Simultaneous linear equations

Junior Cycle · Higher Level

Understand the idea

How to start

We need one x-value and one y-value that work in both equations at the same time.

1. Label equations A and B, and line up the x terms, y terms and right-hand sides.
2. Choose the variable to remove. For addition, make its terms equal in size and opposite in sign.
3. Solve for the remaining variable, substitute back and check both original equations.

Key terms & definitions

Elimination
Combining equations to remove a variable.

Substitution method
Replacing a variable using an equivalent expression from another equation.

Key ideas

• A simultaneous solution is a pair of values that makes both original equations true. We must find both values, then check both equations.

• Label the equations A and B. Write x terms under x terms, y terms under y terms and the constants under the constants on the right of the equals signs.

• To eliminate a variable by addition, make its two terms the same size but opposite in sign. Their sum is zero: for example, $+2x + (-2x) = 0$.

• If a multiplier is needed, multiply every term in that equation, including the right-hand side. Choose a multiplier that creates the opposite term you need.

• Add the aligned equations. One variable disappears, leaving a one-variable equation you can solve.

• Substitute that value into one original equation to find the second variable. State the pair and check it in both original equations.

• You can also subtract equations with equal coefficients. We use addition with opposite terms in the early examples so the cancellation is easy to see.

• On a graph, the solution is the intersection of two lines: the same x-value gives the same y-value on both. Check your answer in both equations.

Quick reminder

• Choose which variable to remove.

• For addition, make equal-sized opposite terms and add.

• Substitute back to find the second value; check both equations.

Worked examples

Example 01 · Opposite terms are already ready to cancel

Solve the simultaneous equations $x + y = 7$ and $x - y = 1$.

Blue follows x terms and purple follows y terms in the aligned equations. Crossed-out terms add to zero.

Why this method?

We will remove y. One equation has $+y$ and the other has $-y$, so adding them removes y without any multiplication.

$x + y = 7$ A $x - y = 1$ B	**Step 1** Label and align the original equations. The y terms already have opposite signs.
$x \cancel{+y} = 7$ A $x \cancel{-y} = 1$ B $2x = 8$ $A + B$	**Step 2** Add A and B. The crossed terms sum to zero $: +y - y = 0$. Add the x terms and the right-hand sides too.
$x = 4$	**Step 3** Divide both sides by 2 because $2x$ means two lots of x.
$4 + y = 7$	**Step 4** Substitute $x = 4$ into original equation A. Now the only unknown is y.
$y = 3$	**Step 5** Subtract 4 from both sides to find y.
$4 + 3 = 7$ and $4 - 3 = 1$. Therefore $(x, y) = (4, 3)$.	**Step 6** Check the pair in both original equations. Both equalities must be true.

Example 02 · Multiply one equation to remove x

Solve $2x + y = 8$ and $x + 2y = 7$.

Blue follows x terms and purple follows y terms in the aligned equations. Crossed-out terms add to zero.

Why this method?

We will remove x. A contains $+2x$. B contains $+x$, so multiplying B by -2 creates $-2x$. Then adding the equations makes the x terms cancel.

$$2x + y = 8 \quad \text{A}$$
$$x + 2y = 7 \quad \text{B}$$

Step 1
Label the equations. Compare $+2x$ in A with $+x$ in B: their sizes are not yet equal.

$-2(x + 2y) = -2(7)$,
so $-2x - 4y = -14$.

Step 2
Keep A unchanged. Multiply every term of B by -2, including the 7 on the right, to make $-2x$.

$$\begin{array}{rrcrl} \cancel{2x} & +y & = & 8 & \text{A unchanged} \\ \cancel{-2x} & -4y & = & -14 & B \times (-2) \\ \hline & -3y & = & -6 & \text{Add} \end{array}$$

Step 3
Add A and $B \times (-2)$. The x terms cancel because $+2x - 2x = 0$. For y, $+y - 4y = -3y$.

$y = 2$

Step 4
Divide both sides by -3. A negative divided by a negative is positive.

$x + 2(2) = 7$

Step 5
Substitute $y = 2$ into original equation B, which has just one x.

$x + 4 = 7$

Step 6
Simplify, then subtract 4 from both sides.

$x = 3$,
so $(x, y) = (3, 2)$.

Step 7
Find x and state the pair in x, y order.

$2(3) + 2 = 8$
and $3 + 2(2) = 7$.

Step 8
Check A and B using both values.

Example 03 · When substitution is simpler

Solve $x = y + 1$ and $2x + y = 11$.

Why this method?

A already tells us what x equals. Replace x in B with the whole expression $y + 1$. This gives one equation containing only y.

$x = y + 1$ A $2x + y = 11$ B	**Step 1** Label the equations so it is clear which expression we will use.
$2(y + 1) + y = 11$	**Step 2** Replace the x in B with a bracket containing $y + 1$. The 2 multiplies that whole bracket.
$2y + 2 + y = 11$	**Step 3** Expand the bracket and collect the y terms.
$3y = 9$	**Step 4** Subtract 2 from both sides.
$y = 3$	**Step 5** Divide both sides by 3 to find y.
$x = 3 + 1 = 4$	**Step 6** Use A to find x from the y-value we have just found.
$4 = 3 + 1$ and $2(4) + 3 = 11$. Therefore $(x, y) = (4, 3)$.	**Step 7** Check the original equations.

Example 04 · Multiply both equations to remove y

Solve $5x + 3y = 67$ and $2x + 4y = 52$.

Why this method?

We will remove y. The coefficients are 3 and 4; their lowest common multiple is 12. Multiply A by 4 to get $+12y$ and B by -3 to get $-12y$. Then add.

$5x + 3y = 67$ A $2x + 4y = 52$ B	**Step 1** Label the original equations and line up the like terms.
$20x + 12y = 268$	**Step 2** Multiply every term of A by 4. We choose 4 because $3y \times 4$ becomes $+12y$.
$-6x - 12y = -156$	**Step 3** Multiply every term of B by -3. We choose -3 because $4y \times (-3)$ becomes $-12y$, the opposite of $+12y$.
$20x + 12y = 268$ $A \times 4$ $-6x - 12y = -156$ $B \times (-3)$ $14x = 112$ Add	**Step 4** Add the two new equations. $+12y - 12y = 0$, so only x remains. On the right, $268 - 156 = 112$.
$x = 8$	**Step 5** Divide both sides by 14 to find one x.
$2(8) + 4y = 52$	**Step 6** Substitute $x = 8$ into original equation B. Using an original equation helps avoid carrying a multiplication error forward.
$16 + 4y = 52$	**Step 7** Calculate 2×8. The equation is now written with a numerical constant and one unknown.
$4y = 36$	**Step 8** Subtract 16 from both sides to remove the constant 16.
$y = 9$, so $(x, y) = (8, 9)$.	**Step 9** Divide both sides by 4 to find y. State both values; finding y completes the solution.

$5(8) + 3(9) = 67$
and $2(8) + 4(9) = 52$.

Step 10

Check the pair in both A and B.

Example 05 · The same method with fractional answers

Solve $4x - 3y = 4$ and $3x - 4y = 6$.

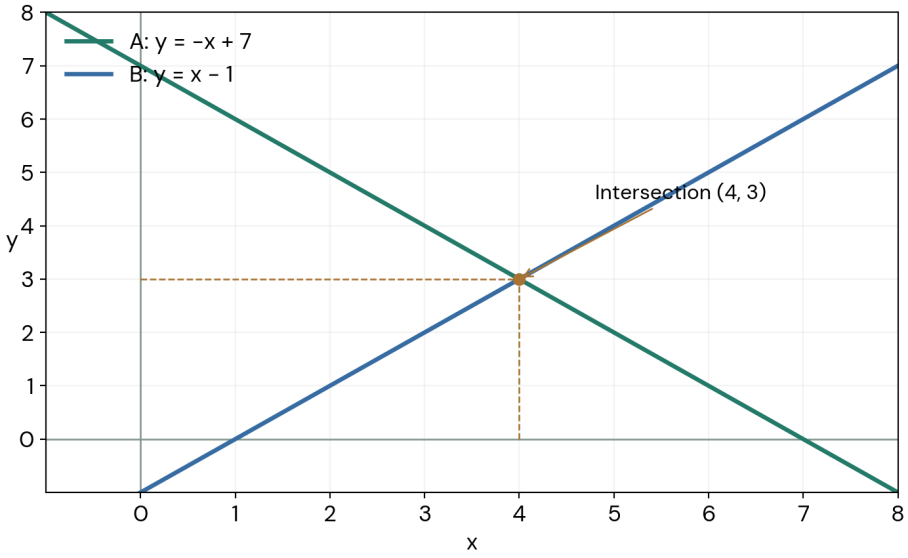
Why this method?

We again remove y using coefficients of size 12. $A \times 4$ gives $-12y$; $B \times (-3)$ gives $+12y$. The method is unchanged even though the answers will be fractions.

$4x - 3y = 4$ A $3x - 4y = 6$ B	**Step 1** Label and align A and B.
$16x - 12y = 16$ and $-9x + 12y = -18$.	**Step 2** Multiply all of A by 4 and all of B by -3 to produce opposite y terms.
$16x - 12y = 16$ $A \times 4$ $-9x + 12y = -18$ $B \times (-3)$ $7x = -2$ Add	**Step 3** Add. The y terms sum to zero, while $16x - 9x = 7x$ and $16 - 18 = -2$.
$x = -\frac{2}{7}$	**Step 4** Divide by 7; keep the exact fraction rather than rounding.
$4(-\frac{2}{7}) - 3y = 4$	**Step 5** Substitute this x-value into original equation A.
$-\frac{8}{7} - 3y = 4$	**Step 6** Multiply, then add $\frac{8}{7}$ to both sides. Write 4 as $\frac{28}{7}$ before adding.
$-3y = \frac{28}{7} + \frac{8}{7} = \frac{36}{7}$	**Step 7** Combine the fractions on the right.
$y = -\frac{12}{7}$; $(x, y) = (-\frac{2}{7}, -\frac{12}{7})$.	**Step 8** Divide by -3 to find y, then state the complete pair.
$4(-\frac{2}{7}) - 3(-\frac{12}{7}) = 4$ and $3(-\frac{2}{7}) - 4(-\frac{12}{7}) = 6$.	**Step 9** Check both original equations exactly.

Example 06 · Why the solution is an intersection

Find where $y = -x + 7$ and $y = x - 1$ meet.



Lines $A : y = -x + 7$ and $B : y = x - 1$ meet at $(4, 3)$. Both the horizontal and vertical coordinates agree.

Why this method?

At an intersection the two lines share both coordinates. Their y-values are equal, so we set their two expressions for y equal to each other.

$-x + 7 = x - 1$	**Step 1** Name the lines A and B. Both expressions equal y, so they must equal each other at the meeting point.
$8 = 2x$	**Step 2** Add x to both sides and add 1 to both sides to collect the variable and constant terms.
$x = 4$	**Step 3** Divide by 2 to find the horizontal coordinate of the intersection.
$y = 4 - 1 = 3$	**Step 4** Use either original line to find the vertical coordinate. We use B.
$3 = -4 + 7$. The intersection is $(4, 3)$.	**Step 5** Check A as well. The same pair must lie on both lines.

Extension: first find where two lines meet, then test that same point in the third line. Lines sharing one point are called concurrent.

Example 07 · Extension: does a third line pass through the intersection?

The three lines are A: $y = -x - 5$, B: $y = -2$, and C: $y = x + 1$. Do all three meet at one point?

Why this method?

First find the intersection of A and B, exactly as in the previous example. Then check that pair in C. The third line is a test of the point we already found.

$-2 = -x - 5$	**Step 1** At the intersection of A and B their y-values agree. Replace y in A with -2 from B.
$3 = -x$, so $x = -3$.	**Step 2** Add 5 to both sides, then multiply both sides by -1.
$(x, y) = (-3, -2)$	**Step 3** B already gives y. State the intersection of the first two lines.
$y = -3 + 1 = -2$	**Step 4** Now test that exact point in C. Put $x = -3$ into $y = x + 1$.
All three lines pass through $(-3, -2)$. They are concurrent.	**Step 5** The result is the same y-value, so the point lies on C too.

Extension: first find where two lines meet, then test that same point in the third line. Lines sharing one point are called concurrent.

Example 08 · Extension: a third line that misses the intersection

The three lines are A: $y = 2$, B: $y = x + 6$, and C: $y = -x + 2$. Are they concurrent?

Why this method?

Find where A and B meet, then substitute that pair into C. If it fails C, there is no single point common to all three lines.

$2 = x + 6$	**Step 1** Set the expressions for y from A and B equal.
$x = -4$ and $y = 2$.	**Step 2** Subtract 6 from both sides. Use A for y.
$y = -(-4) + 2 = 6$	**Step 3** Test $x = -4$ in the third line C.
$6 \neq 2$, so the lines are not concurrent.	**Step 4** C gives 6 rather than 2 at that x-coordinate. The intersection of A and B is not on C.

Common mistake

Stopping after finding x when the question asks for x and y. Substitute x into an original equation to find y, then check the pair in both original equations.

Exam technique

Check the pair in both original equations.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Solve the simultaneous equations $2x + y = 11$ and $x + y = 7$. What is the value of x?

Question 2

Lines $L1$, $L2$ and $L3$ are $y = -1x + (-6)$; $y = 0x + (-4)$; $y = 1x + (-2)$ respectively. Which point, if any, lies on all three lines?

A. $(-2, 4)$

B. $(-2, -4)$

C. $(2, -4)$

D. No single common point

Question 3

Solve these simultaneous equations for x and y : $2x + 3y = 4$ and $3x - 2y = 1$.

A.
$x = \frac{10}{13}, y = \frac{11}{13}$

B.
$x = \frac{5}{13}, y = \frac{10}{13}$

C.
$x = \frac{11}{13}, y = -\frac{14}{13}$

D.
$x = \frac{11}{13}, y = \frac{10}{13}$

Watch an explanation

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Cognito — Solving Simultaneous Equations 2 - Elimination Method

Cognito — Solving Simultaneous Equations 3 - Substitution Method

Cognito — Solving Simultaneous Equations 1 - Using Graphs

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

4

Question 2

$(-2, -4)$

Question 3

$x = \frac{11}{13}, y = \frac{10}{13}$

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