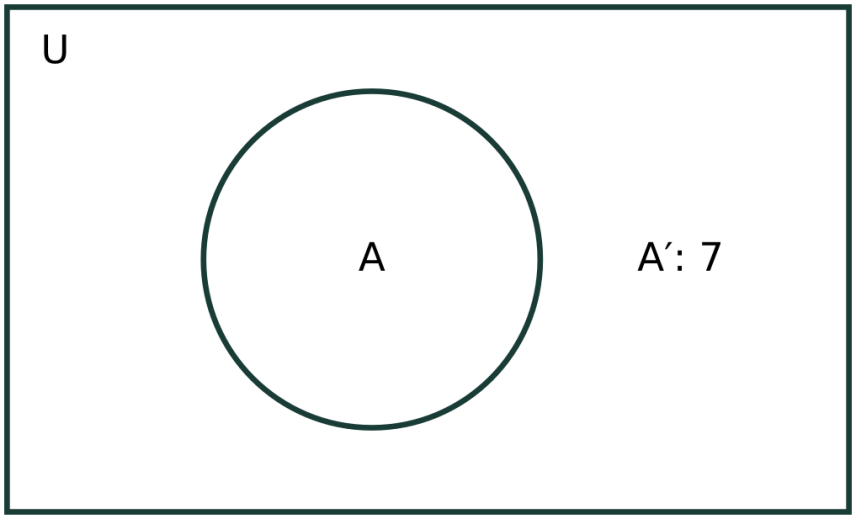
$A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5\}$. How many elements are in $A \cup B$?



Venn diagram matching the sets in this question. A only : [1, 2]. Both : [3, 4]. B only : [5].

Question 3

The universal set is $U = \{1, 2\}$ …, $12\}$. $A = \{1, 3, 5, 7, 9\}$. How many elements are in A□, the complement of A?



Universal set $U = \{1\}$ …, $12\}$ drawn as a rectangle, with $A = \{1, 3, 5, 7, 9\}$ as a circle inside it. The complement A□ is the part of U outside the circle, with 7 elements.

Question 4

The set $B = \{4, 4, 6, 9, 9, 9\}$. What is n(B)?

Question 5

In a club, 20 members play A, 18 play B and 15 play C. 6 play A and B, 4 play B and C, and 3 play A and C. 2 play all three. How many members play at least one of the three sports? The pairwise counts include those who are in all three sets.

A. 66

B. 40

C. 53

D. 42

Question 6

In a survey of students, the universal set contains 73 people. Set A has 27, set B has 19, and set C has 20. There are 2 in A and B, 3 in A and C, and 6 in B and C. Each pairwise count includes everyone in all three sets. There are 2 in all three sets. How many people are in none of A, B or C?

A. 18

B. 22

C. 57

D. 16

Question 7

$A = \{1, 2, 3, 4\}, B = \{3, 4, 5\}$. Is A ∪ B the same as B ∪ A?

A. Yes: the common elements cancel out

B. Yes: the two results have different sizes

C. Yes: the order of the sets never matters

D. No: A ∪ $B = \{1, 2\}$ but B ∪ $A = \{5\}$

Question 8

$A = \{1, 2, 3\}, B = \{2, 3, 4\}, C = \{3, 5\}$. Is (A ∪ B) ∪ C the same as A ∪ (B ∪ C)?

A. Yes: the brackets can be ignored

B. No: (A ∪ B) ∪ $C = \{1\}$ but A ∪ (B ∪ C) = $\{1, 3\}$

C. Yes: the brackets never change the answer

D. Yes: the answer depends on which set is first

Watch an explanation

Cognito — Set Notation & Venn Diagrams

Cognito — Venn Diagrams

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

2

Question 2

5

Question 3

7

Question 4

3

Question 5

42

Question 6

16

Question 7

No: A □ $B = \{1, 2\}$ but B □ $A = \{5\}$

Question 8

No: (A □ B) □ $C = \{1\}$ but A □ (B □ C) = $\{1, 3\}$

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