Symmetry and transformations

Junior Cycle · Higher Level

Understand the idea

How to start

An image is the new position of a shape after a transformation. Each original point must follow the same rule.

1. Read the transformation and any axis, centre, angle or vector given.
2. Apply the rule to each point separately.
3. Label the image points with primes, such as A′, and check lengths stay the same.

Key terms & definitions

Image
The result of applying a transformation.

Axis of symmetry
A line across which a shape reflects onto itself.

Centre of rotation
The fixed point about which a figure turns.

Key ideas

• A translation moves every point the same distance and direction. Add the vector components to the coordinates.

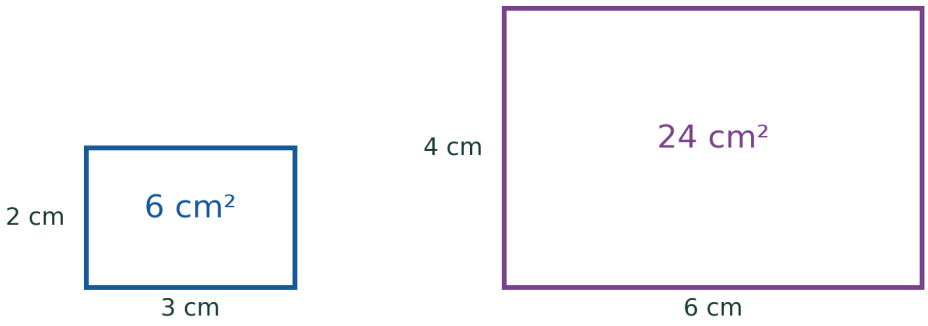
• Reflection in the x-axis maps (x, y) to $(x, -y)$; reflection in the y-axis maps it to $(-x, y)$.

• Central symmetry about the origin maps (x, y) to $(-x, -y)$, the same as a half-turn.

• A 90° anticlockwise rotation about the origin maps (x, y) to $(-y, x)$. State the centre, direction and angle.

• Translations, reflections and rotations preserve lengths and angles. The image remains congruent to the original.

Enlargement: length ×2, area ×4



A 3 cm by 2 cm rectangle has area 6 cm². Doubling both lengths makes a 6 cm by 4 cm rectangle with area 24 cm²: four times as large.

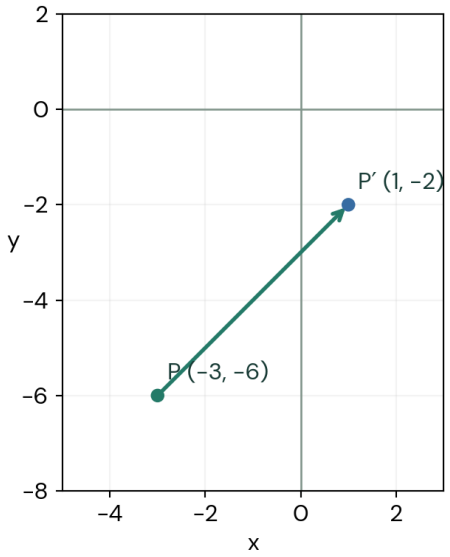
Quick reminder

- State the axis, centre or vector when needed.
- A reflection changes position across an axis, not size.
- For a rotation, check centre, angle and direction.

Worked examples

Example 01 · Translation by a vector

$P(-3, -6)$ is translated by the vector $(4, 4)$ to give P′. Find the x-coordinate of P′.



The transformation in this question maps $P(-3, -6)$ to P′$(1, -2)$.

Why this method?

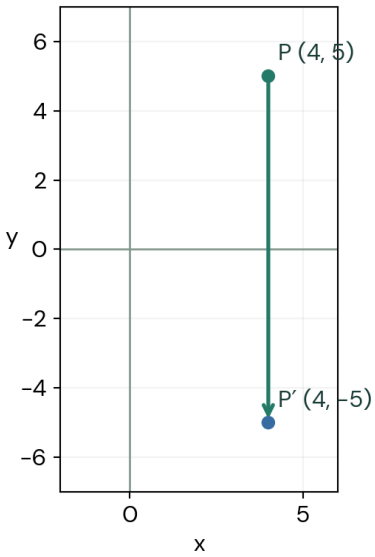
Identify the transformation, its centre or axis, and its direction. Apply the same rule to each coordinate.

$x' = -3 + 4 = 1$	**Step 1** The first vector component adds 4 to the horizontal coordinate.
$y' = -6 + 4 = -2$	**Step 2** The second component adds 4 to the vertical coordinate.
$P' = (1, -2)$	**Step 3** Write the image point in x, y order.
The x-coordinate is 1.	**Step 4** Read the first coordinate, as requested.

Think about it: Before moving on: why does a translation keep the shape the same size?

Example 02 · Reflection in an axis and central symmetry

$P(4, 5)$ is reflected in the x-axis to give P′. Find the y-coordinate of P′.



The transformation in this question maps $P(4, 5)$ to P′$(4, -5)$.

Why this method?

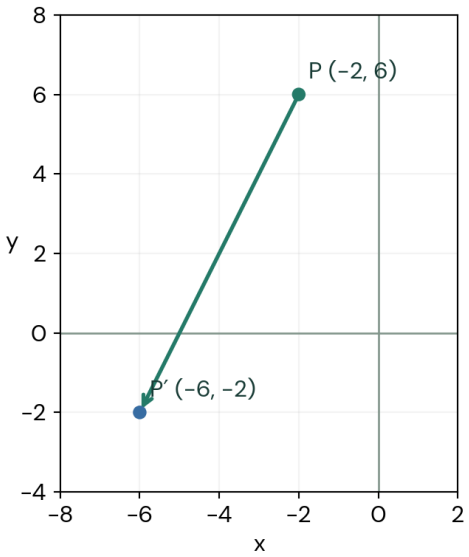
Identify the transformation, its centre or axis, and its direction. Apply the same rule to each coordinate.

P′ $= (4, -5)$	**Step 1** Reflecting in the x-axis: (x, y) → $(x, -y)$
y-coordinate of P′ $= -5$	**Step 2** Take the y-coordinate

Think about it: Before moving on: which coordinate changes when a point is reflected in the x-axis?

Example 03 · Rotation about the origin

$P(-2, 6)$ is rotated 90° anticlockwise about the origin to give P′. Find the x-coordinate of P′.



The transformation in this question maps $P(-2, 6)$ to P′$(-6, -2)$.

Why this method?

Identify the transformation, its centre or axis, and its direction. Apply the same rule to each coordinate.

P′ $= (-6, -2)$	**Step 1** Rotation of 90° anticlockwise about the origin
x-coordinate of P′ $= -6$	**Step 2** Take the x-coordinate

Think about it: Before moving on: why does $a90^\circ$ turn anticlockwise and $a270^\circ$ turn anticlockwise give different images?

Common mistake

Using a clockwise turn when the question asks for anticlockwise. About the origin, $a90^\circ$ anticlockwise turn sends $(1, 0)$ to $(0, 1)$; $a90^\circ$ clockwise turn sends it to $(0, -1)$.

Exam technique

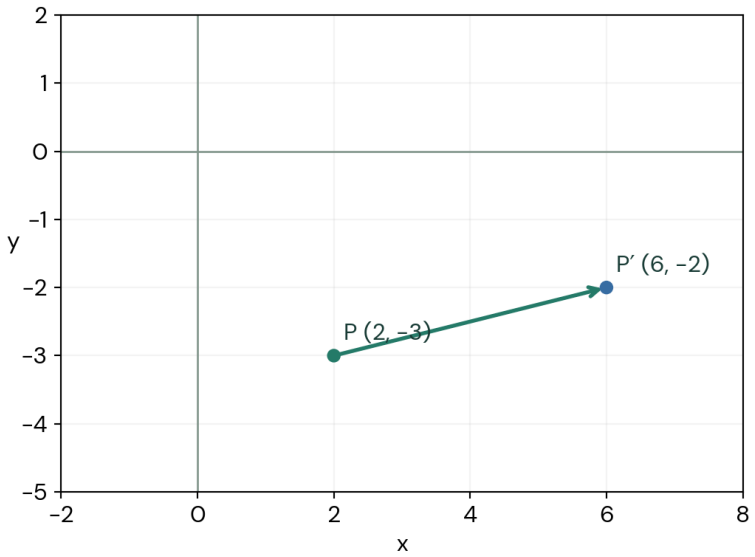
Transform every vertex and join them in the original order.

Try it yourself

Work on paper before checking the answers at the end. These questions are for self-checking, not submitted homework.

Question 1

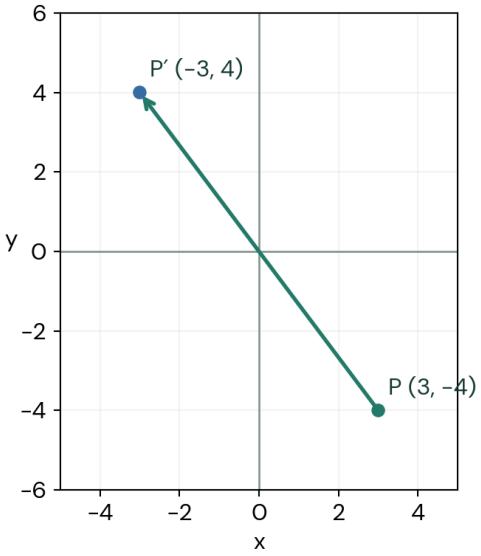
$P(2, -3)$ is translated by the vector $(4, 1)$ to give P′. Find the x-coordinate of P′.



The transformation in this question maps $P(2, -3)$ to P′$(6, -2)$.

Question 2

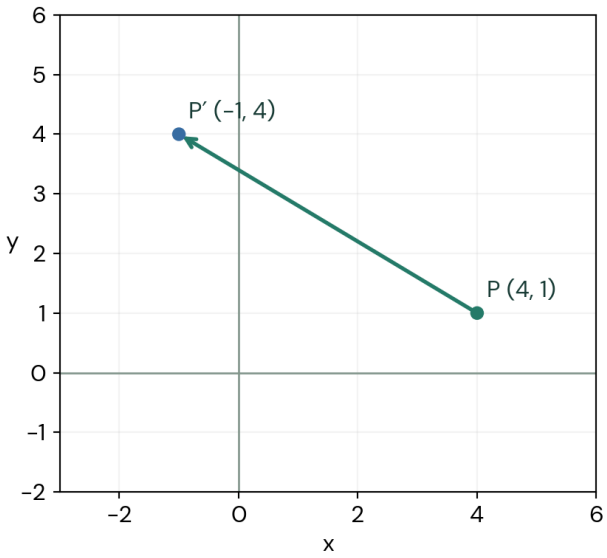
$P(3, -4)$ is reflected in the origin (central symmetry) to give P′. Find the y-coordinate of P′.



The transformation in this question maps $P(3, -4)$ to P′$(-3, 4)$.

Question 3

$P(4, 1)$ is rotated $90°$ anticlockwise about the origin to give P′. Find the x-coordinate of P′.



The transformation in this question maps $P(4, 1)$ to P$(-1, 4)$.

Watch an explanation

Cognito — The Four Transformations

Open the website chapter to load the embedded players. Use the rates and rules in your question for financial examples.

Self-check answers

Question 1

6

Question 2

4

Question 3

-1

Open the full chapter online