Surds and exact answers

Junior Cycle · Higher Level

Understand the idea

How to start

A square root asks which non-negative number squares to give the number inside. For example, $\sqrt{9} = 3$.

1. To simplify a root, look for a square factor such as $4, 9, 16$ or 25.

2. Take the square root of that factor, leaving the other factor inside.

3. Add surds only when their root parts match. Keep exact roots until a decimal is requested.

Key terms & definitions

Exact answer
An answer without rounding, such as $3\sqrt{2}$.

Like surds
Terms with the same simplified root part.

Conjugates
A pair such as $a + \sqrt{b}$ and $a - \sqrt{b}$.

Key ideas

• A surd is an irrational root written exactly. Keep it exact unless a decimal approximation is requested.

• Find a square factor to simplify: $\sqrt{72} = \sqrt{36}\sqrt{2} = 6\sqrt{2}$.

• Collect like surds after simplifying. Different surds cannot usually be added into one term.

• For non-negative real a, b, $\sqrt{a}\sqrt{b} = \sqrt{ab}$. Expand brackets carefully when multiplying surd expressions.

• The positive square root symbol means the non-negative root. Solving $x^2 = 9$ gives $x = \pm 3$.

Quick reminder

• Find a square factor to simplify a root.

• Collect only terms with the same simplified root part.

• Keep exact surds until a decimal answer is requested.

Worked examples

Example 01 · Simplifying a surd

Simplify $\sqrt{8}$.

Why this method?

Keep the answer exact. Look for square factors and matching root parts before combining any terms.

$8 = 4 \times 2$	**Step 1** Look for a square factor. Four is a square because $2 \times 2 = 4$.
$\sqrt{8} = \sqrt{4} \times \sqrt{2}$	**Step 2** The square root of a product of non-negative numbers can be split into the product of the roots.
$\sqrt{8} = 2\sqrt{2}$	**Step 3** The root of 4 is 2. Two has no square factor greater than 1, so its root stays inside.

Think about it: Why can 2 come out of the square root, while the other factor stays inside?

Example 02 · Simplifying a surd — another example

Simplify $\sqrt{12}$.

Why this method?

Keep the answer exact. Look for square factors and matching root parts before combining any terms.

$12 = 2^2 \times 3$	**Step 1** Find the largest square factor of 12
$\sqrt{12} = \sqrt{2^2} \times \sqrt{3} = 2\sqrt{3}$	**Step 2** Take the square root of the square factor out

Example 03 · Multiplying surds

Simplify $\sqrt{2} \times \sqrt{3}$.

Why this method?

Keep the answer exact. Look for square factors and matching root parts before combining any terms.

$\sqrt{2} \times \sqrt{3} = \sqrt{6}$	**Step 1** Multiply the numbers under one root
$\sqrt{6}$	**Step 2** There is no square factor greater than 1 to take outside, so this is already simplified.

Think about it: Before moving on: why can the two numbers be multiplied under one root, but not added?

Example 04 · Multiplying surds — another example

Simplify $\sqrt{2} \times \sqrt{5}$.

Why this method?

Keep the answer exact. Look for square factors and matching root parts before combining any terms.

$\sqrt{2} \times \sqrt{5} = \sqrt{10}$	**Step 1** Multiply the numbers under one root
$\sqrt{10}$	**Step 2** There is no square factor greater than 1 to take outside, so this is already simplified.

Example 05 · Adding and subtracting surds

Simplify $\sqrt{18} + \sqrt{32}$.

Amber marks the shared root part. The coefficients can be added only after those root parts match.

Why this method?

Keep the answer exact. Look for square factors and matching root parts before combining any terms.

$\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$	**Step 1** The square factor 9 has square root 3.

$\sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$	**Step 2** Simplify the other root using the square factor 16.
$3\sqrt{2} + 4\sqrt{2} = (3 + 4)\sqrt{2}$	**Step 3** The root parts now match, so add their coefficients.
$7\sqrt{2}$	**Step 4** Keep the shared root part unchanged.

Think about it: Before moving on: why can the coefficients be combined, but the numbers under the roots cannot be added?

Example 06 · Adding and subtracting surds — another example

Simplify $\sqrt{80} + \sqrt{45}$.

Why this method?

Keep the answer exact. Look for square factors and matching root parts before combining any terms.

$\sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}$	**Step 1** The square factor 16 has square root 4.
$\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$	**Step 2** Simplify the other root using the square factor 9.
$4\sqrt{5} + 3\sqrt{5} = (4 + 3)\sqrt{5}$	**Step 3** The root parts now match, so add their coefficients.
$7\sqrt{5}$	**Step 4** Keep the shared root part unchanged.

Example 07 · Rationalising a denominator with a surd

Rationalise the denominator of $\frac{6}{\sqrt{2}}$.

Why this method?

Keep the answer exact. Look for square factors and matching root parts before combining any terms.

$\frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$	**Step 1** Multiply by $\frac{\sqrt{2}}{\sqrt{2}}$, which is 1. This keeps the value unchanged and gives two matching roots in the denominator.
$\frac{6\sqrt{2}}{2}$	**Step 2** Multiply top by top and bottom by bottom. The product $\sqrt{2} \times \sqrt{2}$ is 2.
$3\sqrt{2}$	**Step 3** Divide the numerical coefficient 6 by 2; the root part remains.

Think about it: Before moving on: why does multiplying the top and bottom by $\sqrt{m}$ leave the value unchanged?

Example 08 · Rationalising a denominator with a surd — another example

Rationalise the denominator of $\frac{1}{\sqrt{3}}$.

Why this method?

Keep the answer exact. Look for square factors and matching root parts before combining any terms.

$\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$	**Step 1** Multiply the fraction by $\frac{\sqrt{m}}{\sqrt{m}}$. That whole factor equals 1, so the value stays unchanged while the denominator becomes a whole number.
$= 1\frac{\sqrt{3}}{3}$	**Step 2** The bottom becomes m, because $\sqrt{m} \times \sqrt{m} = m$

Example 09 · Multiplying binomial surds

Simplify $(4 + 2\sqrt{3})(1 + 3\sqrt{3})$.

Why this method?

Keep the answer exact. Look for square factors and matching root parts before combining any terms.

$4 + 12\sqrt{3} + 2\sqrt{3} + 6\sqrt{3}\cdot\sqrt{3}$	**Step 1** Multiply each term in the first bracket by each term in the second. Remember $\sqrt{n} \times \sqrt{n} = n$
$22 + 14\sqrt{3}$	**Step 2** Collect the whole-number terms and the $\sqrt{n}$ terms

Think about it: Before moving on: what is $\sqrt{5} \times \sqrt{5}$, and why does it not stay as a surd?

Example 10 · Multiplying binomial surds — another example

Simplify $(3 + 3\sqrt{2})(1 + 3\sqrt{2})$.

Why this method?

Keep the answer exact. Look for square factors and matching root parts before combining any terms.

$3 + 9\sqrt{2} + 3\sqrt{2} + 9\sqrt{2}\cdot\sqrt{2}$	**Step 1** Multiply each term in the first bracket by each term in the second. Remember $\sqrt{n} \times \sqrt{n} = n$
$21 + 12\sqrt{2}$	**Step 2** Collect the whole-number terms and the $\sqrt{n}$ terms

Example 11 · Rationalising a binomial denominator

Rationalise the denominator $: \frac{1}{(4+\sqrt{3})}$.

Why this method?

Keep the answer exact. Look for square factors and matching root parts before combining any terms.

$\frac{1}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}}$	**Step 1** Change the sign between the two denominator terms to make a conjugate. Multiply top and bottom by it, so the multiplier is 1.
$\frac{4-\sqrt{3}}{16-3}$	**Step 2** The middle root terms cancel when the two denominator brackets are multiplied.
$\frac{4-\sqrt{3}}{13}$	**Step 3** Subtract in the denominator. The numerator stays together above the fraction bar.

Think about it: Before moving on: why does multiplying by the conjugate remove the surd from the denominator?

Example 12 · Rationalising a binomial denominator — another example

Rationalise the denominator $: \frac{1}{(3+\sqrt{5})}$.

Why this method?

Keep the answer exact. Look for square factors and matching root parts before combining any terms.

$\frac{1}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$	**Step 1** Change the sign between the two denominator terms to make a conjugate. Multiply top and bottom by it, so the multiplier is 1.
$\frac{3-\sqrt{5}}{9-5}$	**Step 2** The middle root terms cancel when the two denominator brackets are multiplied.

$\frac{3-\sqrt{5}}{4}$	**Step 3** Subtract in the denominator. The numerator stays together above the fraction bar.

Example 13 · Products with different surds

Expand and simplify $(4-4\sqrt{2})(2-3\sqrt{7})$.

Why this method?

Keep the answer exact. Look for square factors and matching root parts before combining any terms.

$4\times 2;$ $(-4\sqrt{2})\times 2;$ $4\times(-3\sqrt{7});$ $(-4\sqrt{2})\times(-3\sqrt{7}).$	**Step 1** Form all four products
$(-4\sqrt{2})\times(-3\sqrt{7})=12\sqrt{14}.$	**Step 2** Multiply the radicals and extract square factors
$8-8\sqrt{2}-12\sqrt{7}+12\sqrt{14}$	**Step 3** Collect only like radical terms

Think about it: Why can the two cross terms be collected only when they have the same radical?

Example 14 · Products with different surds — another example

Expand and simplify $(3-4\sqrt{3})(3-4\sqrt{7})$.

Why this method?

Keep the answer exact. Look for square factors and matching root parts before combining any terms.

$3\times 3;$ $(-4\sqrt{3})\times 3;$ $3\times(-4\sqrt{7});$ $(-4\sqrt{3})\times(-4\sqrt{7}).$	**Step 1** Form all four products
$(-4\sqrt{3})\times(-4\sqrt{7})=16\sqrt{21}.$	**Step 2** Multiply the radicals and extract square factors
$9-12\sqrt{3}-12\sqrt{7}+16\sqrt{21}$	**Step 3** Collect only like radical terms

Common mistake

Splitting a square root across addition. $\sqrt{9+16} = \sqrt{25} = 5$, but $\sqrt{9} + \sqrt{16} = 3 + 4 = 7$. These are different calculations.

Exam technique

Simplify each root before deciding whether terms can be collected.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

Simplify $\sqrt{98}$.

A. $\sqrt{14}$

B. $7\sqrt{14}$

C. $49\sqrt{2}$

D. $7\sqrt{2}$

Question 2

Simplify $\sqrt{3} \times \sqrt{12}$.

A. $\sqrt{15}$

B. 36

C. $3\sqrt{12}$

D. 6

Question 3

Simplify $\sqrt{98} + \sqrt{50}$.

A. $35\sqrt{2}$

B. $\sqrt{148}$

C. $\sqrt{24}$

D. $12\sqrt{2}$

Question 4

Rationalise the denominator of $\frac{11}{\sqrt{3}}$.

A. $11\frac{\sqrt{3}}{3}$

B. $11\sqrt{3}$

C. $\frac{11}{3}$

D. $\frac{33}{\sqrt{3}}$

Question 5

Simplify $(5+\sqrt{3})(1+\sqrt{3})$.

A. $6+6\sqrt{3}$

B. $8-6\sqrt{3}$

C. $2+6\sqrt{3}$

D. $8+6\sqrt{3}$

Question 6

Rationalise the denominator
$: \frac{1}{(6+\sqrt{5})}$
.

A.
$\frac{(6+\sqrt{5})}{31}$

B.
$\frac{(6-\sqrt{5})}{31}$

C.
$\frac{(6-\sqrt{5})}{41}$

D.
$\frac{(6-\sqrt{5})}{6}$

Question 7

Expand and simplify $(4+2\sqrt{6})(4-3\sqrt{2})$.

A. $16-12\sqrt{2}-12\sqrt{3}+8\sqrt{6}$

B. $16-12\sqrt{3}$

C. $16 - 12\sqrt{2} + 12\sqrt{3} + 8\sqrt{6}$

D. $16 - 24\sqrt{2} + 8\sqrt{6}$

Watch an explanation

Cognito — Surds - Introduction & how to Simplify

Cognito — Surds - Rationalising the Denominator

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

$7\sqrt{2}$

Question 2

6

Question 3

$12\sqrt{2}$

Question 4

$11\frac{\sqrt{3}}{3}$

Question 5

$8 + 6\sqrt{3}$

Question 6

$$\frac{(6 - \sqrt{5})}{31}$$

Question 7

$16 - 12\sqrt{2} - 12\sqrt{3} + 8\sqrt{6}$

Open the full chapter online