Volume, surface area and nets

Junior Cycle · Higher Level

Understand the idea

How to start

Volume is the space inside a solid. Surface area is the area of its outside faces.

1. Read whether the question asks for volume or surface area.
2. For a prism, find the cross-section area, then multiply it by the prism length.
3. Use cubic units for volume and square units for surface area.

Key terms & definitions

Prism
A solid with a constant polygonal cross-section along its length.

Net
A flat arrangement of the surfaces that fold into a solid.

Exposed surface
A face or curved surface visible on the outside.

Key ideas

• Volume measures space inside a solid. For a prism, $V = A_{\text{cross}-\text{section}}l$.

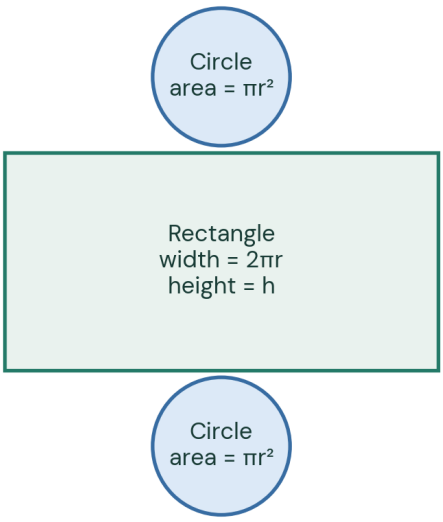
• Cuboid: $V = lwh$. Cylinder: $V = \pi r^2 h$. Sphere: $V = \frac{4}{3}\pi r^3$.

• Surface area is the total exposed area. A net helps identify every face.

• Cylinder curved surface area is $2\pi rh$; add $2\pi r^2$ for two closed circular ends. Sphere surface area is $4\pi r^2$.

• In a composite solid, add or subtract volumes as required. For exposed surface area, exclude joined internal faces.

• A cylinder net has a rectangle of length $2\pi r$ and height h, plus two circles when both ends are closed.



Closed cylinder net: one rectangle and two circular ends. This schematic shows the surfaces separately.

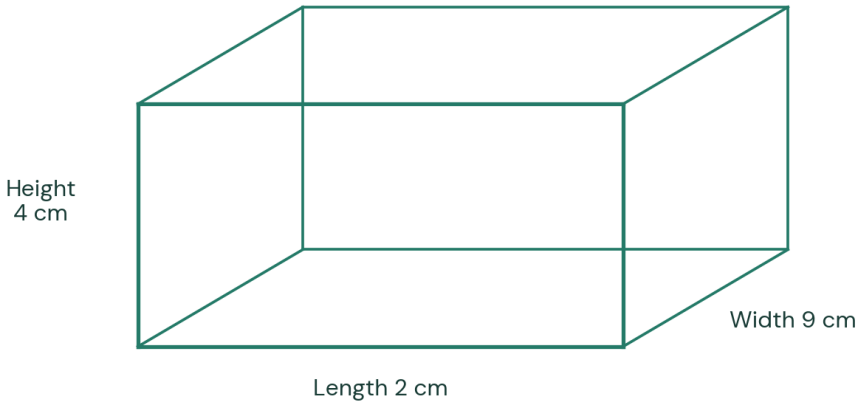
Quick reminder

- Volume uses cubic units; surface area uses square units.
- Prism volume = cross-section area × length.
- Count exposed surfaces only; do not count joined internal faces.

Worked examples

Example 01 · Volume of a cuboid

A cuboid has length 2 cm, width 9 cm and height 4 cm. What is its volume?



Cuboid with length 2 cm, width 9 cm and height 4 cm. Schematic; use stated dimensions.

Why this method?

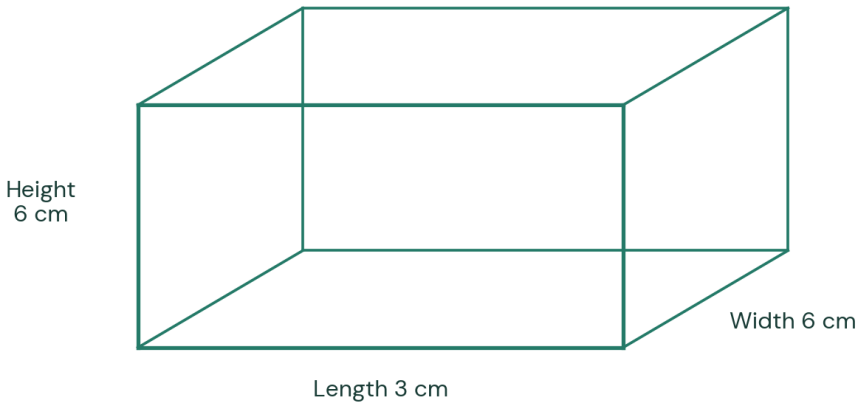
A cuboid has rectangular layers. Its volume is the area of one layer multiplied by the height.

$V = lwh$	**Step 1** V means volume. Here $l = 2$ cm, $w = 9$ cm and $h = 4$ cm.
$V = 2 \times 9 \times 4$	**Step 2** Put each length into the matching place in the formula.
$V = 18 \times 4 = 72$	**Step 3** Multiply the lengths.
$V = 72\text{cm}^3$	**Step 4** Three lengths were multiplied, so the result is in cubic centimetres.

Think about it: Before moving on: why does a volume need all three dimensions?

Example 02 · Total surface area of a cuboid

A cuboid has length 3 cm, width 6 cm and height 6 cm. What is its total surface area?



Cuboid with length 3 cm, width 6 cm and height 6 cm. Schematic; use stated dimensions.

Why this method?

Decide whether the question asks for space inside or area outside. List the required volumes or exposed surfaces before calculating.

$3 \times 6, 3 \times 6, 6 \times 6$	**Step 1** A cuboid has three pairs of equal faces
$2 \times (18 + 18 + 36) = 144$ cm^2	**Step 2** Add the three face areas and double

Think about it: Before moving on: why are the faces counted in pairs?

Example 03 · Volume of a cylinder

A cylinder has radius 2 cm and height 4 cm. Find its volume, giving your answer in terms of π.

Why this method?

Decide whether the question asks for space inside or area outside. List the required volumes or exposed surfaces before calculating.

$V = \pi \times 2^2 \times 4$	**Step 1** Volume of a cylinder $= \pi \times$ radius$^2 \times$ height
$V = 16\pi$ cm^3	**Step 2** Work out the square and the product

Think about it: Before moving on: why do we square the radius, and why is the answer given in terms of π?

Example 04 · Volume of a triangular prism

The cross-section of a triangular prism is a right-angled triangle with sides 9 cm, 12 cm and 15 cm. The prism is 12 cm long. Find its volume in cm^3.

Why this method?

Decide whether the question asks for space inside or area outside. List the required volumes or exposed surfaces before calculating.

$\frac{1}{2} \times 9 \times 12 = 54 \text{ cm}^2$	**Step 1** Area of the triangular cross-section $= \frac{1}{2} \times$ base $\times$ height
$54 \times 12 = 648 \text{ cm}^3$	**Step 2** Volume $=$ area of the cross-section $\times$ length

Think about it: Before moving on: why is the area of the cross-section half the product of the two shorter sides?

Example 05 · Volume of a triangular prism — another example

The cross-section of a triangular prism is a right-angled triangle with sides 12 cm, 16 cm and 20 cm. The prism is 10 cm long. Find its volume in cm^3.

Why this method?

Decide whether the question asks for space inside or area outside. List the required volumes or exposed surfaces before calculating.

$\frac{1}{2} \times 12 \times 16 = 96 \text{ cm}^2$	**Step 1** Area of the triangular cross-section $= \frac{1}{2} \times$ base $\times$ height
$96 \times 10 = 960 \text{ cm}^3$	**Step 2** Volume $=$ area of the cross-section $\times$ length

Example 06 · Surface area of a triangular prism

The cross-section of a triangular prism is a right-angled triangle with sides 6 cm, 8 cm and 10 cm. The prism is 12 cm long. Find its total surface area in cm^2.

Why this method?

Decide whether the question asks for space inside or area outside. List the required volumes or exposed surfaces before calculating.

$2 \times \frac{1}{2} \times 6 \times 8 = 48 \text{ cm}^2$	**Step 1** Two triangular ends, each of area $\frac{1}{2} \times$ base $\times$ height
$(6 + 8 + 10) \times 12 = 288 \text{ cm}^2$	**Step 2** Three rectangular faces: perimeter of the cross-section $\times$ length
$48 + 288 = 336 \text{ cm}^2$	**Step 3** Add the two parts

Think about it: Before moving on: why is the perimeter multiplied by the length, and not added to it?

Example 07 · Surface area of a triangular prism — another example

The cross-section of a triangular prism is a right-angled triangle with sides 9 cm, 12 cm and 15 cm. The prism is 7 cm long. Find its total surface area in cm^2.

Why this method?

Decide whether the question asks for space inside or area outside. List the required volumes or exposed surfaces before calculating.

$2 \times \frac{1}{2} \times 9 \times 12 = 108 \text{ cm}^2$	**Step 1** Two triangular ends, each of area $\frac{1}{2} \times$ base $\times$ height
$(9 + 12 + 15) \times 7 = 252 \text{ cm}^2$	**Step 2** Three rectangular faces: perimeter of the cross-section $\times$ length
$108 + 252 = 360 \text{ cm}^2$	**Step 3** Add the two parts

Example 08 · Volume of a sphere

A sphere has radius 4 cm. What is its volume? Give your answer in terms of π.

Why this method?

Decide whether the question asks for space inside or area outside. List the required volumes or exposed surfaces before calculating.

$$\left(\frac{4}{3}\right) \times \pi \times 4^3 = \left(4 \times \frac{64}{3}\right)\pi$$

Step 1

The volume of a sphere is $\left(\frac{4}{3}\right)\pi r^3$

$\frac{256}{3}\pi$ cm^3

Step 2

Give the exact answer

Think about it: Before moving on: why is the radius cubed for a volume but squared for a surface area?

Example 09 · Volume of a sphere — another example

A sphere has radius 5 cm. What is its volume? Give your answer in terms of π.

Why this method?

Decide whether the question asks for space inside or area outside. List the required volumes or exposed surfaces before calculating.

$$\left(\frac{4}{3}\right) \times \pi \times 5^3 = \left(4 \times \frac{125}{3}\right)\pi$$

Step 1

The volume of a sphere is $\left(\frac{4}{3}\right)\pi r^3$

$\frac{500}{3}\pi$ cm^3

Step 2

Give the exact answer

Example 10 · Curved surface area of a cylinder

A cylinder has radius 7 cm and height 6 cm. What is its curved surface area?

Why this method?

Decide whether the question asks for space inside or area outside. List the required volumes or exposed surfaces before calculating.

$$2\pi \times 7 \times 6 = 84\pi$$

Step 1

The curved surface is a rectangle of height h, wrapped round a circle of circumference $2\pi r$

Think about it: Before moving on: why is the curved surface a rectangle, and what is its length?

Example 11 · Curved surface area of a cylinder — another example

A cylinder has radius 8 cm and height 8 cm. What is its curved surface area?

Why this method?

Decide whether the question asks for space inside or area outside. List the required volumes or exposed surfaces before calculating.

$$2\pi \times 8 \times 8 = 128\pi$$

Step 1
The curved surface is a rectangle of height h, wrapped round a circle of circumference $2\pi r$

Example 12 · Surface area of a sphere

A sphere has radius 9 cm. What is its surface area?

Why this method?

Decide whether the question asks for space inside or area outside. List the required volumes or exposed surfaces before calculating.

$$4\pi \times 9^2 = 324\pi$$

Step 1
The surface area of a sphere is $4\pi r^2$

Think about it: Before moving on: why is the surface area of a sphere four times the area of a circle with the same radius?

Example 13 · Surface area of a sphere — another example

A sphere has radius 10 cm. What is its surface area?

Why this method?

Decide whether the question asks for space inside or area outside. List the required volumes or exposed surfaces before calculating.

$$4\pi \times 10^2 = 400\pi$$

Step 1
The surface area of a sphere is $4\pi r^2$

Example 14 · Composite volumes and exposed surface areas

A closed solid consists of a cylinder of radius 2 cm and cylinder length 8 cm, with 1 hemisphere of the same radius attached at one end; the other end is a flat circular face. The parts meet exactly and do not overlap. Find the total volume in exact form.

Why this method?

Decide whether the question asks for space inside or area outside. List the required volumes or exposed surfaces before calculating.

Cylinder volume : 32π cm^3. Caps volume : $\frac{16}{3}\pi$ cm^3.	**Step 1** Separate the cylinder from its hemispherical caps
No volume is subtracted at a face-only join.	**Step 2** Add the non-overlapping volumes
$\frac{112}{3}\pi$ cm^3	**Step 3** Combine the exact terms

Think about it: Why do the circular faces at an internal join contribute no exposed surface area?

Example 15 · Composite volumes and exposed surface areas — another example

A closed solid consists of a cylinder of radius 4 cm and cylinder length 13 cm, with 2 hemispheres of the same radius attached at opposite ends. The parts meet exactly and do not overlap. Find the total exposed surface area in exact form.

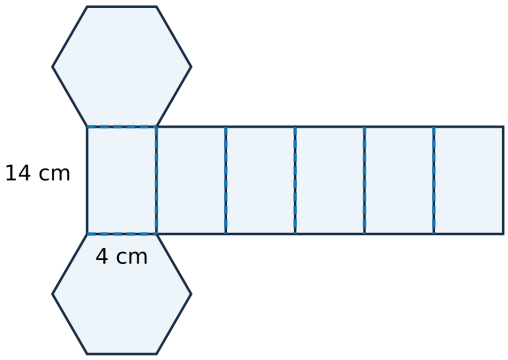
Why this method?

Decide whether the question asks for space inside or area outside. List the required volumes or exposed surfaces before calculating.

Cylinder curved area : 104π cm^2. Caps curved area : 64π cm^2.	**Step 1** Separate the cylinder from its hemispherical caps
Exposed flat area : 0π cm^2.	**Step 2** Count only exposed flat faces
168π cm^2	**Step 3** Combine the exact terms

Example 16 · Nets of prisms and cylinders

A closed right prism has regular $6-$gon bases with side 4 cm and perpendicular height 14 cm. State the number and dimensions of its lateral rectangles and the number of $6-$gon bases in a net.



Blue dashed edges: folds. Not to scale; no glue tabs.

Closed regular prism net; solid outlines and blue dashed fold edges. Dimensions are stated; drawing not to scale.

Why this method?

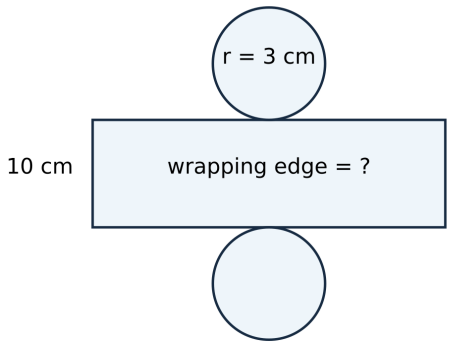
Decide whether the question asks for space inside or area outside. List the required volumes or exposed surfaces before calculating.

The $6-$gon has 6 sides, so use 6 lateral rectangles.	**Step 1** Match side rectangles to base edges
Each rectangle is 4 cm $\times 14$ cm.	**Step 2** Match rectangle dimensions to the solid
Use two regular $6-$gons, each with side 4 cm.	**Step 3** Close both ends with congruent polygons

Think about it: Which edges meet when the net folds, and how do you know their lengths agree?

Example 17 · Nets of prisms and cylinders — another example

A closed cylinder has radius 3 cm and height 10 cm. In its net, what is the rectangle edge that wraps around the cylinder, and how many discs are needed? Give the length exactly in terms of π.



Not to scale; no glue tabs shown.

Closed cylinder net; solid outlines and blue dashed fold edges. Dimensions are stated; drawing not to scale.

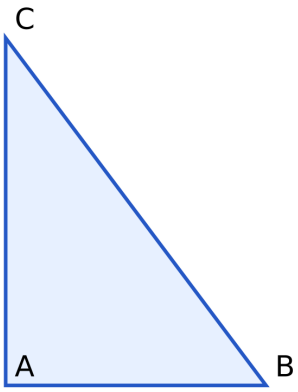
Why this method?

Decide whether the question asks for space inside or area outside. List the required volumes or exposed surfaces before calculating.

The rectangle has height 10 cm.	**Step 1** Cut along a generator and open the curved surface
$2\pi r = 2\pi \times 3 = 6\pi$ cm.	**Step 2** Match the wrapping edge to the base circumference
Use two congruent discs, each of radius 3 cm.	**Step 3** Close both ends

Example 18 · Nets of prisms with irregular polygonal bases

A right prism has length 11 cm. Its base vertices, in boundary order, are $A(0, 0)$; $B(6, 0)$; $C(0, 8)$, with coordinates in cm. Starting with edge AB, list the side rectangles in perimeter order and the number of polygonal ends for a closed net.



Consecutive vertices follow the boundary.

Prism base polygon with vertices in boundary order as listed in the question; drawing in coordinate units.

Why this method?

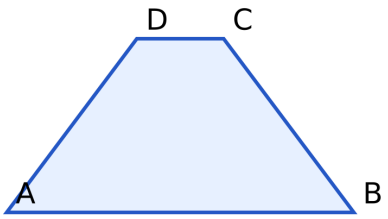
Decide whether the question asks for space inside or area outside. List the required volumes or exposed surfaces before calculating.

Base edge lengths : 6, 10, 8 cm.	**Step 1** Measure every edge, including the closing edge
Common side-face dimension : 11 cm.	**Step 2** Use the prism length as the common rectangle dimension
2 congruent polygonal ends; side rectangles 6 × 11, 10 × 11, 8 × 11 cm.	**Step 3** Add two congruent ends

Think about it: Why does an n-sided prism need n rectangular side faces and two polygonal ends?

Example 19 · Nets of prisms with irregular polygonal bases — another example

A right prism has length 9 cm. Its base vertices, in boundary order, are $A(0, 0)$; $B(8, 0)$; $C(5, 4)$; $D(3, 4)$, with coordinates in cm. Starting with edge AB, list the side rectangles in perimeter order and the number of polygonal ends for a closed net.



Consecutive vertices follow the boundary.

Prism base polygon with vertices in boundary order as listed in the question; drawing in coordinate units.

Why this method?

Decide whether the question asks for space inside or area outside. List the required volumes or exposed surfaces before calculating.

Base edge lengths : 8, 5, 2, 5 cm.	**Step 1** Measure every edge, including the closing edge
Common side-face dimension : 9 cm.	**Step 2** Use the prism length as the common rectangle dimension
2 congruent polygonal ends; side rectangles 8×9, 5×9, 2×9, 5×9 cm.	**Step 3** Add two congruent ends

Common mistake

Giving surface area in cubic units or volume in square units. Surface area measures covering and uses units such as cm^2; volume measures space and uses cm^3.

Exam technique

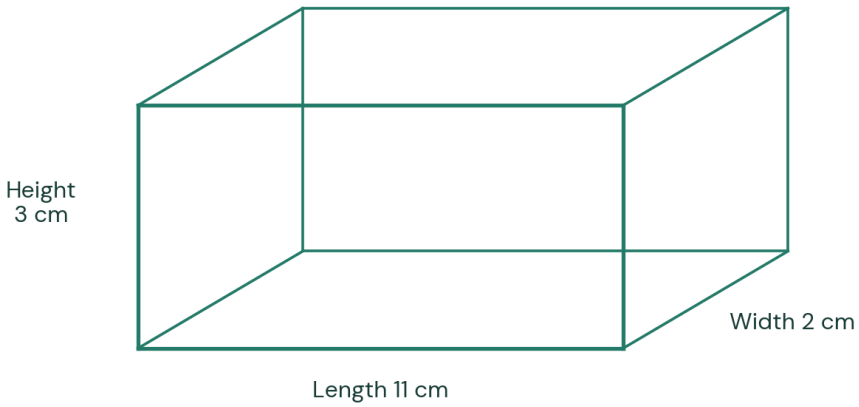
Make a face list and cross off hidden joined surfaces before adding areas.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

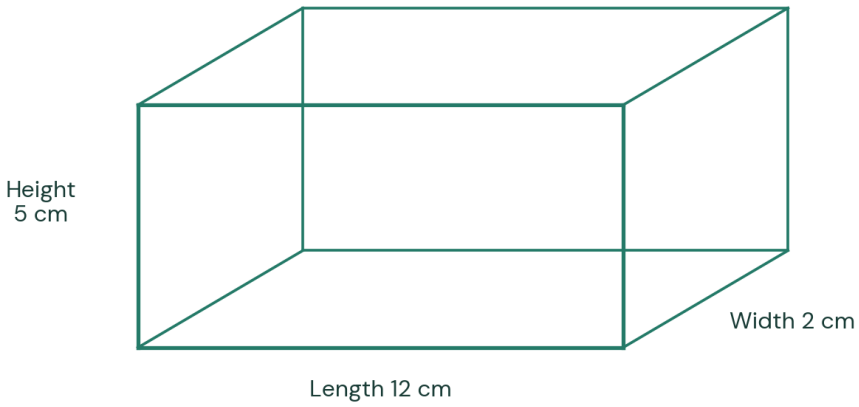
A cuboid has length 11 cm, width 2 cm and height 3 cm. What is its volume?



Cuboid with length 11 cm, width 2 cm and height 3 cm. Schematic; use stated dimensions.

Question 2

A cuboid has length 12 cm, width 2 cm and height 5 cm. What is its total surface area?



Cuboid with length 12 cm, width 2 cm and height 5 cm. Schematic; use stated dimensions.

Question 3

A cylinder has radius 3 cm and height 5 cm. Find its volume, giving your answer in terms of π.

A. 45

B. 15π

C. 45π

D. 180π

Question 4

The cross-section of a triangular prism is a right-angled triangle with sides 9 cm, 12 cm and 15 cm. The prism is 5 cm long. Find its volume in cm^3.

A. 90

B. 540

C. 270

D. 59

Question 5

The cross-section of a triangular prism is a right-angled triangle with sides 9 cm, 12 cm and 15 cm. The prism is 5 cm long. Find its total surface area in cm^2.

A. 234

B. 198

C. 288

D. 149

Question 6

A sphere has radius 18 cm. Find its volume in terms of π.

A. $432\pi\ \text{cm}^3$

B. $3888\pi\ \text{cm}^3$

C. $1296\pi\ \text{cm}^3$

D. $7776\pi\ \text{cm}^3$

Question 7

A cylinder has radius 5 cm and height 9 cm. What is its curved surface area?

A. $225\pi\ \text{cm}^2$

B. $45\pi\ \text{cm}^2$

C. 90π cm^2

D. 180π cm^2

Question 8

A sphere has radius 16 cm. What is its surface area?

A. 1024π cm^2

B. 64π cm^2

C. 512π cm^2

D. 256π cm^2

Question 9

A closed solid consists of a cylinder of radius 3 cm and cylinder length 12 cm, with 2 hemispheres of the same radius attached at opposite ends. The parts meet exactly and do not overlap. Find the total exposed surface area in exact form.

A. 108π cm^2

B. 126π cm^2

C. 144π cm^2

D. 90π cm^2

Question 10

A closed right prism has regular $5-$gon bases with side 5 cm and perpendicular height 17 cm. State the number and dimensions of its lateral rectangles and the number of $5-$gon bases in a net.

A. 5 rectangles of 5 cm $\times 17$ cm; $25-$gon bases

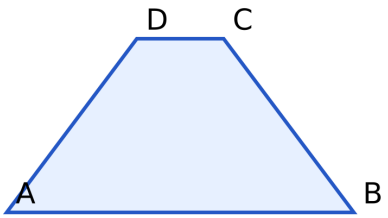
B. 5 rectangles of 85 cm $\times 17$ cm; $25-$gon bases

C. 5 rectangles of 5 cm $\times 17$ cm; $15-$gon bases

D. 7 rectangles of 5 cm $\times 17$ cm; $25-$gon bases

Question 11

A right prism has length 11 cm. Its base vertices, in boundary order, are $A(0,0)$; $B(8,0)$; $C(5,4)$; $D(3,4)$, with coordinates in cm. Starting with edge AB, list the side rectangles in perimeter order and the number of polygonal ends for a closed net.



Consecutive vertices follow the boundary.

Prism base polygon with vertices in boundary order as listed in the question; drawing in coordinate units.

A. 2 congruent polygonal ends; side rectangles 8×11, 5×11, 2×11, 5×11 cm.

B. 2 congruent polygonal ends; side rectangles 8×11, 5×11, 2×11 cm.

C. 1 congruent polygonal ends; side rectangles 8×11, 5×11, 2×11, 5×11 cm.

D. 2 congruent polygonal ends; side rectangles 8×11, 5×11, 2×11, 6×11 cm.

Watch an explanation

Cognito — Volumes of Cylinders & Prisms

Corbettmaths — Surface area: cylinders

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

66

Question 2

188

Question 3

45π

Question 4

270

Question 5

288

Question 6

7776π cm^3

Question 7

90π cm^2

Question 8

1024π cm^2

Question 9

108π cm^2

Question 10

5 rectangles of 5 cm $\times$17 cm; 25 – gon bases

Question 11

2 congruent polygonal ends; side rectangles 8×11, 5×11, 2×11, 5×11 cm.

Open the full chapter online